Earthquake nucleation on model faults with rate- and state-dependent friction: Effects of inertia

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Abstract. Laboratory studies suggest that earthquake nucleation involves a transition from quasi-static slip when inertial effects are negligible to inertia-driven, dynamic motion. This transition occurs via quasi-dynamic motion, during which the effects of inertia become increasingly important. The characteristics of this transition, which depend on frictional properties of the fault, determine the observability of earthquake nucleation by seismic, geodetic, or other means. By investigating the role of inertia during nucleation, we obtain a quantitative definition of the limiting velocity \( V_{in} \), which marks the end of quasi-dynamic motion and the onset of instability. For reasonable friction parameters and fault widths, we obtain estimates of \( V_{in} \) for crustal faults. To study the roles of inertia, stiffness, and friction parameters on preseismic motion, we simulate triggered instabilities in a one-dimensional model of a homogeneous fault, with rate and state variable friction. In most of our simulations, triggering is achieved by applying a stress perturbation to an initially creeping fault under steady state friction. We also investigate triggering on faults which are initially locked and overstressed compared to their nominal frictional strength, due to time-dependent healing. We study the amount, \( U_p \) and duration, \( T_p \), of preseismic slip as a function of system mass \( m \) and other model parameters. For crustal faults, we interpret the relevant mass as a product of density and fault width and find that wider fault zones result in smaller \( U_p \) and larger \( T_p \). Both \( U_p \) and \( T_p \) are proportional to the system stiffness \( K \), the characteristic slip distance \( D_c \), and the friction constitutive parameter \( a \) and inversely proportional to the size of the triggering event. We find greater \( U_p \) and \( T_p \) for constitutive laws which allow strengthening at zero velocity, compared to laws that require slip or a combination of slip and aging for state evolution. In contrast to quasi static modeling, our simulations suggest a minimum stress perturbation criterion for instability, which may be interpreted in terms of a strain threshold for triggered seismicity.

Introduction

There is growing evidence from laboratory data, seismic observations, and theoretical modeling that earthquake nucleation involves an identifiable transition from quasi-static creep to dynamically driven motion [Scholz et al., 1972; Dieterich, 1978, 1981a, 1986, 1992; Okubo and Dieterich, 1984; Ohnaka and Kawai, 1990; Shibazaki and Matsuzawa, 1992, 1995; Ito, 1992, 1995; Ohnaka, 1993; Abercrombie and Mori, 1994; Baumberger et al., 1994, Kato et al., 1994; Ellsworth and Beroza, 1995]. This transition involves the accumulation of finite fault slip, suggesting that time- and state-dependent frictional behavior play a key role in the nucleation process. Laboratory friction experiments show that preinstability motion consists of an initial, steady creep phase during which motion is quasi-static, followed by slow but self-driven acceleration [Scholz et al., 1972; Dieterich, 1981a; Ohnaka and Yamashita, 1989; Kato et al., 1994]. Such quasi-dynamic motion may produce distinct seismic and geodetic signatures; however, these signatures and the extent to which crustal faults reproduce laboratory behavior depend on details of the frictional properties. Thus a central problem in identifying and interpreting characteristics of earthquake nucleation is that of understanding the effect of friction parameters on the transition from quasi-static to dynamic slip.

Previous theoretical studies of earthquake nucleation use arbitrary velocity cutoffs to denote the onset of instability and assume that the nucleation process is purely quasi-static [Dieterich, 1986, 1992, Rice and Tse, 1986], but see Gu and Wong [1991] and Weeks [1993]. As a result, these studies do not fully model quasi-dynamic acceleration prior to the onset of instability. Other studies, which have included the effects of inertia [e.g., Sleep, 1995a,b; Segall and Rice, 1995], have focused primarily on shear heating, compaction, and dilution as they relate to instability. These studies do not define the transition from quasi-static to dynamic slip, nor do they address the role of friction in determining characteristics of preseismic slip and earthquake nucleation.

In this study, we investigate the effect of friction parameters, friction laws, and inertia on preseismic slip and quasi-dynamic motion preceding earthquake-like instability. We derive, via the equations of motion, a quantitative definition of the transition from quasi-static to dynamic slip. To focus on the friction parameters and constitutive formulations and to avoid the complicating effects of geometry, we use a single-degree-of-freedom elastic model of a homogeneous fault (Figure 1). Friction is assumed to obey rate and state variable constitutive laws, and fault normal stress is constant. We do not consider effects of rupture...
growth or dynamic rupture propagation, nor do we account for
dilation, compaction, or shear heating. We investigate the
amount \( U_p \) and duration \( T_p \) of preseismic slip as a function of
mass per unit fault area \( m \), stiffness \( K \), stress perturbation \( \eta \), and the friction parameters and constitutive formulation.
In the majority of our simulations, we trigger instability by
applying a stress perturbation to a fault undergoing slow creep
under steady state friction. In addition, we also consider cases
in which the fault is initially locked and overstressed due to
time-dependent frictional strengthening.

We define two distinct phases of motion during preseismic
nucleation: a quasi-static phase, with a limiting slip velocity of
\( V_{q} \), and a quasi-dynamic phase, which involves slow
acceleration to a limiting velocity \( V_{in} \). The velocity \( V_{in} \)
defines the point at which motion becomes inertia-dominated.
We find that this definition is consistent with unstable motion
as predicted by a linear stability analysis. In the limit of
negligible system mass, the duration of quasi-dynamic motion
approaches zero, and we recover the purely quasi-static
nucleation process assumed by Dietrich [1986, 1992].

In the following section, we describe the rate and state
variable constitutive laws used in the simulations and derive
the limiting velocities \( V_{q} \) and \( V_{in} \). Next, we present the
results of our simulations to illuminate the effects of inertia
and constitutive parameters on preseismic nucleation. By
varying mass with all other parameters fixed, we determine its
role during preseismic nucleation.

