Mechanics of Earthquakes and Faulting

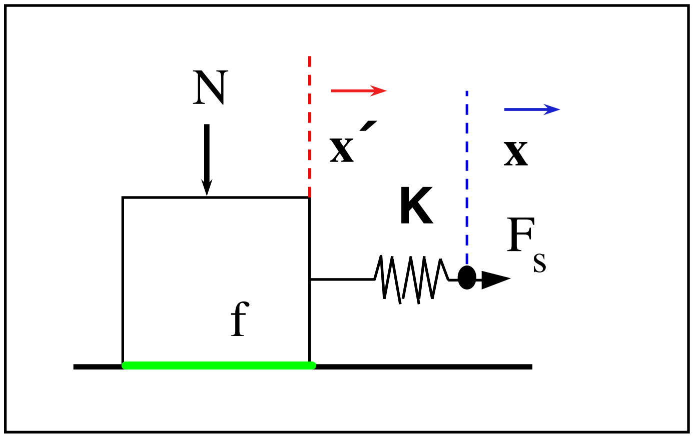
Lecture 9, 25 Feb. 2021

www.geosc.psu.edu/Courses/Geosc508

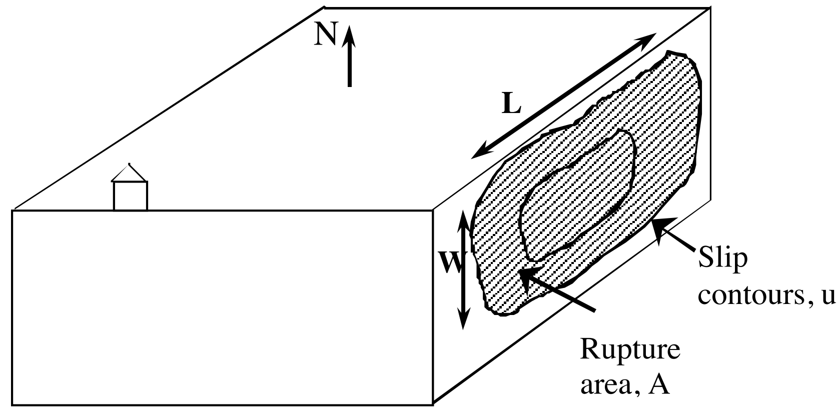
- Stability Criterion
- Rate-State Friction Laws
- Changes in state via normal stress perturbations
- SHS test to measure RSF parameters
- Stability of frictional slip, Stick-slip dynamics

Brittle Friction Mechanics, Stick-slip

- **Stick-slip (unstable) versus stable shear**



1-D fault zone analog, Stiffness K



Why is this a reasonable approach?

How do we get at stiffness?

$$\Delta\sigma = \frac{7\pi}{16} G \frac{\Delta\bar{u}}{r}$$

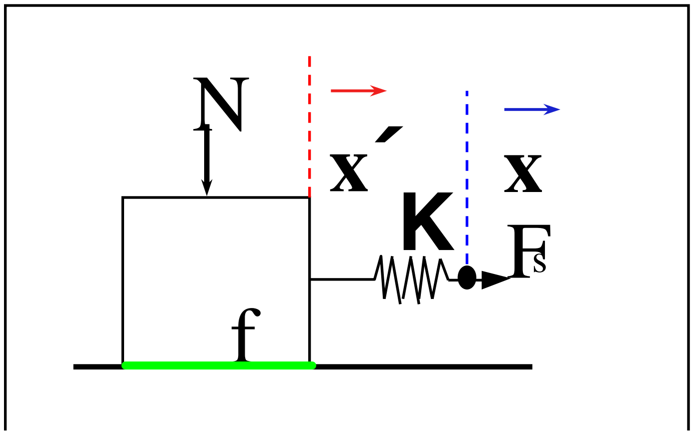
Relation between stress and slip on a dislocation of radius r. Therefore, the local stiffness around the slip patch is:

$$K = \frac{\Delta\sigma}{\Delta\bar{u}} = \frac{7\pi}{16} \frac{G}{r}$$

That is, stiffness decreases as the patch enlarges.

