Mechanisms controlling rupture shape during subcritical growth of joints in layered rocks

Laura Savalli
Terry Engelder†

Department of Geosciences, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

ABSTRACT

During joint propagation, the orientation of plume lines on joint faces allows for mapping the path taken by the crack tip line during rupture growth. Rupture shapes in layered, clastic sediments of the Devonian Catskill Delta (Finger Lakes district, New York) indicate joint growth through three stages, where velocity of the tip line, \( v_t \), varies as a function of the crack-tip stress intensity, \( K_I \). The initial stage of growth is characterized by a rupture of approximately circular or elliptical shape that expands from an initiation flaw with a velocity, \( v_e \). In some cases, primary growth involves self-correction, where an elliptical rupture redistributes the crack-tip stress so that the rupture returns to a more stable circular shape. The crack tip line eventually propagates to a bedding interface where properties of either a plastic or a noncohesive bedding interface cause \( v_e \to 0 \) along a portion of the tip line. The onset of secondary growth occurs when the rupture tip line intersects both bedding interfaces and splits into two discontinuous segments that propagate synchronously as a single, coherent rupture. Nonuniform, nonsystematic \( v_e \) eventually interferes with the rupture’s ability to grow coherently and thus leads to the transition to tertiary growth, characterized by the detachment of the coherent rupture into one or more independently propagating, noncoherent tip lines. Throughout all three stages of rupture growth, the \( K_I \)-dependent \( v_e \) points to subcritical propagation. Variation from a smooth to rough plume morphology is consistent with propagation through region I of the subcritical regime with a transition at \( v_e \approx 10^{-4} \) m/sec. Rupture velocity may enter region II of the subcritical regime, but such a broad range of \( K_I \) must be crossed to reach region III and quasistatic behavior at the critical stress intensity, \( K_{IC} \), that region II acts as a ‘barrier’ through which few joints pass, thus greatly limiting the number of postcritical joints in the Catskill Delta if not in the crust of the Earth.

Keywords: joints, fracture, crack, rupture, crack velocity, plumose morphology.

INTRODUCTION

A number of models support the assertion that stable or quasistatic crack growth is the most common type of joint propagation within the Earth (e.g., Segall, 1984a; Olson, 1993, 2003; Renshaw and Pollard, 1994; Schultz, 2000). Mechanisms promoting stable rupture include subcritical crack growth, which takes place when the host rock is subjected to slow, long-term loading in the presence of chemically reactive pore fluids (Wiederhorn and Bolz, 1970; Wiederhorn, 1972), a changing effective stiffness of the rock as joints propagate (Segall, 1984a), the increasing compressibility of individual cracks as they grow (Lehner, 1990), an interaction of overlapping cracks (Pollard et al., 1982), and a decreasing fluid pressure if the total fluid mass is constant (Schor, 1969). To date, the field evidence verifying the stable crack-growth paradigm is based on the growth pattern of joint sets (e.g., Olson, 1993), the cyclic propagation of natural hydraulic fractures (e.g., Lacazette and Engelder, 1992), and the similarity between the surface morphologies of slow-growing features such as columnar joints (e.g., DeGraff and Aydin, 1987), and mud-cracks (e.g., Weinberger, 1999) and other joints. Here, we present additional evidence for subcritical growth based on the interpretation of rupture patterns on common joints.

Rupture during joint growth is recorded by feathery, plumelike structures of low-relief ridges and valleys found on the surfaces of joints (i.e., opening mode or mode I cracks) (Fig. 1A) (Woodworth, 1896; Hodgson, 1961a, 1961b; Roberts, 1961; Bankwitz, 1966, 1984; Bahat and Engelder, 1984; Kulander and Dean, 1985; Pollard and Aydin, 1988). These subtle ridges are called plumes or bars (Roberts, 1961), striae (Bahat, 1979), inclusion hackles (Kulander and Dean, 1985), or plume lines (Müller and Dahm, 2000). They are thought to form during small-scale, out-of-plane crack propagation as the rupture deviates into pore spaces and along grain boundaries (Kulander et al., 1979; Scott et al., 1992). Plumes (either lines or curves) radiate away from joint initiation sites where local inhomogeneities serve to concentrate the crack driving stress (McConaughey and Engelder, 2001). Plume patterns are maps of the paths taken by ruptures as they cut through intact rock to form a joint (e.g., Bahat and Engelder, 1984; DeGraff and Aydin, 1987; Helgeson and Aydin, 1991; Lacazette and Engelder, 1992).

One strategy for testing the proposition that subcritical joint growth is important in the Earth is to investigate the mechanisms controlling the shape of growing ruptures as defined by plume patterns on planar joint surfaces. In rocks, the rupture shape is a macroscopic average of the small-scale heterogeneities that give rise to plume morphology. The rupture shape is best defined by the crack tip line, a singularity centered within the damage zone that is the rupture. Both ceramicists and geologists conclude that the crack tip line moves normal to the plume traces during fracture propagation, and therefore, the plume lines provide a detailed record of the location of the crack tip line (i.e., rupture shape) at all stages during joint growth (Fig. 1B) (e.g., Kulander and Dean, 1985; Beauchamp, 1996).
crack tip line, the simplest rupture shape. Field examples of a circular tip line include isolated planar joints in granite where \( c \) is the radius of the joint (e.g., Bankwitz and Bankwitz, 1984; Bahat et al., 2003). Of course, a geometrically perfect circular rupture is not expected in rock, a complex, inhomogeneous material, but to a first approximation, simple geometric shapes apply to the initial phases of rupture in rock as demonstrated in this paper.