**Rate and State Variable Constitutive Laws**

Laboratory studies indicate that rock friction is a function
of both instantaneous sliding velocity ("rate") and sliding history ("state") [Dietrich, 1979; 1981; Ruina, 1980, 1983;
Scholz, 1990]. Two types of rate/state friction laws have been
proposed to account for different types of state evolution
[e.g., Beeler et al., 1994; Perrin et al., 1995]. In this paper we
explore three specific rate/state constitutive laws, however,
we focus primarily on the type of law proposed by Dietrich
[1979]:

\[
\mu = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \psi}{D_c} \right)
\]

(1)

Here, \( \mu \) is the coefficient of friction, \( V \) is the instantaneous
sliding velocity, \( V_0 \) is a constant reference velocity for which
steady state friction is \( \mu_0 \), \( \psi \) is the state variable, and \( D_c \) is
a characteristic slip distance. The terms \( a \), \( b \), and \( D_c \) are
constitutive parameters determined experimentally [e.g.,
Dietrich, 1979; Tullis and Weeks, 1986; Marone et al., 1990;
Reinen and Weeks, 1993]. The second and third terms in (1)
represent the instantaneous velocity and state dependence of
friction, respectively. Equation (2) describes friction state
change and indicates that state cannot change instantaneously but
rather evolves with time \( t \) and characteristic displacement \( D_c \). This relation allows for
friction evolution during stationary contact, in contrast to
another law we use and describe more fully below, in which
friction evolution requires finite slip [Ruina, 1983].

Because rate/state friction laws have been discussed in detail
[e.g., Rice and Gu, 1983; Tullis, 1988; Scholz, 1990; Linker and
Dietrich, 1992], we restrict attention here to a few key
points. Figure 2 shows the friction response described by (1)
and (2) to a step change in velocity. The parameter \( a \) scales
the instantaneous friction increase at constant state and \( b \) scales
friction evolution over characteristic slip distance \( D_c \).
The change in steady state friction scales with \( (a-b) \) and from
(2) the steady state value of the state parameter \( \psi \) is \( D_c/V \). If
\( a-b > 0 \), the evolution effect outweighs the direct effect.
Friction exhibits so-called velocity weakening (Figure 2). For
\( b < a \), steady state friction increases with velocity (so-called
velocity strengthening).

Velocity-weakening frictional behavior is a necessary
condition for instability [Rice and Ruina, 1983], and from
numerous seismic observations, such as the depth-frequency
distribution of seismicity [e.g., Marone and Scholz, 1988;
Blanpied et al., 1991], we infer \( (a-b) < 0 \) for the seismicogenic
region of crustal faults. Thus, in our simulations we set \( a-b \)<0
and use parameter ranges consistent with laboratory
results [Dietrich, 1979; Marone and Cox, 1994; Reinen et al.,
1994; Blanpied et al., 1995].

**Figure 2.** Friction response for a step increase in sliding
velocity at time \( t_1 \). At times \( \mu > \mu_0 \), friction is at steady state,
\( \mu = \mu_0 \). The "direct effect" is the immediate jump in \( \mu \) following
the velocity step, and the "evolving effect" is the decay in \( \mu \)
over a slip distance \( D_c \). Velocity weakening frictional behavior, \( (a-b) < 0 \), is shown.

\[
\frac{d\psi}{dt} = 1 - \frac{V\psi}{D_c} \quad (7)
\]
Transition From Quasi-Static to Dynamic Slip

In the context of our model, fault slip and rupture nucleation are determined by the equation of motion of the mass coupled to the constitutive formulation for fault friction. We begin by defining a limiting velocity for quasi-static motion, using solutions to the linearized equations of motion, and then consider the transition from quasi-static to dynamic motion.

Following Rice and Ruina [1983], the equation of motion of a spring-slider system, initially at steady state but subjected to an arbitrary perturbing stress $q(t)$, may be written

$$m\ddot{x}(t) = \tau_0 - Kx(t) - \tau(t) + q(t),$$

where $m$ is the mass per unit area, $\tau$ is shear stress given by $\mu\sigma$, where $\sigma$ is normal stress, $\tau_0$ is steady state shear stress, $K$ is stiffness, $x(t)$ is the change in slider position, and overdots indicate time derivatives. From a linear stability analysis of this equation coupled to the constitutive law (1) and (2) [Rice and Ruina, 1983], the critical stiffness for unstable motion is

$$K_c = \frac{\sigma(b-a)}{D_c} \left[ 1 + \frac{mV_0^2}{\sigma a D_c} \right].$$

Linearized Equations of Motion

Assuming that the perturbing force is a Heaviside function, $q(t) = q_0 \Theta(t)$, the position and velocity of the slider for $t > 0$ are $x(t) = V_0 t + \xi(t)$ and $V(t) = V_0 + \dot{\xi}(t)$, respectively. In addition, the state variable is perturbed from its steady state value, $\eta(t) = D_c V_0 + \xi(t)$. Substituting these expressions into (3) and writing $K = K/\mu$, we obtain linearized equations for the perturbed quantities valid for $t > 0$ (neglecting all terms that are of second or higher order in the perturbations),

$$m\ddot{\xi}(t) = -K \dot{\xi}(t) - \frac{V_0 a}{D_c} \dot{\xi}(t) + \frac{bV_0 \xi(t)}{D_c} + q_0,$$

$$\dot{\xi}(t) = \frac{V_0 a}{D_c} \xi(t) - \frac{\xi(t)}{V_0}.$$  \hspace{1cm} (6)

