ON THE RATE OF FRICTIONAL HEALING AND THE CONSTITUTIVE LAW FOR TIME- AND SLIP-DEPENDENT FRICTION

Chris Marone

Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139

ABSTRACT

The rate of frictional strengthening (healing) during quasi-stationary contact plays an important role in many aspects of repetitive stick-slip sliding. Observations of frictional healing are also a key feature of a class of slip-rate and state-variable friction constitutive laws widely-used to model earthquake faulting. This paper describes a suite of laboratory experiments to measure healing rate in simulated fault gouge. Modeling is carried out to critically evaluate rate and state friction laws. Experiments consisted of both slide-hold-slide tests, in which surfaces were sheared at a specified velocity and held in quasi-stationary contact for a given time, and velocity stepping tests in which step changes in load point velocity were imposed to evaluate friction constitutive parameters. The study addresses two main issues: 1) variation in healing rate with loading velocity, and 2) consistency of friction parameters and constitutive behavior as observed in slide-hold-slide and velocity-step tests. Experiments were performed in the double-direct shear geometry, at room temperature, and at constant normal stress of 25 MPa. Gouge consisted of quartz sand and was sheared within rough Westerly granite surfaces. Surface contact dimensions were 10 cm × 10 cm, the gouge layers were 2.5 mm thick prior to shear, and load point velocities ranged from 0.5–100 μm/s. Velocity stepping tests indicated velocity weakening (the steady-state coefficient of friction decreased with slip rate). In slide-hold-slide tests, friction decayed during holds and exhibited a peak value (taken as the coefficient of static friction) upon reloading. Static friction increased approximately linearly with log hold time. In addition, static friction and the rate of frictional healing varied with loading velocity. Friction relaxation during holds also varied with hold time and loading velocity. The data indicate that healing is not simply a frictional property, but rather is a system response that varies with loading velocity and properties of the elastic loading system. Constitutive modeling is carried out on three aspects of the data: slide-hold-slide tests, velocity stepping tests, and the rate of frictional healing. Friction parameters are determined by modeling each data set and these are compared to evaluate internal consistency. Both slip- and time-dependent state evolution laws (Ruina and Dieterich laws, respectively) are evaluated. The modeling indicates that individual tests are fit well by either the Ruina or Dieterich laws. However, slide-hold-slide tests indicate that the friction parameter a is larger than b, whereas velocity step tests indicate the opposite. Velocity step tests predict less friction relaxation and faster healing than observed in slide-hold-slide tests. These discrepancies may indicate variation in constitutive parameters as a function of slip velocity and/or limitations in the rate and state friction laws as presently written.
KEYWORDS
Friction • healing • constitutive laws • granular fault gouge • fault healing • earthquakes

INTRODUCTION
The rate at which frictional surfaces strengthen is of fundamental importance for many problems in mechanics, particularly those involving repetitive stick-slip. For earthquakes and faulting, the seismic cycle of repeated failure requires that faults restrengthen (heal) during the interseismic period, and the rate of healing plays a key role in determining fault strength, seismic stress drop, earthquake repeat time, and the mode of dynamic rupture propagation (Kanamori, Allen 1986; Scholz et al. 1986; Marone et al. 1995; Perrin et al. 1995). However, the rate of healing and the underlying deformation mechanisms are poorly understood.

Laboratory measurements on rock surfaces and simulated fault gouge indicate that frictional healing (defined as the time dependent increase in static friction) is approximately linearly with log time during quasi-stationary contact (Dieterich 1972; Beeler et al. 1994; Karner et al. 1997). This is generally consistent with seismic estimates of fault healing (Marone et al. 1995). However, in the laboratory the friction increases by only a few percent of its absolute value per decade in time, whereas seismic stress drop increases by a factor of 2 to 5 per decade increase in earthquake recurrence interval (Kanamori, Allen 1986; Scholz et al. 1986; Marone et al. 1995). The apparent discrepancy may arise from a number of sources. For example, it may be attributed to differences in the time scale and physical conditions of laboratory experiments and tectonic faulting (Scholz 1990), assumptions about the application of laboratory measurements of friction velocity dependence to fault healing (Cao, Aki 1986), or to difficulties in evaluating and comparing the healing rate of different seismogenic faults (Marone et al. 1995). Each of these factors may be important, however, previous studies have proceeded by direct comparison of laboratory and seismic estimates of fault healing. This approach assumes that healing and time dependent variations in static friction are intrinsic frictional properties. However, as I show here, healing varies with loading velocity. Similar effects have been demonstrated by Johnson 1981 and Beeler et al. 1994. These data indicate that healing is a system response, for which modeling must be carried out to recover the intrinsic constitutive parameters for comparison between laboratory and field conditions. Such modeling of frictional healing also provides an opportunity for comparison with parameters obtained from velocity stepping tests. Since laboratory measurements of frictional healing played a key role in the development of rate and state friction constitutive laws, this comparison may be a useful approach for critical evaluation of the laws.