Our premise is that the magnitude and distribution of the stress intensity \( K_I \) along the crack tip line is the most important parameter in dictating \( \nu_t \). Such a law was postulated by Charles (1958) for subcritical crack growth

\[
\nu_t = AK_I^n, \tag{1}
\]

where \( n \) is the subcritical fracture growth index and \( A \) is a constant of proportionality (Atkinson and Meredith, 1987a; Olson, 1993). By equation 1, circular ruptures grow only if both \( K_I \) and \( dK_I/dc \) are equal at all positions along the tip line.

We postulate that the circular or penny-shaped rupture is the stable shape from which more complex ruptures evolve. For example, in elastic, interbedded sedimentary sequences, differences in the elastic properties of adjacent siltstone and shale layers, weak bedding planes, or the interaction with nearby crack tip stress concentrations introduce a mechanical heterogeneity that modifies \( K_I \) along the crack tip line. Variation in \( K_I \) brings about differential or nonuniform instantaneous \( \nu_t \) that causes ruptures to evolve from circular to elliptical and then to irregular shapes. However, there is also a tendency for an elliptical shape to redistribute \( K_I \) in a manner that returns the rupture to its stable, circular shape (Broek, 1986). This leads to the phenomenon of a self-correcting rupture and the concomitant cycling of rupture shapes. Eventually, the rupture will become irrevocably complicated at bed boundaries where the moving portion of the tip line may split and head in opposite directions within a single bed, or it may cross into adjacent beds to form a composite joint (Helgeson and Aydin, 1991).

Fracture Surface Morphology and Strain Energy Release Rate, \( G \)

Experiments performed on glass suggest that the initiation of surface features requires a threshold strain energy release rate, \( G \), (e.g., Tsai and Mecholsky, 1992). \( G \) is related to \( K_I \) by:

\[
G = \frac{1 - \nu^2}{E} K_I^2, \tag{2}
\]

where \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio of the rock. In glass, a smooth surface called a ‘mirror plane’ develops as an unstable fracture accelerates away from its initiation point. Once \( G \) exceeds a threshold \( G_m \), a rougher surface called ‘mist’ is produced (Rabinovitch et al., 2000). Here, \( G_m \geq 2\gamma = G_{IC} \) where \( \gamma \) is the work to create additional joint surface in the absence of stress corrosion, and \( G_{IC} \) is the critical strain energy release rate. The presence of fringe hackles and crack branching in glass is indicative of even higher thresholds, \( G_h \) and \( G_b \), respectively (Sommer, 1969; Bahat et al., 2003). Even between thresholds, when \( G \geq 2\gamma \), there is a predictable increase in roughness of fracture surfaces with increasing stress intensity \( K_I \) (Hull, 1999).

Mist and then hackle on glass fractures develop when the postcritical \( K_I \) becomes strong enough to initiate the growth of cracks
from tiny flaws in a process zone ahead of the initial running crack (Congleton and Petch, 1967). Primary or parent cracks branch into secondary cracks and leave a rough surface cut by many small side cracks penetrating the glass (Beauchamp, 1996). Because macroscale process zone cracking only occurs in the region of dynamic crack growth, mist and hackle fringe in glass are both diagnostic of postcritical propagation (Rabinovitch et al., 2000; Bahat et al., 2003). In this context, the term "mirror" should be reserved for those fracture surfaces that are unaffected by process zone cracking.

Although there are similarities between cracks in glass and in porous material like rocks, the surface morphology on cracks in glass and joints differs in three fundamental ways (Rice, 1984). Glass fractures initiate with mirror-smooth planes, whereas joints in porous rock exhibit a plume morphology from the outset. The surface of glass fracture becomes progressively rougher and more complex as the rupture grows unstably (e.g., Hull, 1999), whereas joints in porous rock commonly maintain a more uniformly rough surface as if the propagation were stable. Aside from blast fractures, the presence ofhackle fringes and hence, unstable propagation, is rare on rock joints but common on glass fractures.

The differences between the surface morphology of glass and joints are a consequence of the size of internal flaws before crack propagation. The surface roughness in glass results from postcritical secondary crack branching in the process zone ahead of the crack tip, whereas the uniform roughness (i.e., the plume pattern) on rock joints is due to out-of-plane propagation of the crack tip in response to a much weaker subcritical $K_c$ interacting with much larger pores and grain boundaries within a process zone about the crack tip (e.g., Kulander et al., 1979; Scott et al., 1992; McConaughy and Engelder, 2001). In contrast to relatively large grain boundaries and pores spaces in rock ($10^{-2}–10^{-3}$ mm), flaws in glass are more than 3 orders of magnitude smaller and, therefore, require a much larger $K_c$ to induce process zone cracking. In brief, the distinction between rough surfaces in rocks and rough surfaces in glass is largely dependent on whether the onset of process zone cracking occurs in the subcritical or postcritical regimes.

The Geologic Context for a Study of Rupture Shape

Joints within the Devonian Catskill Delta complex located on the Appalachian Plateau of New York and Valley and Ridge Province of Pennsylvania were chosen for this study because they exhibit a well-developed, well-preserved joint surface morphology (Sheldon, 1912; Parker, 1942; Engelder and Geiser, 1980). The best examples are found within the Ithaca Formation (Genesee Group) in the Finger Lakes District of New York (McConaughy and Engelder, 2001) and in its stratigraphic equivalent, the Brallier Formation, located in central Pennsylvania (Ruf et al., 1998). For this study, we collected surface morphology data from over 250 joints located in and around Huntingdon, Pennsylvania, Watkins Glen, New York, and further east in Broome and Cortland counties, New York.