Quasi-Static Limit

In the limit of quasi-static motion the acceleration term in (5) is small, so $0 = Kx(t) + \left[ \frac{\sigma a x(t)}{V_0} + \frac{bV_0 \xi(t)}{D_c} \right] - q_0$. The critical stiffness is then $K_c = \sigma(b-a)/D_c$ [Ruina, 1980; Gu et al., 1984], which is equivalent to (4) when $mV_0^2/\sigma a D_c << 1$, i.e., at small steady state velocities. As steady state velocity increases, when $mV_0^2/\sigma a D_c = 1$, motion is no longer quasi-static. Thus we define the limiting velocity for quasi-static motion $V_{qs}$ as

$$V_{qs} = \sqrt{\frac{\sigma a D_c}{m}}.$$ \hspace{1cm} (7)

Equation (7) indicates that for initial, steady state velocity $V < V_{qs}$, motion will begin quasi-statically. The velocity $V_{qs}$ is defined in terms of steady state velocities, and we use it as an upper bound for the quasi-static phase of motion. For $\sigma, a$, and $D_c$ values consistent with field and laboratory estimates, and taking $m = \rho r$, where $\rho$ is density and $r$ is a relevant fault dimension (such as the nucleation patch radius), $V_{qs}$ is much greater than creep velocities on geologic faults ($< 10^{-9}$ m/s).

Thus a fault undergoing interseismic, creep-type motion driven by plate tectonic loading satisfies our condition for quasi-static motion.

Inertia-Dominated Limit

At the onset of inertia-dominated motion the friction terms in (5) are negligible compared to the applied stress and acceleration terms. In this case, the equation for the slip perturbation $\dot{\xi}$ is approximately that of a simple harmonic oscillator, $m\ddot{\xi}(t) = K\dot{\xi}(t)$, where $\ddot{x}(t) = \omega_0^2 \exp(i\omega t)$ with frequency $\omega_0 = \sqrt{K/m}$. We do not consider full dynamic oscillations of the system but instead are concerned with the velocity at which the frictional terms in (5) are negligible compared to the inertial ones. Setting the sum of the friction terms in (5) to zero, we have $-a\dot{\xi}(t)/V_0 = b\xi(t)/V_0$.

Substituting this into (6) yields the state variable evolution when motion is inertia-dominated:

$$\ddot{\xi}(t) = \frac{V_0 a \xi(t)}{D_c} \frac{(b-a)}{a}.$$ \hspace{1cm} (8)

From (8), the timescale over which the state variable changes is $T^* = a D_c/V_0(b-a)$. On the other hand, the timescale over which velocity changes is $T_m = \sqrt{mK}$. When the motion is inertia-dominated, $T^* \ll T_m$ or $V_0 \gg a D_c \sqrt{K/m(b-a)}$. Thus we define $V_{in}$ as the limiting velocity at which motion first becomes inertia-dominated:

$$V_{in} = \frac{a D_c}{(b-a)} \sqrt{K/m}.$$ \hspace{1cm} (9)

Note that this relation differs from estimates based on comparison of the natural period of vibration $T = \sqrt{mK}$ with the characteristic friction evolution time $D_c/V_0$ [e.g., Rice and Tse, 1986]. In that case, $V_{in} = D_c \sqrt{K/m}$. Since laboratory data generally show $a \approx 2(b-a)$, the two estimates do not differ greatly. However, (9) has the feature that for given stiffness, mass, and characteristic friction distance, inertia-dominated motion is reached earlier when friction exhibits greater velocity weakening and later for cases in which friction increases more strongly upon an increase in velocity (larger $a$). Thus our estimate has the advantage that it relates $V_{in}$, which is closely related to seismic or geodetic thresholds of observability of earthquake nucleation, with the friction parameters controlling preseismic behavior and earthquake nucleation.

Preeismic Motion and Validity of the Definition of $V_{in}$

Figure 3 illustrates the development of unstable slip in our model. For $V < V_{in}$, slip is either quasi-static ($V < V_{qs}$) or quasi-dynamic ($V_{qs} < V < V_{in}$). The time at which $V = V_{in}$ marks the end of preeismic slip. To trigger instability, we apply a stress perturbation defined as the ratio of the perturbed stress to the initial stress, $\eta = q(t)/\sigma_0$. The perturbation is applied at $t = 0$ after which the load point velocity is fixed at $V_0$. Initially, frictional and applied stresses are approximately equal, acceleration is negligible, and motion is quasi-static (quasi-static region in Figure 3). The onset of quasi-dynamic motion begins when velocity weakening causes friction to fall below the applied stress. In this paper, we refer to inertia dominated motion ($V > V_{in}$) as seismic or dynamic motion.

To verify our definition of the limiting velocity $V_{in}$, we check whether our numerical simulations exhibit instability
figures that follow, we use nondimensional slip, time, and velocity, \( U' = UD_c \), \( T' = TV_0 /D_c \), and \( V' = V /V_0 \), respectively, with primes indicating dimensionless variables. In each case, velocity increases at about the same rate until \( T' = 4x10^{-6} \), after which it increases sharply for \( K < K_c \) (Figure 4b). For \( K > K_c \), \( V \) never exceeds \( V_{in} \) and slip never becomes dynamic. Note that \( V_{in} \) increases with \( K \) and thus \( V_{in} \) is lowest in the most unstable case. For \( K/K_c < 1 \), \( V \) eventually exceeds \( V_{in} \), and thus cases that satisfy the criteria for unstable slip also exhibit dynamic motion according to our definition.

Simulations

We first present simulations of triggered instabilities on faults undergoing steady, creep-type motion with steady state friction according to (1) and (2). In these cases, the question addressed is: under what conditions will a stress change, for example due to a nearby earthquake, trigger instability on a fault driven at steady state. On the other hand, faults may be locked and overstressed due to time-dependent frictional strengthening. On such faults, triggering may involve speeding an instability that is already slowly growing. To investigate these cases, we also present simulations in which the initial friction is higher than the steady state value at the initial loading rate.