The purpose of this paper is 1) to present laboratory measurements of the effect of loading velocity on static friction and the healing rate of simulated faults and 2) to use these data to critically evaluate rate and state dependent friction laws. The laboratory data indicate a trade-off between healing time and initial slip rate, with faster slip producing larger static friction for a given contact time. Such behavior is predicted by rate/state friction laws. However, detailed comparison of the friction parameters determined from different types of tests indicates a lack of internal consistency. The healing rate predicted by constitutive parameters obtained from velocity-step tests is faster than observed during relaxation tests. This may be attributed to changes in the underlying deformation mechanisms as a function of velocity or to limitations in the form of the friction laws as they are currently written.

EXPERIMENTS
Friction experiments were carried out on rock and granular fault gouge (used to simulate breccia and wear material within fault zones) in a servocontrolled testing apparatus. The apparatus consists of two perpendicular load frames fitted with hydraulic rams capable of producing forces up to 1 MN. Each ram is controlled by a high-speed servo capable of running in load or displacement feedback. The experiments described here were performed in displacement feedback control to produce a constant loading rate.

To simulate natural fault roughness, gouge layers of granular quartz powder (initial particle size 50–150 μm) were sheared within granite surfaces roughened by sandblasting (rms. roughness =200 μm). Shear was imposed in the double-direct-shear configuration by controlling displacement of a load point in contact with a central sample block as it was forced between two contacting blocks (Figure 1). Nominal frictional contact dimensions were 10 cm × 10 cm and gouge layers were initially 2.1 mm thick. Normal stress on the fault planes was held constant at 25 ±0.1 MPa and changes in layer thickness were recorded continuously to a precision of 0.1 μm. Shear stiffness of the apparatus is 225.0 MPa/cm expressed as shear stress per unit shear displacement, or 9.0×10^4/μm expressed as a change in the coefficient of friction per shear displacement for the normal stress used.

RESULTS

General characteristics of the friction and porosity data are consistent with previous studies (Dieterich 1981; Biegel et al. 1989; Marone et al. 1990; Marone, Kilgore 1993; Beeler et al. 1996). Sliding friction evolved to a steady value over 3–6 mm of slip and underwent a transition from velocity strengthening (the steady-state coefficient of sliding friction increased with slip velocity) to velocity weakening at engineering shear strains γ of 4–5. The critical slip distance, D_c (a measure of the slip or shear strain needed to reach a new steady-state friction upon a perturbation in slip rate) also evolved over this interval. Gouge layers thinned rapidly during initial shear due to comminution and compaction (Scott et al. 1994), but once friction reached steady-steady, thinning became linear with slip reflecting geometric spreading (Biegel et al. 1989; Scott et al. 1994). In the experiments described here, loading velocities ranged from 1–100 μm/s and hold times from 5 s to 10^4 s. The shortest hold time was set by precision of the shear stress measurement, which limited resolution of the coefficient of friction to about 1×10^-4, and the longest by transducer drift and thermal fluctuations, since beyond about 10^5 s it was not possible to obtain data of the same quality and reproducibility as for shorter hold times (see Figure 1).

Frictional healing was measured in “slide-hold-slide” experiments in which: 1) shear is imposed at a given velocity until sliding friction reaches steady-state, 2) the load point velocity is set to zero for a given time (hold), and 3) loading resumes at the initial velocity (Figure 1). Thus, prior to holds surface slip velocity was equal to load point velocity. Upon reloading, slip velocity lags load point velocity and, as discussed below, elastic modeling must be carried out to recover slip history. Shear stress and frictional strength decayed during holds (Figure 1). Upon renewed shearing, friction exhibited a distinct maximum before evolving back to its initial value (Figure 1). This friction maximum is traditionally taken as the coefficient of “static” friction (e.g., Dieterich 1972), despite the fact that creep occurs during the hold.