The Ithaca and Brallier Formations are made up of Devonian-aged interbedded siltstone and shale turbidite deposits. A combination of abnormal pore pressure and tectonic stress during the Alleghanian Orogeny produced regional jointing in the foreland fold-thrust belt. The joints formed at a burial depth of ~3 km and were driven by natural gas generated from the underlying organic-rich Genesee shale (Lacazette and Engelder, 1992; Gerlach and Cercone, 1993; Evans, 1994). The dominant sets of joints, which were most frequently measured during this study, are oriented approximately perpendicular to the strike of the Alleghanian folds and reflect a clockwise rotation of the horizontal stress field during Alleghanian deformation (Nickelsen and Hough, 1967; Engelder and Geiser, 1980; Zhao and Jacobi, 1997; Younes and Engelder, 1999).

EVOlUtion of Rupture SHapes as a Function of Relative $v_p$: Field Observations

Although plumes are a record of the joint rupture from initiation through propagation and arrest, some interpretation is necessary to capitalize on this record. Under certain field conditions, $G$ will decrease enough to allow the rupture to hesitate or possibly arrest before continuing through the rock (Fig. 1A). Hesitation and arrest are expressed as small rib marks or zones of increased surface topography that coincide with the outline or profile of the crack tip during a pause in the growth of a rupture (Lacazette and Engelder, 1992). These subtle marks or zones are invariably normal to the plume lines of the joint (Kulander and Dean, 1985; Rummel, 1987). This observation, in addition to experiments performed in ceramics, implies that the crack tip line moves normal to the plume lines during fracture propagation (e.g., Beauchamp, 1996). Consequently, the shape of the crack tip-line profile can be mapped progressively along the length of a joint to track the evolution of a rupture during all stages of joint development (Fig. 1B). By mapping incremental rupture growth, it becomes apparent that $v_p$ often varies along the crack tip line in a consistent manner through three predictable stages of rupture evolution.

Rupture Shapes During Primary Growth

During primary growth in layered rocks, a rupture initiates either from the interior of a bed or from the bed boundary. The rupture can propagate away from the initiation point along either straight or curved trajectories, as recorded by the plume lines. Maps of tip-line profiles during early joint growth delineate circular and elliptical rupture shapes (Fig. 1B). The degree of ellipticity and the position of the rupture shape within the bed provide information regarding the crack-tip stress field, relative $v_p$ and effect of the bed boundary on rupture growth. If straight plume lines radiate away from an initiation point located within the interior of a bed, the rupture is circular, $v_p$ is equal along the tip line, and the center of the rupture remains fixed over the initiation point until the tip line intersects a bed boundary (Fig. 2). In this example, the initial rupture is also closed, which means that the crack tip line is continuous and $v_p > 0$ at every point on the tip line.

Initiation in layered rock is more common at bed boundaries than in the central portion of beds (McConaughy and Engelder, 2001). One end member for bed-boundary initiation leads to a closed, circular rupture whose center is not stationary over the initiation point. Such ruptures leave behind curved plumes that cut into the bed and then circle back toward the bed boundary (Figs. 3 and 4A). In this case, $v_p = 0$ at the bed boundary initiation point and varies along the remainder of the tip line, $v_p > 0$. The other end member for rupture growth from bed boundaries produces an open, semicircular rupture where tip-line continuity is interrupted at the bed boundary (Figs. 4B and 5). An open rupture is distinguished by having a portion of the tip line motionless (i.e., $v_p = 0$), often at the bed boundary. The straight plume lines of a semicircular rupture emanate away from a rupture whose center is stationary over the initiation point. Crack tip $v_p$ is equal at all points along the moving portions of the open tip line.

Joints in the Ithaca siltstone often initiate along irregularly shaped sedimentary structures, such as flute casts (McConaughy and Engelder, 2001). The asymmetry of these structures can produce sharp, angular contacts with the base of the bed. Such sharp curvature concentrates stress in an irregular manner and as a result, preferentially drives either a closed (Fig. 6) or open elliptical rupture (Fig. 7) at an oblique angle relative to bedding. Both cases are indicated by plumes curving up into
MECHANISMS CONTROLLING RUPTURE SHAPE DURING SUBCRITICAL GROWTH OF JOINTS

A.

B.

the bed and then back toward the bed boundary. The central point of the elliptical rupture moves away from the initiation point in both examples, a necessary condition for growth of an initially closed rupture from a bed boundary. Generally, the displacement of the central point of the rupture toward the center of the bed is not sufficient to maintain a closed rupture for long after the tip line intersects a bed boundary.

Cycling of Rupture Shapes During Primary Growth

Occasionally primary growth involves an initial growth spurt producing an elliptical shape followed by a decrease in ellipticity as the rupture moves into the central portion of a bed. Such behavior indicates that \( v_{\parallel} \) becomes faster in the direction of the short axis. The short axis catches up in length to the long axis, thereby returning the rupture to a circular shape (Figs. 6 and 7). In some instances the rupture continues to grow in what was the direction of the short axis. The effect is that an elliptical rupture seems to turn at right angles to itself in the interior of a bed (Figs. 6 and 7). In these cases, the rupture evolves seamlessly from elliptical to circular and back to elliptical.

Although primary rupture growth in layered rock may initially involve a closed tip line, evolution usually takes the rupture toward a bed boundary where \( v_{\parallel} \rightarrow 0 \) along the portion of the tip line impinging on the boundary. If \( v_{\parallel} = 0 \) along that part of the tip line, the rupture becomes open because only part of the tip line is moving. The continuity of the tip line is usually first interrupted by intersection with the bed boundary containing the joint initiation site. In this case, a small portion of the tip line is hung up along the bedding boundary, while propagation continues through the bed. Primary growth of an open or closed rupture will continue until the tip line intersects the opposite bed boundary where \( v_{\parallel} \rightarrow 0 \) along a second portion of the tip line. Here the rupture splits into two distinct moving segments and enters a stage of secondary growth.