Parameter Ranges

The initial velocity and normal stress for all simulations is \( V_0 = 10^{-2} \) m/s and \( \sigma = 100 \) MPa, respectively, chosen to be comparable to geologic slip rates and a nominal effective stress value in the seismogenic region of crustal faults. We use a range of stiffnesses from 1 to 5 GPa/m, corresponding to \( K/K_c \) values of 0.2 to 0.9. Mass per unit area \( m \) is varied from \( 10^1 \) to \( 10^3 \) kg/m². As we discuss more fully below, the range of \( m \) values may be considered to represent either a range of rupture nucleation dimensions \( r \) (taking \( m = \rho r \) and noting that \( K = G/r \), where \( \rho \) is density and \( G \) is shear modulus) or a range of fault zone widths \( w \) (taking \( m = \rho w \)). We use values of the friction constitutive parameters consistent with laboratory data and fault zone modeling estimates: \( a = 0.005 \) to 0.007, \( b = 0.006 \) to 0.008, and \( D_f \) from \( 2x10^{-3} \) to \( 2x10^{-3} \) m. Higher field-based estimates of \( D_c \) exist, based on its mechanical interpretation and scaling properties [e.g., Marone and Kilgore, 1993]; however, our upper limit is fixed by computational time constraints.

Definition of \( \eta_{min} \) and \( \eta_{max} \)

Although the instability criteria \( K < K_c \) and \( \eta < \eta_{min} \) must be satisfied in order to achieve dynamic motion (e.g., Figure 4), inertia and the initial frictional state also play a role in determining whether slip velocity exceeds \( V_{in} \). Stress perturbations smaller than a critical size \( \eta < \eta_{min} \) will not result in dynamic motion because \( V_{max} < V_{in} \), and hence the transition from preseismic to seismic slip is undefined (case C of Figure 5). Case B of Figure 5 shows \( \eta = \eta_{min} \) for which \( V_{max} \) just exceeds \( V_{in} \). Larger stress perturbations result in higher maximum velocities and shorter preseismic durations (case A, Figure 5). For stress perturbations larger than an upper limit \( \eta > \eta_{max} \), the system is immediately unstable with \( V > V_{in} \). This occurs when the applied stress jump exceeds the instantaneous friction direct effect, and in this case, preseismic slip and duration are zero. Thus, in our simulations we limit the range of stress perturbations to \( \eta_{min} < \eta < \eta_{max} \).
Figure 4. (a) Slip versus time curves for numerical simulations using identical initial velocities, constitutive parameters and slider masses, hence identical $K_c$, but differing values of system stiffness, i.e., differing $K/K_c$. All simulations use $\eta = 1.120$ and $m = 10^7$ kg/m². Inset shows the same data at larger scale to illustrate the unstable slip versus time function when $K/K_c < 1$, and stable motion when $K/K_c > 1$. (b) Velocity versus time curves for the simulations in Figure 4a. Dashed lines show $V/\sqrt{V_0}$. Note that velocity exceeds $V_{in}$ only in cases where $K/K_c < 1$, i.e., when unstable motion is predicted by a linear stability analysis of the equation of motion. Thus our definition of $V = V_{in}$ as the onset of dynamic motion is consistent with these results.

Results

System Mass and Stiffness

Figure 6 shows normalized velocity and slip versus time for simulations with the same perturbation $\eta$ but different masses. A dot marks the point at which motion changes from quasi-dynamic to inertia-dominated and thus denotes the end of preseismic slip. Larger $m$ causes longer preseismic duration and smaller preseismic slip (Figure 6). This is because for fixed $\eta$, acceleration is inversely proportional to mass, and thus for larger masses the velocity is lower during preseismic slip (Figure 6b). As a result, slip-dependent evolution of the state variable requires a longer time, leading to slower velocity weakening and a longer preseismic duration $T_p$ (Figures 6 and 7). In addition, $V_{in}$ decreases as $m^{-1/2}$, so larger $m$ yields dynamic motion at lower velocities (Figure 6b and inset to Figure 6a), leading to smaller preseismic slip $U_p$ (Figures 6a and 7). Note that both $V_{qs}$ and $V_{in}$ are proportional to $m^{-1/2}$, so as mass approaches zero $V_{qs}$ and $V_{in}$ approach

Figure 5. Illustration of minimum and maximum stress perturbations used in simulations. The smallest value of the perturbation, $\eta_{min}$ is defined as the minimum value required to produce seismic slip, case B. At the maximum value, $\eta_{max}$, the applied stress immediately exceeds the friction direct effect, and thus the preseismic slip and duration are zero (case A). Normalized velocity versus time for three cases illustrate the definition of $\eta_{min}$. The maximum velocity for case B just reaches $V_{in}$, and thus the stress perturbation is just large enough to cause dynamic slip. In case A, $V_{in}$ is exceeded, and in case C, $V_{in}$ is never reached. The friction parameters for each case are identical, but the stress perturbations vary, $\eta_A > \eta_B \approx \eta_{min} > \eta_C$. 


infinity, consistent with fully quasi-static motion in this limit.

Preseismic behavior is also a function of stiffness and the degree of instability, via $K/K_c$. For a range of $m$, $\eta$, and $K$ values, Figure 7 shows the preseismic duration and slip determined from curves such as in Figure 6. At a fixed $\eta$, higher stiffnesses yield longer preseismic durations and larger preseismic slips. The parameter $\eta_{\text{min}}$ increases with $K/K_c$ (Figure 7), indicating that larger stress perturbations are required to trigger instability when the system is inherently more stable. In addition, $\eta_{\text{min}}$ decreases with increasing mass, which indicates that larger masses require smaller perturbations to trigger instability (Figures 6 and 7).

