
Effects of Normal Stress and Humidity on Frictional Strength of Granular Fault Concrete, Type and Rate Posters.
The Stick-Slip Mechanism

Rate & Static Friction Constitutive Laws

Motivation, recall:

\[
\begin{align*}
& \text{Time-dependent static friction} \\
& \text{Velocity-dependent sliding friction} \\
& \text{Memory-effects, static dependence} \\
& \text{Repetitive stick-slip instability}
\end{align*}
\]

Generalization of slip weakening friction model:

\[
\mu_s - \mu_d \Rightarrow \mu(v, \phi) \Rightarrow \mu(v, \phi)
\]

\Rightarrow \text{Distinguish between } \mu_s \text{ & } \mu_d \text{ artificial}

Laboratory Observations, Constitutive Laws \Rightarrow \text{One set of relations to describe complete seismic cycle}

Velocity jump tests show transient strengthening for a velocity increase, weakening for a decrease, and steady-state changes that can be either velocity weakening or velocity strengthening for different materials.

![Graph showing coefficients of friction and porosity as a function of shear strain.](image)
Rate & State Friction Laws

1) \[ \mu(V, \theta) = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V \theta}{D_e} \right) \]

Reference value of base friction

State Variable:
Characterizes physical state of the surface

Time evolution, Dieterich Law

One of several possible laws

\[ \frac{d\theta}{dt} = \frac{1 - \frac{V \theta}{D_e}}{t} \]

(1) Implies

\[ \frac{d\theta}{dt} = \left( \frac{V}{V_0} \right) \frac{d\mu}{d\ln V} \]

Direct Effect
Evolution Effect
Slip
Fading memory of past state

Steady-state sliding: \[ \theta_{ss} = \frac{D_e}{V} \]
\[ \Rightarrow \frac{d\theta}{dt} = 0 \]

Then (1) becomes

\[ \mu - \mu_0 = (a - b) \ln \left( \frac{V}{V_0} \right) \]

\[ (a - b) = \frac{\Delta \mu}{\Delta \ln V} \]
Convention is to use \( a, b \) for friction of \( \Theta, \beta \) for stress, so that (1) could be written as

\[
\tau = \tau_0 + A \ln \left( \frac{V}{V_0} \right) + B \ln \left( \frac{V_0 \Theta}{D_c} \right) \quad ; \quad \tau \equiv \text{Frictional stress}
\]

\[
A - B = \frac{\alpha}{\Delta \ln v}
\]

steady-state velocity strengthening

if \( a - b > 0 \), velocity weakening for \( a - b < 0 \)

Rate Dependence of Steady-State Sliding Friction

\[
\begin{align*}
\tau & \text{ vs. } \log v \\
\uparrow & \quad \text{velocity strengthening} \\
\downarrow & \quad \text{velocity weakening}
\end{align*}
\]

\( a \neq b \) are small, dimensionless constants determined from experimental measurements. Both in the range \( 10^{-3} \) to \( 10^{-2} \)

\( D_c \) has units of length.

Modelling Experimental data:

1) \( \mu = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \Theta}{D_c} \right) \)

2) \( \frac{d\Theta}{dt} = 1 - \frac{V \Theta}{D_c} \)

Elastic Coupling

3) \( \frac{d\Delta}{dt} = K (V_p - V) \)

Solve 3 & 2 as simultaneous equations, with 1 as a constraint.
Rearrange (1) & use in (7) to determine $\Theta$ history from (2):

$$V = V_o \exp \left[ \frac{-\mu_o - b \ln \left( \frac{V_o \Theta}{D_o} \right)}{a} \right]$$

Solve:

$$\frac{\partial (\tau / \Theta)}{\partial t} = \frac{\mu}{V} \left( V_p - V_o \exp \left[ \frac{-\mu_o - b \ln \left( \frac{V_o \Theta}{D_o} \right)}{a} \right] \right)$$

$$\frac{\partial \Theta}{\partial t} = 1 - \frac{V \Theta}{D_o}$$

Fit data

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Parameter values
- same in both cases.