Figure 2. (A) Joint surface exposed in the Devonian Ithaca Formation along Highway 414 south of the intersection with Highway 14, Watkins Glen, New York. Initiation point is an irregular, weather-resistant inclusion inside the bed. (B) Interpretation of the joint surface showing the growth of an initially circular, closed rupture from a small inclusion (thick solid lines indicate shape of closed rupture), followed by the transition to an open rupture (thick solid and dashed tip lines indicate shape of open rupture), and then followed by the detachment from the initial rupture (thin, dashed tip lines indicate crack tip lines).
Rupture Shape During Secondary Growth and Rupture Detachment

The onset of secondary growth is marked by the splitting of the tip line following intersection with both the upper and lower mechanical layer boundaries. Even after the tip line has split, the two segments of the tip line move synchronously. Synchronous motion is a manifestation of coherent rupture as long as the growth along one segment affects $K_I$ along the other growing segment. During secondary growth the joint continues propagating as a split but coherent rupture where both tip lines move synchronously away from one another.

Figure 3. (A) Joint surface exposed in the Devonian Ithaca Formation along Highway 14 between Watkins Glen and Montour Falls, New York. (B) Interpretation of plumose morphology showing the evolution of the rupture shape and the onset of rupture detachment (rupture shapes and tip lines indicated as in Fig. 2).

Figure 4. (A) Crack-tip stress field for a crack initiating along a plastic, cohesive bed boundary across which there is a large contrast in the plastic properties of the materials. (B) Crack-tip stress field for a crack initiating along a brittle, noncohesive bed boundary. Bed-parallel splitting and slip along the interface accommodate joint propagation and generate a free surface across which the crack-tip stress field cannot penetrate.
When the host bed is sandwiched between two strongly plastic layers, such as a siltstone between shale, the properties of the bedding interface cause the crack tip to slow so that $v_r \to 0$. This nonuniform $v_r$, leads to shape change of the rupture from a circle to an ellipse, with the long axis of the ellipse growing parallel to bedding, a manifestation of secondary growth (Fig. 3).

Any time the tip line is subject to variation in $v_r$, caused by layering, the growing crack eventually loses its ability to maintain a coherent rupture with synchronous growth of multiple segments. Coherence is lost with the initial rupture detaching into two or more nonsynchronous ruptures that propagate in opposite directions (Fig. 2). For example, a rupture in one direction may hesitate at an arrest line, whereas the rupture in the opposite direction continues independently (Figs. 3 and 6) (Lacazette and Engelder, 1992). At this point, the nonsynchronous rupture is best described as a short blade crack with an open tip line on each end (Fig. 8).

The transition from the growth of a coherent rupture to the separation and growth of two or more independent or incoherent open ruptures is called rupture detachment. This marks the transition into the tertiary stage of growth.

### Postdetachment Rupture Shapes During Tertiary Growth

Postdetachment ruptures generally propagate according to one of three general schemes. First, the rupture forms a bedding-contained, short-blade crack that propagates, leaving a single plume axis between bedding interfaces. Second, the rupture forms as an irregularly shaped crack tip advancing along several small ruptures that detached from preexisting ruptures. In this case, the small rupture fronts grow independently in multiple directions (>2), leaving several small plume axes between bedding interfaces. Third, the rupture forms large composite joints that cross vertically stacked siltstone beds and thin shale layers, leaving multiple plume axes.

The simplest postdetachment rupture geometry is that of a bedding-constrained short blade crack propagating along a single siltstone layer. This type of crack propagation is characterized by a plume axis that runs along the center of bedding and plume lines that diverge from the axis and bend to intersect the upper and lower bed boundaries at a high angle. Some detached ruptures with a central plume axis in the Ithaca Formation display a surface roughness that makes a transition from smooth to rough topography in cycles (Fig. 9).

The angle of intersection between the plumes and the bed boundaries reflects the difference in $v_r$ along the plume axis as compared to that near the bed boundaries. Plume lines that intersect the bed boundary in the direction of propagation indicate the presence of an interface that retards $v_r$. Plumes that curve to intersect the bed boundary opposite the initial direction of propagation indicate that the crack tip line is held within the bed until a secondary rupture comes backward along the top of the bed to finish driving the rupture.

---

**Figure 5.** (A) Joint surface exposed in the Devonian Ithaca Formation near Whitney Point, Broome County, New York. (B) Interpretation of plumose morphology showing the growth of a semicircular rupture (rupture shapes and tip lines indicated as in Fig. 2).
Figure 6. (A) Joint initiation at an asymmetrical stress concentration (flute cast) in the Devonian Ithaca Formation exposed along Highway 14 between Watkins Glen and Montour Falls, New York. (B) Interpretation of the joint surface (rupture shapes and tip lines indicated as in Fig. 2).

Figure 7. (A) Joint surface in the Devonian Ithaca Formation exposed along Highway 14 between Watkins Glen and Montour Falls, New York. (B) Interpretation of the joint surface (rupture shapes and tip lines indicated as in Fig. 2). ss—siltstone, sh—shale.
MECHANISMS CONTROLLING RUPTURE SHAPE DURING SUBCRITICAL GROWTH OF JOINTS

Figure 8. Schematic of a short-blade crack.