**Perturbation Size**

The size of the stress perturbation controls the initial acceleration and average preseismic velocity, and thus both preseismic slip and duration decrease with increasing $\eta$ (Figure 7). For fixed $K/K_c$ and $m$, we find a log-linear relationship between $T_p$ and $\eta$ (Figure 7, left). As $m$ increases the range of stress perturbations producing preseismic slip decreases (Figure 7). This is due to a both a decrease in $\eta_{\text{min}}$ and a faster decrease in $\eta_{\text{max}}$. The maximum allowed perturbation decreases with higher $m$, since the instantaneous increase in friction decreases with mass (e.g., Figure 2).

**Characteristic Slip $D_c$**

For fixed $\eta$, preseismic slip and duration increase with increasing $D_c$ (Figure 8). Note that both $V_{\text{q}}$ and $V_{\text{in}}$ increase with $D_c$ and that as $D_c$ goes to zero, $V_{\text{in}}$ becomes small, and thus motion becomes unstable instantaneously, i.e., $U_p$ and $T_p$ are negligible. The normalized preseismic duration $T'$ is independent of $D_c$ (Figure 8a), indicating linear increase in $T_p$ with $D_c$. Preseismic slip increases more rapidly than linearly with $D_c$ (Figure 8b). The effect of $D_c$ on $U_p$ varies with the size of the stress perturbation.

**Friction Parameter $a$ (Direct Effect)**

We explore the role of the friction direct effect in simulations where we vary $a$ for fixed values of the other parameters, $(a-b, \sigma, \eta, D_c, K/K_c, \alpha, \text{and } m)$. Increasing $a$ leads to an increase in the duration and amount of preseismic slip (Figures 9 and 10). The net stress immediately following the perturbation decreases with increasing $a$ and thus average preseismic velocity decreases (Figure 9b). This effect leads to an increase in the preseismic duration (Figure 9b), whereas the fact that $V_{\text{in}}$ scales as $a$ leads to an increase in preseismic slip (Figure 9a). For fixed $a$, $U_p$, and $T_p$ decrease with $\eta$ (Figure 10). Note that by definition the maximum allowed stress perturbation $\eta_{\text{max}}$ increases with $a$.

**Rate and State Constitutive Formulation**

The above simulations use the constitutive formulation in equations (1) and (2), which we refer to as the Dieterich law. We also consider two other rate and state variable formulations, which we refer to as the Ruina law [Ruina, 1982], and the Perrin-Rice-Zheng law [Perrin et al., 1995]:

Ruina law

$$\mu = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \psi}{D_c} \right)$$

$$\psi = \left( \frac{V \psi}{D_c} \right) \ln \left( \frac{V \psi}{D_c} \right)$$
For fixed constitutive parameters the PRZ law evolves the fastest, followed by the Ruina and Dieterich laws (Figure 11a). Preseismic slip and duration are smallest for the PRZ law, followed by the Ruina and Dieterich laws (Figures 11b and 11c). An approximately log-linear relation between $T_p$ and stress perturbation is observed for each law below an upper limit in $\eta$ that is lowest for the PRZ law (Figure 11b). Preseismic slip is largest for the Dieterich law although $T_p$ decreases more rapidly with increasing $\eta$ in this case.

Locked and Overstressed Faults

The simulations above start with the fault creeping steadily under steady state friction. To investigate the effects of overstress, such as for a partially or fully locked fault that has undergone frictional healing, we start with a higher frictional strength than the steady state value for a given creep rate. For these simulations, we use the Dieterich friction law. In this case, $d\psi/d\tau=1$ when the fault is stationary and thus overpressuring corresponds to increasing the initial value of $\psi$ by the interseismic locking time. We studied interseismic times of $10^7$ s and $3 \times 10^9$ s (=100 years). All other aspects of the simulations were identical to the cases described above, including the initial velocity $V_0$. For the overstressed cases, we find lower preseismic duration and slip compared to faults with steady state friction (Figure 12). However, the effect is not large. For example, 100 years of overpressuring results in a reduction in preseismic duration of $1.30$ s for a nucleation patch roughly 4 km in radius ($\eta=10^7$ kg/m²) with $D_c=10^{-2}$ m and $V_0=10^{-9}$ m/s (Figure 12). Overstressed faults have lower $\eta_{\text{min}}$, but the minimum stress perturbation capable of triggering seismic slip is still appreciable (Figure 12), in contrast to results that do not include inertia [Dieterich, 1986].

Discussion

Our numerical simulations show that the preseismic duration and time to instability decreases with increasing perturbation size. In particular, $\log{(T_p)}$ decreases linearly with the size of the triggering stress perturbation (Figures 7, 8, 10, 11, and 12). The slope of this decay is fairly constant, $(\Delta \log{(T_p)}/\Delta \eta) \approx -50$, varying somewhat with friction parameter $a$ (Figure 10) and with constitutive formulation (Figure 11).

Scaling of Preseismic Duration With Perturbation Size and Aftershock Occurrence Rate

Our model simulates a fault with homogeneous frictional properties and stress conditions. Therefore, for a given set of parameters, it yields a single time to failure for a given stress perturbation. Following Dieterich [1986], our results can be applied to more complex faults if the initial stress is heterogeneous. In that case, a given triggering event will produce a range of stress perturbation sizes and potential nucleation sites. Assuming a uniform distribution of stress perturbations, the number of nucleation sites $N$ triggered by a perturbation $\Delta \eta$ is proportional to $\eta$; hence $N \sim \log{(T_p)}$. This indicates that the rate of triggered events varies inversely with time, $dN/d\tau \sim 1/T_p$, and thus our relationship between $T_p$ and $\eta$ is consistent with the well-known inverse relationship between aftershock occurrence rate and time from the mainshock (Omori's law). Our results indicate that overstressed regions of the fault may fail earlier than regions driven at the steady

\[
\mu = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \psi}{2D_c} \right)
\]

\[
\psi = 1 - \left( \frac{V \psi}{2D_c} \right)^2
\]
state creep rate and that instability will be delayed in regions for which the friction parameters \(a\) and/or \(D_c\) are larger, such as those of higher roughness or that contain granular fault gouge [Marone and Kilgore, 1993; Marone and Cox, 1994].