I measured frictional healing Δμ as the difference between static and initial friction (Figure 1). Static friction and Δμ increase with hold time and loading velocity. The slip needed to reestablish steady-state friction scales with Δμ (Figure 1). Figure 1 also shows the effect of an instantaneous change in load point velocity. Friction first decreased upon a reduction in load point velocity and then increased, evolving to a
new, higher steady-state value and hence indicating frictional velocity weakening. The effect of loading velocity on static friction during holds was significantly larger than on steady-state sliding friction (Figure 1).

The data of Figure 1 are typical of the steady state portion of the experiments. A number of repeat experiments were performed to test for systematic effects of load history and net displacement. These indicated that $\Delta \mu$ and $\Delta \phi$ did not depend on the order in which hold time and loading velocity were varied. The data show that healing rate and static friction (for a given hold time) increase systematically during initial slip, but reach a steady-state condition after $\gamma$ of 4–5. To further reduce the possible affects of net displacement, detailed measurements were carried out within a narrow displacement range after steady-state friction conditions had been reached in each experiment.

Frictional healing varied systematically with initial slip velocity over a range of hold times spanning three orders of magnitude (Figure 2). I define the healing rate as $\beta = \Delta \mu / \Delta \log t_h$, where $t_h$ is hold time, and find that it increased systematically with loading velocity. Fluctuations in the absolute value of friction and in the displacement necessary to reach steady state preclude comparison between experiments, consistent with previous results (Dieterich 1972; Scholz 1990 Fig. 2.18). For a given hold time, $\Delta \mu$ and $\beta$ increased by 0.005–0.01 and 0.001–0.002, respectively, for a 10x increase in load velocity. For the data of Figure 2, healing rate $\beta$ was in the range 0.007–0.012.

In addition to slide-hold-slide tests, velocity step tests were performed to interrogate friction constitutive behavior (Figures 1 and 3). In these tests (e.g., Dieterich 1981), the load point velocity is suddenly changed and the transient and steady-state friction response modeled. Figure 3 shows the friction displacement record for a portion of the data shown in Figure 2c. At a displacement of about 17.2 mm the load point velocity was increased from 1 to 10 $\mu$m/s and, after the healing tests at 10 $\mu$m/s, changed back to 1 $\mu$m/s at a displacement of about 23 mm (Figure 3). Velocity step tests indicated steady-state velocity weakening (Figures 1 and 3).

**MODELING AND EVALUATION OF FRICTION CONSTITUTIVE LAWS**

Since the pioneering work of Dieterich 1978; 1979 and Ruina 1983, laboratory studies have focused almost exclusively on velocity step tests, and the transient change from one steady-state friction to another, for the determination of rock friction constitutive parameters (e.g., Tullis , Weeks 1986; Marone et al. 1990; Beeler et al. 1996). While this is a valid approach, which handles the important problem of quantifying friction behavior independent of the experimental apparatus used, equivalent information can be obtained from slide-hold-slide tests. In this case, frictional decay and strengthening are modeled as the system is perturbed first away from steady state (during the hold) and then back toward steady state (during the reload). Slide-hold-slide tests provided some of initial data that led to rate and state friction laws (e.g., Dieterich 1972) and recently to an improved understanding of state evolution (Beeler et al. 1994), however, these studies have traditionally focused solely on static friction and $\Delta \mu$, without elastic modeling to account for finite apparatus stiffness and to recover individual constitutive parameters. In this section, I introduce such modeling and compare the results with those obtained from velocity step tests.