Figure 9. Propagation cycles on a joint surface in the Devonian Ithaca Formation exposed along Highway 414 just south of the intersection with Highway 14 near Watkins Glen, New York (see Lacazette and Engelder, 1992).
crack tip line to the bed boundary (Fig. 10). A slight change in grain size in the top portion of some distal turbidites is often enough to retard the upward growth of a detached rupture. Two sets of systematic joints cutting the same bed may exhibit different rupture styles (Ruf et al., 1998). One example is found near Hunt-
ingdon, Pennsylvania, where joints oriented parallel to the strike of bedding formed prior to dip-oriented joints, as inferred from cross-cutting relationships (Fig. 11). The strike joints typically have a surface morphology consistent with that of a short blade crack (Fig. 11A), whereas the dip joints exhibit a more complex morphology (Fig. 11B). The earlier joints have surfaces with a typical plume-related topography (i.e., 1–3 mm within any cm²) that greatly exceeds the grain size (≤0.125 mm) of the host bed, whereas the later joints have surfaces that are smooth to the touch and a topography on the order of the grain size of the host.

The complex, irregular surface morphology on dip joints resembles a frosty window (Fig. 11B). Joint surfaces often contain one or more irregular primary plume axes with several small secondary detachment ruptures (as indicated by secondary plume axes) branching off of them. The detached ruptures behave as individual crack tips, each propagating independently and each having a unique $v_r$. One detached rupture may outrun an adjacent rupture. It is common for such detached ruptures to terminate against or cut off other ruptures. As a result, the bed-bounded joint surface is a composite of numerous secondary ruptures whose growth direction and $v_r$ were impacted by nearby crack-tip stress concentrations.

Large, meter-scale composite joints cross through vertically stacked siltstone beds and generally have a combination of rupture geometries (e.g., Helgeson and Aydin, 1991). They start as circular or elliptical ruptures that eventually detach to run along a bed as a short-blade crack. At many locations, the short-blade cracks detached again to produce higher-order ruptures propagating normal to the primary plume axis (Fig. 12). Although the ruptures have already detached, indicating tertiary growth, composite joints give the appearance of a secondary type of detachment propagating in rapid spurts from the short axis of an elliptical rupture (i.e., the initial short-blade crack). Propagation orthogonal to the horizon-
tal plume axis yields a sense of vertical growth for composite joints that might be classed as a fourth stage of rupture evolution.

THE BEDDING INTERFACE

Closed and open ruptures growing from bed-boundary initiation imply something important about fracture aperture in layered rock. A closed rupture indicated by smoothly curving plume lines of varying lengths that propagate up into the bed and then bend back to intersect the bed boundary suggests a cohesive bed interface with the maximum aperture in the middle of the bed and a nonuniform $v_r$ along the crack tip line (Fig. 4A). An open rupture indicated by straight plume lines emanating from a bed-boundary initiation point suggests joint propagation along a noncohesive bed interface where the maximum aperture occurs along the bed boundary and where $v_r$ is everywhere equal (Fig. 4B).

Cohesive, Plastic Bedding Interface

A joint that initiates at a cohesive bedding interface develops under conditions where there is a large contrast in the plastic properties of the materials across the interface (Fig. 4A). Under these conditions, a rupture grows almost exclusively up into the brittle, higher modulus material. Adjacent to the bedding interface, the crack-tip stress field is blunted by plastic yield so that a stress concentration may just barely project across the interface into the shale. This blunting of the crack-tip stress field has the same effect as a clamp, decreasing $K_I$ on the bottom portion of the closed rupture. Under conditions where $v_r = f(K_I)$, upward growth outruns bed-parallel growth. As a result, the joint grows into the bed with maximum aperture centered there. In this case, the central portion of the rupture is nonstationary.

The plumes are smoothly curving lines that propagate up into the competent bed and then bend back to intersect the plastic bed boundary at an angle rather than propagating parallel to the bed boundary itself. A curving path reflects a lag in $v_r$ along one path relative to neighboring paths. The longest plume line coincides with the portion of the crack tip that propagated with the highest $v_r$.

Noncohesive, Brittle Bedding Interface

A brittle or noncohesive bed interface may split to allow slip along the bedding boundary (Cook and Erdogan, 1972; Renshaw and Pol-
lard, 1995). The bed-parallel split behaves as a free surface, prohibiting the crack-tip stress field

Figure 10. (A) Joint surface in the Devonian Ithaca Formation exposed along Hwy 414 just south of the intersection with Highway 79 near Watkins Glen, New York. (B) Interpretation of surface morphology showing bed-parallel propagation of a short-blade crack between bed boundaries (tip lines indicated as in Fig. 2).
from penetrating into the underlying or overly-
ing medium (Fig. 4B). During rupture growth, the distance of rupture along the interface is equal to the distance of rupture into the brittle material. The surface morphology of the resulting rupture is made up of straight plume lines emanating from the initiation point both along the bed boundary and up into a bed. Maximum joint aperture occurs at the bedding interface and increases with joint growth as a result of bed-parallel slip. If the maximum aperture coincides with the bed boundary, the central portion of the rupture is stationary.

A MECHANICAL BASIS FOR SELF-
CORRECTING RUPTURE

Field evidence indicates that during primary growth initially elliptical ruptures evolve into circular ruptures in a process called self-correcting crack propagation (Figs. 6 and 7). We postulate that in geological media, a circular rupture front is the most stable geometry. Although a number of mechanisms can drive the rupture front into a more complex shape, the crack system is endowed with an innate property that favors evolution toward a circular (i.e., penny-shaped) geometry from an irregularly shaped tip line. However, evolution toward a stable rupture geometry requires a necessary condition that \( v_\ell \) is a function of crack tip \( K_I \). Such a condition is not found during unstable rupture where \( K_I \geq K_{IC} \) (Fig. 13). Rather, \( K_I \)-dependent \( v_\ell \) is found during stable rupture in regions I and III of the subcritical regime.