Our modeling results are also consistent with observations showing that the rate of foreshocks increases rapidly in the days preceding a mainshock [Jones and Molnar, 1979; Jones, 1984, 1994]. Jones and Molnar [1979] presented a model in which foreshocks are generated by static fatigue of contact asperities. They assume that static fatigue is a random process, so that the rate of failure, \(dN/dt\), is proportional to the number of intact asperities, \(N\), and inversely proportional to the average time to failure, \(T_f\). For static fatigue, the time to failure decreases exponentially with differential stress \(\tau\): \(T_f \propto\)

Figure 8. (a) Preseismic duration and (b) preseismic slip versus \(\eta\) for a range of \(D_c\) values. \((K/K_c)\) is fixed at 0.6 in each case by varying the system stiffness \(K\) with other parameters fixed.) The normalized duration is independent of \(D_c\), implying that the duration \(T_p\) scales with \(D_c\). Preseismic slip \(U_p\) increases faster than linearly with \(D_c\).

Figure 9. (a) Slip versus time and (b) velocity versus time curves for simulations with varying \(a\). The parameters used are \(U_i = 2 \times 10^{-5}\) m, \(\eta = 1.080\), \(m = 10^7\) kg/m², and \(K/K_c = 0.6\). We use three different values of the parameter \(a\) (and change \(b\) to keep \(a-b\) constant). The dots in Figure 9a indicate the times at which velocity exceeds \(V_{in}\) and dynamic motion begins. The dashed lines in Figure 9b indicate \(V_{in}/V_0\). The limiting velocity for dynamic motion, \(V_{in}\), increases with \(a\), as do preseismic slip and duration.
Figure 10. (a) Preseismic duration and (b) preseismic slip versus $\eta$ for different values of parameter $a$. For a given stress perturbation, both $U_p$ and $T_p$ increase with $a$. Note that by definition, $\eta_{max}$ increases with $a$ because of the increase in the friction direct effect.

Figure 11. Comparison among different rate/state variable friction laws. (a) The friction response to a step change in velocity for the laws is determined by the rate of state variable evolution. Here the initial steady state $\mu$ and the instantaneous direct effect, are chosen to be the same for all laws. (b) Normalized preseismic duration and (c) normalized preseismic slip are greatest for the Dieterich law, which has the slowest state evolution, followed by the Ruina and the Perrin-Rice-Zheng laws.
This is consistent with our relation between preseismic duration and stress perturbation size $-\log(T_p) \approx \eta$ and $dN/dt \approx 1/T_p$, since $\eta$ determines the differential stress on a given fault patch. Therefore our modeling indicates that cumulative failure of asperities within the nucleation region of an impending earthquake would result in larger stress perturbations and thus progressively higher rates of seismicity. Thus it appears that the relationship between the time to instability and the size of the perturbation is a general feature of triggered instabilities.

A Model of Nucleation Incorporating Mass

From our numerical simulations, preseismic duration increases with $m$ and both preseismic slip and average velocity decrease with $m$. In addition, the limiting velocities $V_{up}$ and $V_{in}$ vary with $m$. If one takes $m$ as the product of density and a spatial dimension, then the problem of applying these results to faults reduces to a problem of identifying the spatial dimension. For a slipping fault patch, this dimension is the width $w$ perpendicular to the fault plane, $m=\rho w$. Thus the range of $m$ values considered in our simulations, $10^6$ to $10^7$ kg/m$^2$, corresponds to normal widths of 4 mm to 4 km. This would indicate a simple scaling between $w$ and preseismic duration; however, $w$ is not independent of nucleation dimension. From continuity considerations, $w$ must scale with rupture size $w=r^a$, so that $m=\rho r^a$, where $r$ is fault radius and $n$ is a constant depending on factors such as gouge zone width and microstructure or fault roughness. For example, a scaling between $w$ and $r$ is predicted by the fractal character of fault roughness, since the maximum topographic variation on a slipping patch is proportional to the patch radius [Brown and Scholz, 1985; Power and Tallis, 1992]. These studies would indicate $n=1$ to 1.5. Thus the inertial parameter in our model scales with nucleation zone size.

Although our simulations do not explicitly incorporate growth of the nucleation patch, such growth may be considered in the context of stiffness variations, taking $K=G/r$. A requirement for instability and seismic slip is $K<K_c$, and since $K_c$ is defined in terms of the constitutive parameters (equation (4)), the instability condition is assumed to be met by a reduction in the local stiffness surrounding an expanding nucleation patch. Following Dieterich [1986], and ignoring for the moment inertial effects, the critical nucleation patch size $r_c$ can be defined in terms of the friction parameters as follows. For a circular patch of radius $r$, the relationship between the stress drop $\Delta\sigma$ and the maximum offset $U$ at the center is, $\Delta\sigma=(7\pi/24)(UG/r)$ [Chinnery, 1969], where $G$ is the shear modulus. Interpreting $\Delta\sigma/U$ as a local stiffness, $K$ varies as $r^{-1}$, $K=(7\pi G/24)(G/r)$ [Dieterich, 1986]. Therefore growth of the slipping patch reduces $K$ until the onset of instability, when $K=K_c$, and the minimum patch radius for instability is [Dieterich, 1986]

$$r_c = \frac{7\pi G}{24 K_c}.$$  

(14)

