GEOSCI 561: Homework #6 (The BLAG model)

Due: Fri., Mar. 6

A slightly simplified version of the BLAG model of the carbonate-silicate geochemical cycle (Amer. J. Sci. 283, 641, 1983) can be written in the form of 4 ODE's:

1) \[ \frac{dM_{Mg}}{dt} = F_{WD} + F_{WMgSi} - k_{V-SW}M_{Mg} \]
2) \[ \frac{dM_{Ca}}{dt} = F_{WD} + F_{WC} + F_{WCaSi} + k_{V-SW}M_{Mg} - \Phi_C \]
3) \[ \frac{dM_{HCO_3}}{dt} = 4F_{WD} + 2F_{WC} + 2F_{WCaSi} + 2F_{WMgSi} - 2\Phi_C \]
4) \[ \frac{dA_{CO_2}}{dt} = \Phi_C + 2F_{MD} + F_{MC} - 2F_{WD} - F_{WC} - 2F_{WMgSi} - 2F_{WCaSi} \]

where \( M_i \) is the number of moles of species 'i' dissolved in the ocean, and \( A_{CO_2} \) is the number of moles of CO2 in the atmosphere. The current sizes of these reservoirs in units of \( 10^{18} \) moles are: \( M_{Mg} = 75, M_{Ca} = 14, M_{HCO_3} = 2.8, \) and \( A_{CO_2} = 0.055 \). The \( F_W \)'s and \( F_M \)'s are fluxes arising from weathering and metamorphism, respectively. Their current values in units of \( 10^{18} \) moles/m.y. are: \( F_{WD} = 1.98, F_{WC} = 7.84, F_{WMgSi} = 3.02, F_{WCaSi} = 2.72, F_{MD} = 1.44, \) and \( F_{MC} = 2.86 \). The present value of the volcanic-seawater interaction coefficient \( k_{V-SW} \) is 0.0667 m.y.\(^{-1}\).

The calcite deposition flux \( \Phi_C \) is given by the expression

5) \[ \Phi_C = k_{prep} \ast (M_{Ca}M_{HCO_3}^2 - K_{eq}A_{CO_2}) \]

where \( k_{prep} = 1.162 \) (\( 10^{18} \) moles\(^{-2}\)) m.y.\(^{-1}\) and \( K_{eq} = 1721 \) (\( 10^{18} \) moles\(^2\)).

All metamorphic fluxes \( F_M \) are multiplied by a spreading rate correction factor \( f_{SR}(t) \), which is specified by the user. This same correction factor is applied to the coefficient \( k_{V-SW} \). All weathering fluxes are multiplied by a land area correction factor \( f_A(t) \), defined by the user, and by a CO2 correction factor \( f_b(CO_2) \), defined by the expression

6) \[ f_b(CO_2) = 1.0 + 0.252y + 0.0156y^2 \]

where \( y = ln(A_{CO_2}/0.055) \).
Assignment

1. a) Write a program to solve the BLAG system using the reverse Euler method. Make it easy on yourself -- calculate your Jacobian elements numerically. Starting from the present reservoir sizes and an initial time step of 100 years (i.e., \( \Delta t = 10^{-4} \)), set both \( f_A(t) \) and \( f_{SR}(t) \) equal to unity and integrate the equations for one billion years, i.e. long enough to reach a pretty good steady state. Do not go any further in time! (Can anybody figure out why? Hint: The steady-state system of equations has an analytic solution!) Print out the resulting reservoir sizes to five significant digits. Use the following automatic time-stepper or one of your own choosing:

After each time step:

\[
DTS = DT \\
IF (CHG .LT. 0.1) DT = DTS * 1.5 \\
IF (CHG .LT. 1.E-2) DT = DTS * 10. \\
IF (CHG .GT. 0.2) DT = DTS/2.
\]

where DT is the current time step and CHG is the maximum percentage change of any of the four variables during that step.

b) Does your answer change if you start from different initial values? Can you figure out why (or why not)?

2. a) Set \( f_{SR}(t) = 2 \) and integrate to steady state, starting from the present reservoir sizes. How many PAL of CO\(_2\) do you get? (‘PAL’ means ‘times the present atmospheric level.’)

b) Set \( f_A(t) = 0.5 \) (letting \( f_{SR}(t) = 1 \) again) and integrate to steady state, starting from present sizes. How does the answer compare with part (a)? Why do you get this result?

c) Set \( f_{SR} = 2 \) and \( f_A = 0.5 \) and integrate to steady state. How many PAL of CO\(_2\) do you get this time?

3. Simulate a return to mid-Cretaceous times over the next 100 m.y. by letting

\[
\begin{align*}
f_{SR}(t) &= 1 + 0.2(t/100) \\
f_A(t) &= 1 - 0.2(t/100)
\end{align*}
\]

for \( 0 < t < 100 \) m.y. For this simulation, use a constant time step of 1 m.y. Print out your answer at every tenth time step. How far out of equilibrium is atmospheric CO\(_2\) at the end of this time? (Hint: To answer this last question, you will need to understand the answers to questions 1 and 2.)