Stable Rupture Shape

Many rocks, including interbedded sedimentary sequences, are neither homogeneous nor isotropic, particularly in and around initiation points. An inhomogeneous, anisotropic media may cause \( K_I \) to be nonuniform along the crack tip line. Under these conditions, a rupture will grow nonuniformly when propagating in region I of the subcritical regime where \( v_\ell \) is a function of \( K_I \) (Fig. 13). Although conditions, particularly at bed boundaries, may favor initial elliptical crack growth, the crack grows into the more isotropic, homogeneous interior of the bed. The field observation is that under isotropic conditions the \( v_\ell \) moves faster in the direction of the short axis of an elliptical rupture (Figs. 6 and 7). To explain this phenomenon, we return to our thesis that \( v_\ell \) is a function of \( K_I \) along the crack tip line as expressed in equation 1.

In growing from a bed boundary toward the center of a bed, an initially elliptical rupture moves away from bedding anisotropy and enters a locally isotropic medium. There, \( K_I \) varies along an elliptical crack tip line according to the following relationship:

\[
K_I = \frac{\sigma \sqrt{\pi \alpha}}{8} \left( \sin^2 \phi + \frac{\alpha^2}{c^2} \cos^2 \phi \right)^{\frac{1}{2}},
\]  

\( \text{Equation 3} \)

Figure 11. Two abutting joint surfaces in the Devonian Bral-lier Formation located in the same bed, exposed at Hunting-don, Pennsylvania. (A) Strike joint surface that illustrates the rough surface morphology characteristic of the strike-oriented joint set. Abutting dip joint surface shown at an oblique angle. (B) A direct view of the abutting dip joint surface showing the characteristic smooth, complex surface morphology of the dip joint faces.
where \( a \) is the length of the short axis of the ellipse, \( c \) is the length of the long axis, and \( \varphi \) is the angle between the long axis of the ellipse and a point along the elliptical crack tip (Broek, 1986). For an elliptical rupture, \( K_I \) is largest at the end of the minor axis (\( \varphi = \pi/2 \)) and smallest at the end of the major axis (\( \varphi = 0 \)).

The nonuniform distribution of \( K_I \) along the crack tip makes an elliptical rupture inherently unstable in the subcritical regime. By substituting equation 3 into equation 1

\[
v_A = A \left[ \frac{\sigma \sqrt{\pi a}}{3\pi \frac{\pi a}{8} \frac{\pi a}{8} \frac{\pi a}{8} \frac{\pi a}{8}} \left( \frac{\sin^2 \varphi + \frac{a^2}{c^2} \cos^2 \varphi}{\frac{a^2}{c^2}} \right) \right]^{\frac{1}{2}}.
\]  

(4)

Here we see that the larger \( K_I \) along the short axis of the ellipse will drive the rupture at a higher subcritical velocity in short-axis direction according to Charles Law, thus leading to a self-correcting process where the rupture evolves toward a circular shape with a uniform \( K_I \) along the crack tip. In the absence of bedding anisotropy, conditions are such that a coherent circular (i.e., penny-shaped) rupture will evolve from any initial shape. This behavior is substantiated by the observation that circular ruptures emerge from some very irregular, odd-shaped initiation flaws (Figs. 6 and 7).

Once a circular rupture evolves

\[
K_I = \sigma Y \sqrt{\pi c},
\] 

(5)

where \( \sigma \) is the remote stress, \( Y \) is the shape factor of a penny-shaped crack, and \( c \) is the radius of the crack (Lawn, 1993). By substituting equation 3 into equation 1

\[
v_A = A (\sigma Y \sqrt{\pi c})^{-\frac{1}{2}},
\] 

(6)

which is the equation for a stable circular rupture shape because \( v_A \) is equal at all points on the crack tip line. As long as the driving stress on the crack remains constant, \( v_A \) will increase during growth of a circular rupture until the upper limit of region I of the subcritical regime is reached (Fig. 13). After this point, \( K_I \) will continue to increase while \( v_A \) is maintained. In summary, as long as the host medium is an isotropic, homogeneous rock, the most stable shape for a subcritically propagating crack tip line is circular.

Although field evidence supports the proposition that an elliptical rupture evolves toward a circular tip line, some plumose patterns reflect a rupture that returns to an elliptical shape (Figs. 6 and 7). The short axis overshoots its ideal length to become the long axis of a newly enlarged elliptical rupture. Often this overshoot is associated with the inhomogeneous bed interfaces when the rupture has again grown out of the isotropic bed interior.

**DISCUSSION**

This study of the surface morphology of joints bolsters the notion that subcritical crack growth is
a dominant mechanism for fracture in the brittle crust (Segall, 1984a; Olson, 1993; Renshaw and Pollard, 1994). Subcritical growth produces a distinct rupture pattern on joint surfaces with five major characteristics that are indicative of subcritical propagation of joints in layered rock.

First, postcritical rupture leads to out-of-plane cracking and produces a surface topography that far exceeds the small-scale, local inhomogeneities seen on the plume morphology of joints in layered rock. Wholesale out-of-plane cracking includes hackle fringe and branching, both of which result in very irregular joint surfaces (Bahat et al., 2003). The joints described here are planar and without a hackle fringe, thus devoid of any structures characteristic of postcritical rupture (Segall, 1984b).

Second, all evidence from joint surface morphology points to a self-correcting rupture shape in an isotropic, homogeneous rock. A $K_s$-dependent $v_d$ is a necessary condition for self-correction. $K_s$-dependent $v_d$ is one of the characteristics for rupture growth within region I of the subcritical regime but not a characteristic of postcritical propagation (Fig. 13).