For a quasi-static analysis, without inertia, (14) provides a functional relationship between nucleation size and friction parameters. When inertia is included, $K_c$ also depends on $m$, which from the above discussion, is related to $r$. However, for reasonable values of initial fault creep rates, $K_c$ is approximately independent of $m$, that is, the inertial term in equation (4) is $\ll 1$ (of order $10^{-12}$). Making this approximation, $K_c = 0.1/D_c$ (in units of MPa/m) for $\tau=100$ MPa, $V_0=10^{-9}$ m/s, and $(b-a)=10^{-3}$ (equation (4)), where we note that the scaling between $D_c$ and $w$ suggested by Marone and Kilgore [1993] is not included. For a circular fault patch and $G=30$ GPa, this indicates a range of $r_c$ values from 5 m to 0.5 km for $D_c$ from $2 \times 10^{-5}$ m to $2 \times 10^{-3}$ m, which is
consistent with seismic estimates of nucleation source dimensions.

Quasi-Dynamic Motion

In the context of our model, preseismic slip and duration are determined by both the quasi-static and quasi-dynamic phases of motion (Figure 3). However, in nature, quasi-dynamic processes are likely to be much more important for the identification of earthquake nucleation. Seismic and geodetic detectability of nucleation will scale with slip velocity, and thus the limiting velocity $V_{\text{in}}$ is a key factor in the interpretation of our results. From equation (9), as the patch radius grows, the local stiffness decreases as $r^{-3}$ and the effective mass per unit area increases as $r$. Thus $V_{\text{in}}$ varies as $r^{-1}$, and as slip progresses toward instability the limiting velocity for dynamic motion decreases.

We can use our above estimates of $r_c$ to estimate $V_{\text{in}}$ for a large-scale fault. At the onset of instability when $r=r_c$, we assume that the width of the fault zone, $w$, is approximately $r_c$. Therefore the effective mass per unit area ranges from $10^4$ to $10^6$ kg/m$^2$ for $w=r_c$ of 5 m to 0.5 km. Using these values for $m$, and setting $K=K_c$ in equation (9), $V_{\text{in}}$ ranges from $9 \times 10^{-3}$ to $8 \times 10^{-3}$ m/s. These velocities are reasonable since they are significantly higher than background creep rates on geologic faults (about $10^{-9}$ m/s) and slightly lower than typical earthquake particle velocities, which involve $V>V_{\text{in}}$.

Minimum Perturbation Size

The parameter $\eta_{\text{min}}$ sets the lowest perturbation for which sliding velocity reaches $V_{\text{in}}$, smaller stress perturbations result in model creep events. In our numerical simulations, $\eta_{\text{min}}$ was determined empirically; however, an analytic solution would allow one to relate fault zone frictional properties to observations of triggered seismicity thresholds [e.g., Hill et al., 1993; Gomberg and Bodin, 1994; Gomberg and Davis, 1996, Beresnev and Wen, 1995]. We derive here an approximate analytic form for $\eta_{\text{min}}$.

By definition, $V_{\text{max}}-V_{\text{in}}$ for $\eta=\eta_{\text{min}}$. When $V>V_{\text{max}}$, state variable evolution is fast relative to velocity changes, and thus $\dot{y}$ is approximately equal to the steady state value $\dot{y}=D_{c}^{-1}/V_{\text{in}}$. Using this approximation in the constitutive law (equation (1)) and combining with the equation of motion (3), we have (setting $m=0$ in equation (3))

$$ (\eta_{\text{min}}-1)\mu_0 = -k(V_0 T_P - U_P) + (a - b) \ln \left( \frac{V_{\text{in}}}{V_0} \right) \tag{15} $$

Taking an initial fault slip rate of $V_0=10^{-9}$ m/s and noting the magnitude of $T_P$ from our simulations: $V_0 T_P \ll U_P$. Thus $\eta_{\text{min}}$ can be written

$$ \eta_{\text{min}} = 1 + \frac{1}{\mu_0} \left[ k U_P + (a - b) \ln \left( \frac{V_{\text{in}}}{V_0} \right) \right]. \tag{16} $$

This can be simplified further by noting that in the numerical simulations, when $\eta=\eta_{\text{min}} U_P / D_{c}^{-1}=100$; thus

$$ \eta_{\text{min}} \approx 1 + \frac{1}{\mu_0} \left[ 100 k D_{c} \ln \left( \frac{V_{\text{in}}}{V_0} \right) \right]. \tag{17} $$

The utility of (17) is that given a value of $\eta_{\text{min}}$, for example, from estimates of coseismic static stress changes, one can estimate the fault zone friction parameters and $V_{\text{in}}$. Comparison of the analytic approximations for $\eta_{\text{min}}$ with empirically derived values from our numerical simulations indicates reasonable agreement (Figure 13). The predicted $\eta_{\text{min}}$ values are given for a range of constitutive parameters, using the numerical preseismic slip values.

Interpretation of $\eta_{\text{min}}$ in Terms of Triggered Seismicity

In the context of slip on large-scale faults, our stress perturbation represents loading by a neighboring earthquake. Observations of triggered seismicity can be used to evaluate $\eta_{\text{min}}$ if $\eta_{\text{min}}$ is interpreted as a static stress (strain) threshold required for nucleation: $\Delta \tau_{\text{t}}=\eta_{\text{min}} \Delta \tau$. Here $\Delta \tau$ is the change in shear stress on the fault and $\epsilon$ is the elastic strain. Taking $\tau_0=60$ MPa as a nominal frictional strength at seismogenic depths, nominal $\eta_{\text{min}}$ values from our modeling (10.1-10.5) give stress thresholds of 0.6 to 3 MPa and a strain threshold of $\epsilon=\eta_{\text{min}}^{-1}(10^{-5})$. These stress changes are higher than field estimates $0.05$ to 0.1 MPa [Reasenberg and Simpson, 1992; Stein et al., 1992; King et al., 1994]. This indicates that our estimate of fault strength is too high, although $\eta_{\text{min}}$ would have to be reduced by an order of magnitude, or that $\eta_{\text{min}}$ for crustal faults is significantly lower than the nominal values we obtained, perhaps due to lower effective stiffness than used in the numerical simulations (Figures 13d and 13e).