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All at 25 MPa $\tau_{\text{in}}$

Typical $\mu$ is $10^{-3} \text{ mm}^{-1}$

Example of 2-side variable behavior

$D_o \sim 1-5 \mu$m for bare rock

$\mu \sim 10^{-6} \text{ Pa}$ for granular fault gouge
State Evolution Laws

Slip vs. Time evolution: evolution at \( v = 0 \)

**Slip-dependent evolution**

**Ruina Law**

\[
\frac{d\theta}{dt} = -\frac{v\theta}{D_c} \ln \left(\frac{v\theta}{D_c}\right)
\]

No evolution at \( v = 0 \).

\[
\lambda = 1 \times 10^{-11} \text{ m}^{-1}
\]

\( a = 0.01 \)

\( b = 0.015 \)

\( D_c = 1 \times 10^{-6} \text{ m} \)

Symmetry in displacement record for Ruina law, most data show such symmetry although some variation is not always possible to distinguish.

Come back to this after we look at time-dependent changes in state healing.

**Mair and Marone: Friction of Fault Gouge at High Velocity**


\[
\mu = \mu_0 + a \ln\left(\frac{v}{v_0}\right) + b \ln\left(\frac{v\theta}{D_c}\right)
\]

\[
\frac{d\theta}{dt} = 1 - \left(\frac{v\theta}{2D_c}\right)^2
\]
Time (& Velocity) Dependent Healing

Observations:

Stressed vs. Unstressed Aging:

Slide-hold-slide Experiment

Coulomb (1785) studied unstressed aging (and very few since then). Karrer & Maute, 1998, 2000, most studies consider stressed aging.

Typical data

Healing

$\frac{\Delta \mu}{\log t}$
Figure 4: Karner and Marone: submitted to JGR 16 Aug. 2000
Figure 5: Karner and Marone: submitted to JGR 16 Aug. 2000
Modeling Frictional Healing.

- Simulations, phase plane \( \mu - \ln V \)
- Healing rate, slip rate dependence, Derivation from rate/steady laws
- Details of time, slip history during a slide- hold- slide

Phase plane plot shows variation in slip velocity, friction and state during a hold test

Friction Law
\[ \mu = \mu_o + a \ln(\frac{V}{V_o}) + c \ln(\frac{V_o \theta}{D_c}) \]

State Evolution
\[ \frac{d\theta}{dt} = 1 - V\theta/D_c \]

Elastic Coupling
\[ \frac{d\mu}{dt} = k(V_p - V) \]

Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution.
Numerical simulations.
Effect of loading rate on healing

\[ V_{slip} \]

- Derivation of healing rate from rate/state laws.

Friction Law
\[ \mu = \mu_0 + a \ln(V/V_p) + b \ln(V/V_0/D_c) \]

State Evolution
\[ \frac{d\theta}{dt} = 1 \cdot \frac{V \theta}{D_c} \]
\[ \frac{d\theta}{dt} = -\frac{V \theta}{D_c} \ln\left(\frac{V \theta}{D_c}\right) \]

Elastic Coupling
\[ \frac{d\mu}{dt} = k(V_p - V) \]

Normalize by \( t \), then slope should be \( (1 \text{ in } 10) \)

Current rate/state laws include the effect of loading rate on frictional healing

Peak Friction (static value)
\[ \frac{d\mu}{dt} = \mu_p(V_p - V) = 0 \text{ at peak} \]
\[ V = V_p \text{ (see phase plane plot)} \]

Take \( V_0 = V_p \)

\[ \mu = \mu_p + a \ln\left(\frac{V}{V_p}\right) + b \ln\left(\frac{V}{V_0/D_c}\right) \]

\[ \Delta \mu = \mu - \mu_p = b \ln \left( \frac{V}{D_c} \right) \text{ constant} = \frac{V}{D_c} \]

But \( \frac{d\theta}{dt} = 1 - \frac{V \theta}{V_0/D_c} \approx 0 \text{ for } V \ll 1 \)
thus, \[ \Delta u = b \ln(t) \] - slope of healing curves is b for Dieterich law

Can also model full friction behavior during hold, including creep, relaxation, peak friction and re-adjustment to steady sliding.