Third, often the surface roughness of joints does not increase with growth away from an initiation point. Postcritical propagation, as shown through experiments in glass and ceramics, is characterized by an increase in roughness with growth. A non evolving surface morphology is indicative of a steady-state behavior and not consistent with postcritical growth.

Fourth, the surface roughness on a single joint may vary cyclically (Figs. 6 and 9). This behavior is consistent with alternating periods of propagation followed by hesitation or arrest (Lacaze and Engelder, 1992). Hesitation or arrest ($v_d \rightarrow 0$) occurs when the crack tip $K_s$ drops below a certain threshold in the subcritical regime.

Fifth, orthogonal joints within the same bed can have two different surface morphologies where both morphologies are consistent with stable, $K_s$-dependent $v_d$ growth (Fig. 11). Because they form within the same bed, the difference in roughness cannot be explained by differences in the rock properties, either within the bed or within adjacent beds. In the next section we argue that the morphological differences result from $K_s$ values innate to rupture growth within two different subcritical regions.

**Surface Roughness**

In the laboratory, process zone development is a subcritical phenomenon as indicated by the cloud of acoustic emissions that surround a crack tip moving at subcritical velocities (Swanson, 1981). At higher crack tip velocities ($v_d > 10^{-3}$ m/sec), acoustic emissions associated with the growth and coalescence of a process zone were found to occur not only in the vicinity of the crack tip line but also ahead and behind the tip line (Swanson, 1984). Acoustic emission activity decreases at slower crack tip velocities ($v_d < 10^{-5}$ m/sec), indicating a smaller process zone (Swanson, 1984). The loss of acoustic emissions may mean that no precursory microcracking is taking place or that chemically assisted microcracking is silent.

One important premise of this study is that joint roughness scales directly with the size of a crack tip process zone that reflects the magnitude of $K_s$ during rupture growth (e.g., Broek, 1986). If so, roughness may be used as a proxy for $v_d$ during joints propagation in the subcritical regime. A smoother surface morphology is indicative of a smaller process zone and, hence, a lower $K_s$ and concomitantly slower $v_d$ (Figs. 9, 11, and 13).

The subcritical roughening of joint surfaces and the postcritical roughening of glass, both as a function of $K_s$, are close analogues with the only difference being the magnitude of $K_s$. Subcritical roughening takes place in a process zone that is best developed within several mm of the future joint surface. Subcritical growth ($v_d = 10^{-3}$ m/sec) occurs through a combination of transgranular (through grains) and intergranular (between grains) fracturing (Swanson, 1984), which leads to a surface topography of 10–100 grain diameters depending on the grain size of the host. Mist on glass is the postcritical equivalent, but it is produced in a much stronger crack tip stress field. A combination of slower crack velocity ($v_d = 10^{-7}$ m/sec) and high moisture content produces an increase in the amount of intergranular fracturing and therefore a smoother surface. Mirror on glass is the postcritical equivalent of this latter behavior.

There is a well-defined transition between a rupture in rock that leaves a smooth surface, here defined as a surface with an immediate topography on the order of the grain size of the rock (e.g., Fig. 11B) and a rupture leading to a rough surface (10–100 grain diameters of relief) generated in process zone much larger than grain scale (i.e.,Fig. 11A). Assuming that
laboratory results extrapolate directly to the field, the transition between these fundamentally different surfaces takes place in the range of $10^{-7} < \nu_f < 10^{-3}$ m/sec. By incorporating the major characteristics of rupture into a scheme for $\nu_f$ in the subcritical regime, we can estimate $\nu_f$ to a first approximation and then correlate surface roughness with subcritical $\nu_f$.

**The Subcritical Regime**

In the subcritical regime, $\nu_f$ varies depending on one of three different chemically assisted crack-growth mechanisms (Wiederhorn, 1967; Atkinson, 1984; Atkinson and Meredith, 1987a). $\nu_f$ in subcritical region I is controlled by the rate of stress corrosion reactions at the crack tip and obeys the relationship expressed by Charles Law (i.e., equation 1) (Fig. 13A). $\nu_f$ is independent of $K_I$ in subcritical region II because there is a balance between the rate at which reactive species diffuse to the crack tip and the rate at which they are absorbed to break down atomic bonds at the crack tip. Region III behavior is controlled by the dielectric properties of the environment, where $\nu_f$ depends on $K_I$ (Wiederhorn et al., 1982). This effect is well documented up to $\nu_f = 10^{-1}$ m/sec. At higher velocities ($10^{-3} < \nu_f < 10^{-1}$ m/sec), subcritical region III is characterized by the transition to mechanically induced cracking in the quasistatic regime where $K_I = K_c$ (Costin, 1987). The exact relationship between $\nu_f$ and $K_I$ is not understood at $\nu_f > 1$ m/sec, nor is much known about the boundary between region III behavior and quasistatic propagation at $K_I = K_c$ (Schultz, 2000).

The behavior of clastic rocks in region I is well documented (e.g., Atkinson and Meredith, 1987b; Holder et al., 2001). Experiments performed in clastic rocks with water-saturated pore spaces indicate that the low velocity limit for rupture growth in region I occurs at $\nu_f < 10^{-4}$ m/sec, and the subcritical index ranges between 80 < $n$ < 20. Generally, the transition to region II rupture takes place at $\nu_f = 10^{-2}$ to $10^{-1}$ m/sec (e.g., Atkinson and Meredith, 1987a). What is not known is the nature of the transition from region II to region III behavior. Region III crack growth has been evaluated (e.g., Yoshida et al., 1999; Ciccotti et al., 2000) and appears to extend up to at least $\nu_f = 10^{-1}$ m/sec with a subcritical index $n > 100$. If region III follows a Charles law to critical behavior, the subcritical index is so steep (i.e., $n > 20$) that the curve defining region III must be tight against the boundary for quasistatic behavior. The literature differs on what happens at $\nu_f > 10^{-1}$ m/sec with some research showing region III behavior right up to the seismic velocity of rock (e.g., Bahat et al., 2003), while others show a transition to $K_I$-independent $\nu_f$ at $\nu_f = 10^{-3}$ m/sec (e.g., Schultz, 2000; Fig. 13).