This comparison is based on static stress perturbations; however, recent observations indicate a frequency-dependent strain threshold for seismic triggering [Gomberg and Davis, 1996]. We may also compare our results to these observations. In the frequency domain our Heaviside stress perturbation applied at $\tau\theta$ has a $1/f$ spectrum. Using the above shear stress estimate, the corresponding frequency-dependent strain perturbation is

$$ \epsilon(f) = \frac{2 \times 10^{-3}(\eta_{\text{min}}^{-1})}{2\pi f}. \tag{18} $$

Although the specific form of (18) is dictated by our assumption of an instantaneous stress perturbation, it has the feature that the threshold triggering strain $\epsilon_T$ is inversely related to perturbation frequency. Equation (18) is compared to the observations of Gomberg and Davis [1996] by assuming that our $\eta_{\text{min}}$ value from static loading applies for all $f$ (Figure 14). The static and total strain thresholds of Gomberg and Davis [1996] are derived from observations of triggered seismicity and by a lack of tidal triggering. Their thresholds are inversely proportional to $f$, with a proportionality constant equivalent to $\eta_{\text{min}}=1.0003$. We note that the inverse scaling between $\epsilon_T$ and $f$ is consistent with higher $\eta_{\text{min}}$ for lower $f$. This may indicate higher effective $D_{c}$ and/or lower $m$ for lower $f$.

Using (17) with $\eta_{\text{min}}=1.0003$, we can calculate the ranges of $K, a-b$, and $D_{c}$ values which would give rise to a threshold close to that of Gomberg and Davis [1996] (Figure 14). To obtain an $\eta_{\text{min}}$ value consistent with field estimates for $D_{c}$ and $(a-b)$ (1 mm to 10 cm and 1.5x10^{-7}), respectively, stiffnesses must be in the range 10^{-1} - 10^{-2} MPa/m (Figures 13d and 13e), which is consistent with geologic faults having low effective stiffness [e.g., Walsh, 1971].
Figure 13. (a)-(c) Comparison of predicted \( \eta_{\text{min}} \) values from equation (16) (open circles), with empirically determined values from simulations (solid circles). The solid circles in Figure 13a correspond to \( \eta_{\text{min}} \) with fixed stiffness and constitutive parameters but changing mass (data for \( K/K_0 = 0.9 \) in Figure 7). In Figure 13b all parameters except the friction parameter \( a \) are fixed (data in Figure 10). In Figure 13c all parameters except the critical slip distance \( D_c \) are fixed (data in Figure 8). The friction parameters and preseismic slip \( U_p \) used in equation (16) to obtain predicted \( \eta_{\text{min}} \) correspond to those in the simulations. (d) and (e) Predicted values of \( \eta_{\text{min}} \) from equation (17) for a range of stiffnesses \( K \) (in MPa/m), with varying friction parameter \( a \) (Figure 15d) and characteristic slip distance \( D_c \) (Figure 13c). The parameter values in Figure 13d are \( D_c = 0.2 \) m, \( b = 0.01 \), \( \sigma = 100 \) MPa, and \( m=10^7 \) kg/m². The parameter values in Figure 13e are \( a = 0.006 \), \( b = 0.007 \), \( \sigma = 100 \) MPa, and \( m=10^7 \) kg/m². The hatched lines show field estimates of a coseismic static stress threshold for triggered seismicity [Reusenberg and Simpson, 1992]. Note that to obtain \( \eta_{\text{min}} \) values comparable to the field estimates, we would require low effective stiffnesses for a wide range of \( a \) and \( D_c \) values.

Conclusions

We investigate the effect of friction parameters, constitutive laws, and inertia on earthquake nucleation through simulations of instabilities in a one-dimensional model of a homogeneous fault. To trigger instabilities, we use a stress perturbation which corresponds to static stress changes due to nearby earthquakes. Preseismic motion consists of two distinct phases: quasi-static slip when velocity is less than a limiting value, \( V_{\text{q-s}} \), and quasi-dynamic slip as inertial effects become more important. During dynamic motion, velocity exceeds the limiting velocity for inertia-driven motion, \( V_{\text{in}} \). From the equations of motion of the system, we obtain quantitative definitions and estimates of these velocities for crustal faults: \( V_{\text{q-s}} = 10^{-9} \) m/s and \( V_{\text{in}} = 10^{-3} \) m/s. In the context of our modeling, the amount and duration of preseismic slip are governed by fault frictional properties, stiffness, and inertia. The preseismic duration scales directly with mass, and the amount of preseismic slip decreases with mass. Both the amount and duration of preseismic slip increase with the ratio of system stiffness to a critical stiffness and the friction constitutive parameters \( a \) and \( D_c \). For a wide range of parameters, we find a log-linear relationship between preseismic duration and the stress perturbation. This result is consistent with empirically observed relationships between the rate of aftershock (or foreshock) occurrence and time since (or prior to) the mainshock [Jones and Molnar, 1979], and with simulations of instabilities in systems without inertia [Dieterich, 1986]. In contrast to purely quasi-static analysis, our modeling indicates that triggered instabilities require a minimum static stress threshold, which depends on the system stiffness and constitutive parameters. We find that the stress threshold decreases and preseismic slip and duration are longer for an initially locked and overstressed fault. In order to match field-based observations of stress thresholds for triggered seismicity, we require fault stiffnesses of order \( 10^{-1} \) to \( 10^{-2} \) MPa/m.
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