The friction laws are rate- and state-dependent constitutive relations in which the coefficient of friction $\mu$ is written in terms of a constant $\mu_o$, appropriate for steady-state slip at velocity $V_o$, a velocity dependent term, and a term to describe hysteresis or memory effects, involving the critical slip distance $D_c$ and an
evolving state variable $\theta$:

$$
\mu = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V \theta}{D_c} \right),
$$

(1)

where $a$ and $b$ are empirical constants. The full constitutive relation is specified by coupling Eqn. 1 with time and velocity-dependent evolution of frictional state $\theta$. Two such state evolution laws have been proposed. In the first, proposed by Dieterich (Dieterich 1978; 1979), friction evolves during truly stationary contact, whereas in the second, proposed by Ruina (Ruina 1983), friction evolves only during slip at finite velocity:

$$
\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c},
$$

Dieterich Law

(2a)

$$
\frac{d\theta}{dt} = -\frac{V \theta}{D_c} \ln \left( \frac{V \theta}{D_c} \right),
$$

Ruina Law

(2b)

To model friction data, Eqn. 1 and 2 are coupled with a relation describing elastic loading of the sliding surfaces. The stiffness of the laboratory testing apparatus and sample is sufficiently high (and the frictional surfaces sufficiently small) that the surfaces may be assumed to slip as rigid units. Also, loading velocities are sufficiently low that inertia may be ignored and thus a single-degree-of-freedom elastic relation is used:

$$
d\mu/dt = k (V_i - V),
$$

(3)

where $k$ is stiffness divided by normal stress and $V_i$ is load point velocity. The resulting set of equations (1–3) is solved numerically, with $V_o$ taken equal to $V_{ip}$, to yield friction curves that vary with the empirical parameters $a$, $b$, and $D_c$. At steady-state $\theta=D_c/V$, and thus the steady-state velocity dependence of friction is, from Eqn. 1, $a-b=\Delta \mu/\ln(V/V_o)$. Depending on the sign of $a-b$, steady-state friction either increases (velocity strengthening) or decreases (velocity weakening) with increasing slip velocity. In some cases it is found that two state variables are required to fit data and then $\theta$ and $D_c$ take on the subscript $i=1, 2$ in Eqn. 1 and 2. For convenience, I omit subscripts when only one state variable is used and thus in the figures and discussion that follows, $D_c$ and $b$ are equivalent to $D_{ci}$ and $b_i$, respectively.

The lower panels of Figure 3 show data from the first slide-hold-slide test at each hold time from Figure 3a, along with model fits obtained by solution of equations (1–3). An iterative, non-linear inversion was performed to obtain least-squares best fit models. For velocity-step experiments friction-displacement records are typically modelled. Equivalently, one could model friction-time records and I use this approach for slide-hold-slide tests. Both the Dieterich and Ruina laws provide good fits to the data, including the relaxation and reload portions of the data (Figure 3). However, in each case the fits indicate $a > b$. This is contrary to the velocity step tests conducted both before and after the hold tests, which indicate velocity weakening and thus $b > a$ (Figure 3). Forward modeling and a series of repeat inversions using different starting models indicate that an acceptable fit to the slide-hold-slide data
cannot be obtained with \( b > a \).

To compare constitutive parameters for the two types of tests, least-squares inversions were also performed for the velocity steps (Figure 4). In Figure 4a, I show data for a 10x velocity increase along with the best-fit Dieterich and Ruina models. One-state-variable models provide reasonable overall fits to the data, however, neither the initial peak nor the first oscillation are well fit (Figure 4a). Also shown in Figure 4a are model fits obtained using the constitutive parameters determined from modeling of slide-hold-slide data. As expected, the slide-hold-slide parameters do not produce acceptable fits, missing the post-peak drop in friction and predicting steady-state velocity strengthening. Two-state-variable models fit the data significantly better. For these inversions, the parameters \( a, b_f \) and \( D_{cf} \) are consistent with those obtained from the slide-hold-slide tests.

The constitutive parameters obtained from modeling of velocity-step and slide-hold-slide tests can also be used to simulate measurements of the healing parameters \( \Delta \mu \) and \( \beta \). In Figure 5 I show the data of Figure 2 along with simulations obtained by numerical solution of equations 1–3. Simulations were carried out using the averaged slide-hold-slide parameters and the best-fit one- and two-state-variable model parameters from velocity step tests. In each case, frictional healing is greater for the higher velocity (Figure 5). For the shorter hold times, \( \Delta \mu \) is fairly well matched, but \( \beta \) is not well matched anywhere, with the possible exception of the Ruina law simulation for 10 \( \mu m/s \) obtained using the one-state-variable parameters (Figure 5b). Another feature of the data to be fit is the velocity effect on \( \Delta \mu \) at a given hold time. This is fairly well fit by the slide-hold-slide parameter simulations (Figure 5a), however, this is to be expected given that the parameters were obtained from this type of test. In general, the Ruina law provides better fits than the Dieterich law, which tends to predict significantly higher \( \beta \) and \( \Delta \mu \) than observed.