Joints that display uniform roughness and the same postdetachment tip-line shape are interesting because they are so common (e.g., Fig. 10 and top bed in Fig. 12). The impression is that neither $K_I$ nor $\nu_f$ vary during growth of these detached ruptures. Presumably, these ruptures are held at a stable $K_I$ and $\nu_f$ by a combination of mechanisms, the most important of which may be hydraulic diffusivity, $d$ (Segall, 1984b). Fluid flow through the rock matrix is sufficiently rapid so that fluid pressure within the joint remains constant. Calculations show that given the range of $d$ for sandstone, the average $\nu_f$ for joints in clastic rocks fall in the range of $10^{-4}$ to $10^{-1}$ m/sec. This places joints of the Ithaca Formation at the upper end of region I behavior, where $\nu_f = 10^{-1}$ to $10^{-4}$ m/sec and plumes are generated in an active process zone ahead of the crack tip.

The break between smooth and rough surfaces for joints of the Ithaca Formation appears to correlate with the laboratory transition into active acoustic emission at the crack tip (Swanson, 1984). The smooth joints are candidates for low region I behavior with $\nu_f \approx 10^{-4}$ m/sec (Figs. 9, 11B, and 13B). The rough surfaces are indicative of rupture at the high end of region I ($10^{-3} < \nu_f < 10^{-2}$ m/sec). Some joints cycle through the transition between low and high region I behavior (Figs. 6 and 9). This cyclic behavior has been interpreted as a manifestation of natural hydraulic fracturing (Secor, 1965). In the Catskill Delta, cycles are consistent with compressibility-limited propagation driven by a gas rather than brine (Lacazette and Engelder, 1992). The exact cause of this cycling is unknown, although it is likely that fluid flow from the matrix does not keep pace with joint growth at faster $\nu_f$, whereas at slower $\nu_f$, fluid flow exceeds growth so that pressure on the interior of the joint increases with time.

Joints with variable roughness are also likely candidates for the transition from region I to region II behavior ($\nu_f = 10^{-3}$ to $10^{-2}$ m/sec) (Atkinson and Meredith, 1987a). However, high $n$ ($> 20$) for both region I and region III means that there is a large separation in $K_I$ between region I and region III behavior (Fig. 13B). If joint growth makes the transition to region II behavior at $\nu_f = 10^{-2}$ m/sec, $K_I$ can increase a great deal while $\Delta K_I = 0$. A crack tip in Devonian siltstone passing from region I behavior into region II behavior requires a significant increase in $K_I$ before reaching critical behavior. In fact, joint propagation has a much broader range $K_I$ to cross before critical behavior at $K_I$ than was required to transition through six orders of magnitude in $\nu_f$ to reach region II behavior (Fig. 13B). If there were a mechanism to hold the driving stress constant in Devonian shale, for example, the joint radius would have to increase 50 times in order to pass the full distance through region II to reach critical behavior. Joints of the Catskill Delta do not grow this large following region I behavior. For the Ithaca Formation, the region II plateau serves as a large ‘barrier’ separating subcritical and postcritical behavior during joint growth. Apparently, postcritical behavior in the Ithaca Formation is rare as a consequence of this ‘barrier.’

**CONCLUSIONS**

In layered clastic sediments, ruptures evolve from a basic shape, the circle. Systematic evolution toward more complex shapes is a function of the $K_I$-dependent $\nu_f$ along the crack tip line. Evolution of a rupture takes place in three growth stages. The primary stage is characterized by a coherent rupture consisting of a closed or open tip line that propagates across a single bed as a circular, semicircular, or elliptical rupture. Primary growth may involve self-correction driven by the higher $K_I$ found along the short axis of an elliptical rupture. The onset of the secondary growth stage is marked by the splitting of a coherent rupture tip line into two discontinuous moving segments following the intersection of the tip line with the opposing bed boundary. The expanding rupture eventually loses its ability to maintain a coherent rupture where both tip lines move synchronously away from each other. The transition from the growth of a coherent rupture to the detachment and growth of one or more independently propagating ruptures marks the transition into the tertiary stage of growth.

Each of the rupture shapes reflects a $K_I$-dependent $\nu_f$ that is either uniformly distributed or varies systematically along the rupture. A simple, organized crack-tip shape reflects a $K_I$-dependent velocity that is consistent with subcritical propagation. Variation from a smooth to rough plume morphology is consistent with propagation through region I of the subcritical regime with a transition at $\nu_f = 10^{-4}$ m/sec. Rupture velocity may enter region II of the subcritical regime, but such a broad range of $K_I$ must be crossed to reach critical behavior at $K_I$ that region II behavior may act as a ‘barrier’ across which few joints cross, thus greatly limiting the number of postcritical joints in at least the Ithaca Formation, if not the brittle crust of the Earth.

**ACKNOWLEDGMENTS**

This project was supported by the Penn State Seal Evaluation Consortium (SEC) and a grant from the Department of Geosciences Krynine Fund. John Bartley, Bill Dunne, Don Fisher, David Green, Chris Marone, and John Olson are thanked for their time and effort in reviewing early versions of this paper.