Statement about authorship and working together on this exam:

For our problem sets, my sense is that there is significant value in working together at some level. Explaining things to someone else, or having them explain things to you is useful. Of course there’s a limit, given that copying down someone else’s answers and/or paraphrasing their thinking, even if you use your own approach, is generally much less useful than doing your own work. Getting a bit of help to get started or past a sticking point is fine, with the idea that you move on from there on your own.

On the other hand, exams are different. This is an ‘open-book’ exam, which means that you are welcome to use any source you like. Books, web sites, papers and your own notes are all fair game. The only exception is other people, including present members of the class, past members of the class, postdocs, faculty, your sister, etc. You are expected to complete this exam on your own, without receiving help from anyone and without talking to anyone about it. Please let me know if you need further clarification.

If you have questions of any sort, including clarification of what is being asked, please see me.

1) [15] Rabinowicz [1951] predicted a relationship between contact junction dimension and sliding velocity for rough surfaces in contact. Using the analysis presented by Johnson et al. [Proceedings of the Royal Society of London, 342, 301-313, 1971] and assuming a simple model in which roughness is characterized by hemispherical asperities of radius 0.1 mm, calculate: A) the asperity contact radius for a material with Poisson ratio = 0.25 and Young’s modulus = 70 GPa subject to a load of 0.1 kN, B) the deformation and contact stress at the asperity, and C) the contact lifetime for such a surface sliding at velocity of 100 µm/s.

2) [10] Starting with rate/state friction theory and assuming a Dieterich evolution law, determine the value of the frictional state variable for: A) a surface contact with critical slip distance of 35 µm sliding at 100 µm/s and B) the same contact sliding at 30 µm/s.

3) [10] Calculate frictional healing on a sliding surface. Assume rate/state friction parameters of a=0.02, b=0.015 and Dc = 25µm, 1D elastic coupling with k=1e-3 µm⁻¹, and Dieterich state evolution. For an initial sliding velocity of 100 µm/s, determine the value of the frictional state at the end of a 100-second hold period, where the load point velocity was set to zero. State any assumptions you make in deriving the answer.

3.1 [5] What would happen if the stiffness were higher? Would the final state, at the end of the hold, be smaller, the same or larger? Explain your answer.

4) You are investigating the role of earthquake recurrence interval as related to seismic source properties. Your data set includes a rare repeating series of M ~2 events for which the recurrence interval on a single fault ranges from 3 years to 10,000 years. Assume that the fault obeys rate and state friction with a Dieterich evolution law, that the faulted area is the same for each event (radius = 40 m), that shear modulus is 30 GPa.

4.1) [15] For an event in this series with repeat time of 3 years, the seismic moment is 1.5x10¹² Nm. What is the coseismic slip?

4.2) [10] Consider the implications of frictional healing for the relation between seismic moment and recurrence interval. What is the expected asymptotic rate of fault healing if the friction parameters are well described by a=0.01, b=0.015, and Dc = 1 mm, and effective normal stress is 100 MPa. Briefly explain the units you decide to use for your answer.
5) Consider a quasi-statically slipping region of a fault plane as it enlarges sufficiently to begin propagating as a dynamic rupture. At the point of dynamic nucleation, the energy release rate for crack extension is just sufficient to balance dissipation associated with surface production, frictional work, seismic radiation, etc.

5.1) [10] Assuming a slip-weakening frictional model, sketch the stress field in front of the crack, in the crack tip region, and behind the crack. On the same plot, show slip on the failure plane. Label all significant dimensions, stresses, etc.

5.2) [10] Assuming a peak yield stress of 50 MPa, a stress drop of 3 MPa, and a critical slip weakening distance $L_p$ of 1 mm, calculate the fracture energy $G$.

5.3) [5] Calculate the cohesive zone length for the values given in 5.2.

5.4) [10] Assume that the shear modulus and Poisson ratio in the region around the fault are 30 GPa and 0.25, respectively, and that the expanding rupture obeys rate/state friction with $a=0.01$, $b=0.015$, and $D_c=1\text{mm}$. Calculate the rupture nucleation dimension assuming effective normal stress = 100 MPa. Briefly discuss any formulae you use, including constants.