As seen in the friction data of Figures 1 and 3, healing affects not only static friction and \( \Delta \mu \) but also the amount of frictional relaxation during holds \( \Delta \mu_{min} \). In Figure 6 I show the measured frictional relaxation \( \Delta \mu_{min} \) as a function of hold time for two experimental runs. For comparison, the \( \Delta \mu \) data are also shown at the same scale. \( \Delta \mu_{min} \) increases linearly with log time and is similar in magnitude to \( \Delta \mu \) (Figure 6).

These data may also be simulated using the rate/state friction laws (Figure 7). Simulations based on the fits to slide-hold-slide tests match \( \Delta \mu_{min} \) and its change with hold-time fairly well (Figure 7a), which is to be expected given that slide-hold-slide tests are dominated (in time) by relaxation data. However, simulations based on the velocity-step parameters do not fit data well. Both laws predict lower \( \Delta \mu_{min} \) and faster healing than observed, consistent with their overprediction of \( \Delta \mu \) (Figure 5). The mismatch between observed and predicted \( \Delta \mu_{min} \) indicates that creep during simulated holds is arrested too quickly and thus healing is too abrupt.

**DISCUSSION AND CONCLUSIONS**

It is evident from the data presented that frictional healing and “static” frictional strength are not single-valued functions of the time of quasi-stationary contact, but rather vary with hold time and slip velocity in a systematic way. The data indicate a trade-off between hold time and loading velocity, such that a given static friction is attained in a shorter time for higher loading velocity. What would produce such behavior and what friction laws can describe it? Certainly this is not a feature of static-dynamic or slip-weakening friction laws, which do not predict any type of healing. Simple time-dependent healing
could be produced with a purely-velocity dependent friction law, however, this would not yield velocity-dependent healing nor the transient effects observed during frictional relaxation and subsequent reloading. On the other hand, such behavior is included in rate and state friction laws. These laws were initially introduced to model velocity-step tests and steady-state friction velocity dependence (Dieterich 1979, 1981; Ruina 1983) but, as shown here, they are also capable of describing time- and velocity-dependent frictional healing.

Both the friction data and the rate/state friction simulations indicate higher static friction and healing rates for larger loading velocity. From a glance at Eqn. 1 this may appear to arise from the $\ln(V/V_o)$ term in a manner similar to the direct change in friction observed in velocity step experiments. However, $\Delta \mu$ is measured at the peak of the friction curve and thus at a point where $d\mu/dt=0$, which from Eqn. 3 indicates that load point and slider velocity are equivalent. Since $V_o$ may be taken as $V_{lp}$, the $\ln(V/V_o)$ term of Eqn. 1 vanishes at peak friction. Thus the velocity dependence of healing is not akin to the direct effect for velocity steps. Rather, the effect arises from state evolution during the hold and reload. Frictional state evolves via slip, and in the case of the Dieterich law time, and thus the velocity effect on $\Delta \mu$ arises from having greater creep during holds that start with larger velocity, and an opposite effect upon reloading. That is, degradation in frictional state upon reloading is minimized for larger loading velocity and thus peak friction is larger. The concave upward curvature of the healing curves (e.g., Figure 5) also arises in this manner. For hold times that are short compared to the time of the reload (from minimum to peak friction), time- and slip-dependent increases in frictional state during holds are erased during reloading and thus the peak friction does not accurately reflect frictional healing. This effect diminishes with increasing hold time and thus $\beta$ increases.

The constitutive modeling carried out here indicates that individual slide-hold-slide tests and velocity-step tests are well fit by rate and state friction laws. However, under the assumption that the underlying deformation mechanisms and constitutive behavior are the same for the two types of tests, the comparative modeling indicates a lack of internal consistency. Two basic problems are evident. 1) Constitutive parameters determined from short duration hold tests ($10^1-10^2$ s) cannot be successfully extrapolated to longer hold times. 2) The steady-state velocity dependence of friction, $a-b$, as determined from velocity step tests and slide-hold-slide tests is opposite in sign, with the former indicating velocity weakening and the latter velocity strengthening.