Dieterich law simulation: \[ a > b \] Implies velocity strengthening

Ruina law simulation: \[ a < b \] Implies velocity weakening

Best fits for a range of hold times.

\[ a = 0.0122 \]
\[ b = 0.0104 \]
\[ D_c = 7.2 \mu m \]

Figure 8. Maione, IJRMS 1991

Comparison of data using a single set of constitutive parameters.
Healing rate is $b \times 2.70 \ldots$

$b's$ are avg. values from previous page:
- $7.8 \times 10^{-7}$ Dicknick
- $9.9 \times 10^{-7}$ Ruma

Static Frictional Strength and Healing Rate Vary with Loading Rate

Experimental measurement of slip rate effect on static friction & healing.
Comparison Between Data and Predictions of Rate and State Friction Laws

Constitutive Parameters from Velocity-Step Tests

- Data
- $V_v$ (µm/s)
  - 1
  - 3
  - 10
  - 30
  - 100

Disterich, Ruina
- $a = 0.0081$
- $b = 0.0089$
- $D_0 = 6.1$ µm

Relaxation, $\Delta \mu$

Constitutive Parameters from Hold-Slide Tests

- Data
- $V_v$ (µm/s)
  - 1
  - 3
  - 10
  - 30
  - 100

Disterich, Ruina
- $a = 0.0087$
- $b = 0.0075$
- $D_0 = 4.6$ µm
- $D_0 = 20.3$ µm

- Humidity Dependence of Healing
- $D_0$, interpretation for bare surface, gauge
- Time - Inverse Velocity: Duality for healing data / rate dependence.
Figure 6. (a) Friction velocity dependence ($a-b$) as a function of shear displacement for velocities $0.001-10$ mm/s and $\sigma_n = 25$ MPa. Several tests are plotted for each condition. Data from Marone and Kilgore [1993] for velocity $0.001-0.01$ mm/s and $\sigma_n = 25$ MPa are superimposed. Note the transition from velocity strengthening to weakening with progressive slip. (b) Plot of ($a-b$) for the discrete displacement ranges shown as a function of upstep velocity. Lines indicate the trends at initial (2-4 mm, solid line) and final (18-20 mm, dashed line) displacements. Velocity has little influence on weakening for slip $> 5$ mm.

Figure 4. Critical slip displacement ($D_c$) as a function of upstep velocity for a range of experiments at $\sigma_n = 25$ MPa. $D_c$ is systematically larger as a function of increasing velocity.
Fig. 2.18 Comparison of the $\mu_s$-$t$ and $\mu_s$-$v$ relations by assuming that $\mu_s$ at holding at $t$ is equivalent to $\mu_s$ at a steady $v = V/t$. Data are from Dieterich (1978) and Scholz and Engelder (1976): (A) $\mu_s$-$t$, quartz sandstone, $\sigma = 18.7$ MPa, (B) $\mu_s$-$t$, Westerly granite, 1.96 MPa, (C) $\mu_s$-$v$, Westerly granite, 20 MPa, $\gamma = 5 \mu$m (Dieterich, 1978). Friction scales are offset because base friction is not reproducible.

Relation between time-dependent healing, and velocity-dependent sliding friction.

Healing scales as

$$\Delta \mu = b \ln(t)$$

Whereas steady state rate dependence scales as

$$\Delta \mu = (\sigma - b) \ln(v)$$

$\sigma' = 100$ MPa

$\Delta \mu = 1$ MPa/decade

Quartz gauge, $\sigma' = 25$ MPa

Rough granite surfaces
Origin of Aging Effect in Granular Quartz at 20°C: Humidity Effect

Humidity dependence of friction

Healing rate and steady-state friction dependence are affected by humidity.

Frye & Marone, MS in prep, 2000