A key feature of the discrepancy between the two types of modeling involves the frictional relaxation $\Delta \mu_{\min}$ during holds. The data indicate that $\Delta \mu_{\min}$ is similar in magnitude to $\Delta \mu$ for a given hold time. This is in agreement with previous observations from rock-on-rock sliding experiments (Johnson 1981; Dieterich, Conrad 1984; Beele et al. 1994) and thus is unlikely to be an artifact of experimental design. However, in order for $\Delta \mu_{\min}$ and $\Delta \mu$ to be of similar magnitude, the friction parameter $a$ must be larger than $b$. This is because friction evolves fairly quickly for both the Dieterich and Ruina evolution relations (Eqn. 2) and thus for $b > a$, the frictional surfaces strengthen quickly with slip, arresting creep. The same problem is indicated by the discrepancy between extrapolations of the healing data. Simulations based on short holds predict greater healing than observed. This may indicate variations in the underlying deformation mechanisms and friction constitutive parameters as a function of slip velocity and/or limitations in the rate and state friction laws as presently written. These issues cannot be readily addressed with the currently data and are being investigated with additional experiments and modeling.

Despite these limitations, constitutive modeling using the rate and state laws provides a useful framework for analysis of frictional healing. The modeling presented here indicates that healing, like frictional
stability, is a system response, dependent upon the combined effects of friction properties, slip velocity and loading rate, and elastic coupling to the surroundings. Thus, measurements of healing $\Delta \mu$ and healing rate $\beta$ must be scaled appropriately for comparison to different conditions. For the problem of healing on seismogenic faults, this will likely involve incorporating the effects of both seismic slip rates, during earthquake rupture, and tectonic loading rates.

ACKNOWLEDGEMENTS

This work was supported by NSF grants EAR-9316082 and EAR-9627895 and by the Kerr-McGee foundation through a career development chair at MIT.

FIGURES
Paper 187, Figure 1.

Figure 1. Typical friction data for "slide-hold-slide" tests. Data show healing for two load point velocities (1 and 3 \( \mu \text{m/s} \)) and two hold times (10 and 100 s). Times shown below data indicate positions at which loading was stopped and surfaces were held in quasi-stationary contact. Shear strength relaxes during holds due to creep and finite apparatus stiffness. Frictional healing \( \Delta \mu \) is taken as the increase in peak friction relative to the initial value of sliding-friction. Healing increases with hold time and initial sliding velocity. The decrease in friction during relaxation \( \Delta \mu_{\text{min}} \) also scales with hold time and velocity. Inset shows experimental configuration in which two gouge layers are sheared simultaneously in double-direct-shear. The testing apparatus consists of two independently servocontrolled load axes.
Paper 187, Figure 2.

Figure 2. Increase in static friction during holds is shown for three experiments. Plot symbols indicate loading velocity (see Figure 1). The data indicate that frictional healing and static friction are a function of hold time and initial slip velocity. (a) Note that several data points plot over one another due to reproducibility, see Figure 1. Lines indicate best fit log-linear relations. Healing rates $\beta = \Delta \mu / \Delta \log t_h$ are: (a) 0.010 for 1 $\mu$m/s, 0.0114 for 3 $\mu$m/s, 0.0125 for 10 $\mu$m/s; (b) 0.0066 for 1 $\mu$m/s, 0.0077 for 10 $\mu$m/s; (c) 0.0093 for 1 $\mu$m/s, and 0.0096 for 10 $\mu$m/s.
Paper 187, Figure 3.

Figure 3. (a) Friction data showing velocity-step and slide-hold-slide tests. The velocity step tests indicate velocity weakening (the steady-state coefficient of friction decreases with velocity). Static friction increases with hold time. Continuous data are shown from 17 to 19 mm of displacement. The velocity step from 10 to 1 μm/s occurred at a displacement of 23 mm. (Panels b and c) Friction data are plotted vs. time for the first hold at each time in (a). Arrows indicate the beginning and end points of the hold. Plotted over the data are best-fit models for the Dieterich and Ruina friction laws determined by an iterative non-linear inversion. Constitutive parameters are given for each fit. Both the Dieterich and Ruina laws are capable of fitting the data quite well. However, note that the values indicate $a > b$, which would imply steady-state velocity strengthening in contrast to the data of (a). For the Dieterich law, the parameter $b$ varies somewhat with hold time, whereas $b$ is independent of hold-time for the Ruina law. Error estimates from the inversions indicate that one std is $< 7 \times 10^{-4}$ for $a$ and $b$ and $< 0.2$ μm for $D_c$. 
Figure 4. Friction data for a 1–10 μm/s velocity step (shown in Figure 3a) are modeled using three sets of constitutive parameters. (a) Upper models show fits based on the average constitutive parameters derived from the slide-hold-slide modeling shown in Figures 3b and 3c. The parameters used were: (Dieterich law) \( a=0.0123, b=0.0094, D_c=2.0 \) μm; (Ruina law) \( a=0.0122, b=0.0104, D_c=3.2 \) μm. Each model over-estimates the friction direct effect and indicates steady-state velocity strengthening in contrast to the data. Also shown in panel (a) are least-squares, best-fit models determined from an iterative non-linear inversion. These models fit the data reasonably well, although they underestimate the direct effect somewhat and miss the subsequent oscillations. The best fitting parameters are: (Dieterich law) \( a=0.0077 \pm 0.8 \times 10^{-4}, b=0.0086 \pm 0.7 \times 10^{-4}, D_c=4.0 \) μm ± 0.1 μm; (Ruina law) \( a=0.0075 \pm 0.9 \times 10^{-4}, b=0.0082 \pm 0.8 \times 10^{-4}, D_c=6.7 \) μm ± 0.1 μm. (b) The best fitting 2 state variable models are shown. The inversion was started with the 1-state variable parameters of the slide-hold-slide modeling given above. The data are fit well by the 2 state variable models. The best fitting parameters are: (Dieterich law) \( a=0.0126 \pm 9.7 \times 10^{-4}, b_1=0.0098 \pm 8.0 \times 10^{-4}, D_{c_1}=1.5 \) μm ± 0.2 μm, \( b_2=0.0036 \pm 2.6 \times 10^{-4}, D_{c_2}=6.8 \) μm ± 0.2 μm; (Ruina law) \( a=0.0110, b_1=0.0104 \pm 1.4 \times 10^{-4}, D_{c_1}=3.2 \) μm ± 0.1 μm, \( b_2=0.0017 \pm 0.4 \times 10^{-4}, D_{c_2}=60.2 \) μm ± 4.1 μm.
Paper 187, Figure 5.

Figure 5. Frictional healing data are compared with numerical simulations for the Dieterich and Ruina laws using three sets of constitutive parameters. In each case the upper set of lines are for a loading velocity of 10 μm/s and the lower lines are for 1 μm/s. Data and symbols are the same as in Figure 2c. (a) Numerical simulations using the average constitutive parameters derived from the slide-hold-slide modeling of Figure 3. Both laws fit the healing data at 10 μm/s for short hold times (10–100 s). However, the laws over-estimate healing and static friction increase for longer hold times. (b) Numerical simulations using the best-fit, 1-state-variable models as shown in Figure 4a. (c) Numerical simulations using the best-fit, 2-state-variable models as shown in Figure 4b.
Paper 187, Figure 6.

Figure 6. Comparison of velocity- and hold-time-dependence of $\Delta \mu$ and $\Delta \mu_{\text{min}}$ for two experiments. Plot symbols indicate slip velocity prior to the hold. Note that $\Delta \mu_{\text{min}}$ increases linearly with log hold time, that it varies with initial slip velocity, and that it is similar in magnitude to $\Delta \mu$. 
Paper 187, Figure 7.

Figure 7. The minimum friction values of Figure 6a are compared with numerical simulations for the Dieterich and Ruina laws using three sets of constitutive parameters. In each case the upper set of lines are for an initial sliding velocity of 10 μm/s and the lower lines are for 1 μm/s. (a) Numerical simulations using the average constitutive parameters derived from the slide-hold-slide modeling of Figure 3. Both laws fit the Δμ min data reasonably well for short hold times (10–300 s). However, the laws underestimate the frictional relaxation rate and thus do not match the data for longer hold times. (b) Numerical simulations using the best-fit, 1-state-variable models as shown in Figure 4a. (c) Numerical simulations using the best-fit, 2-state-variable models as shown in Figure 4b. In panels (a)-(c) the mismatch between data and simulation indicates that the constitutive laws predict faster healing than observed.
References