Brittle Friction Mechanics, Stick-slip

Stick-slip (unstable) versus stable shear



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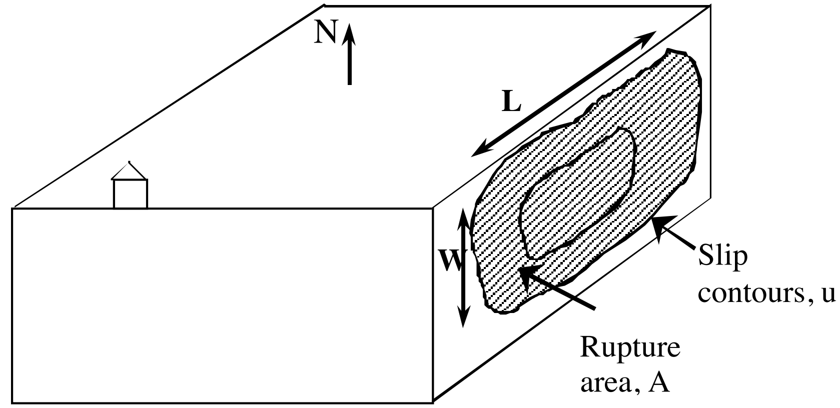
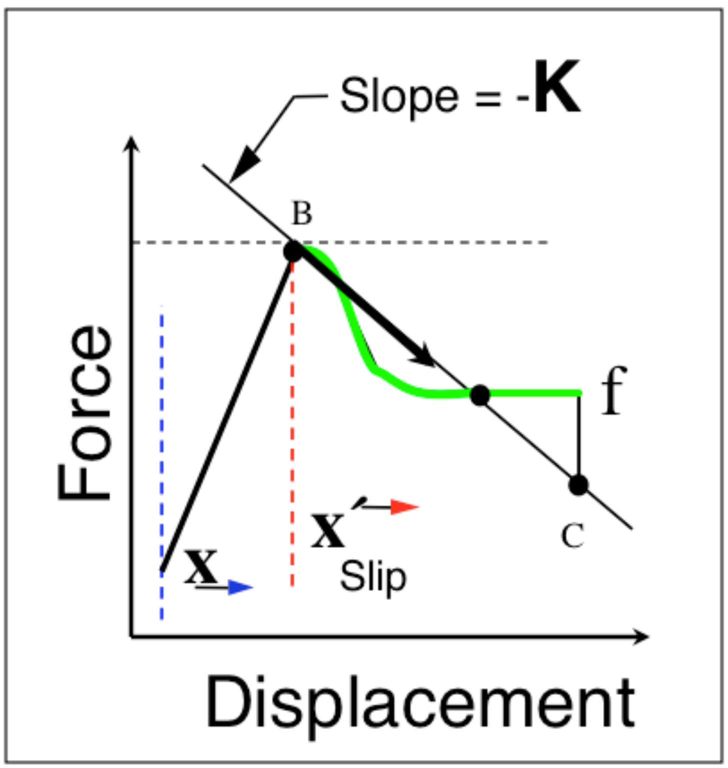
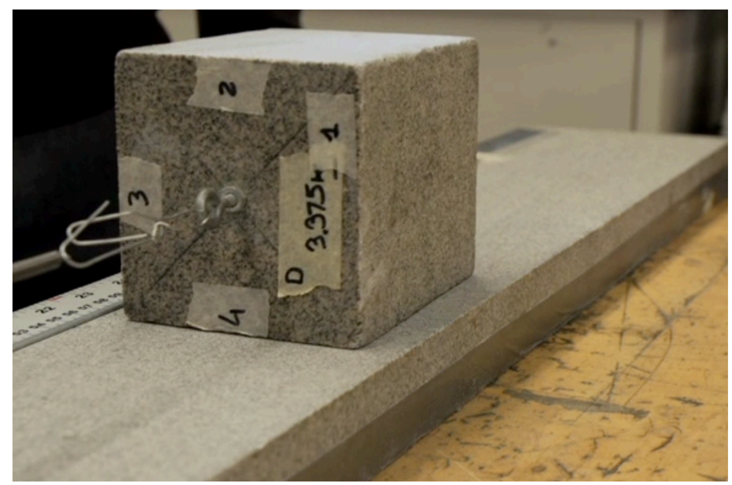
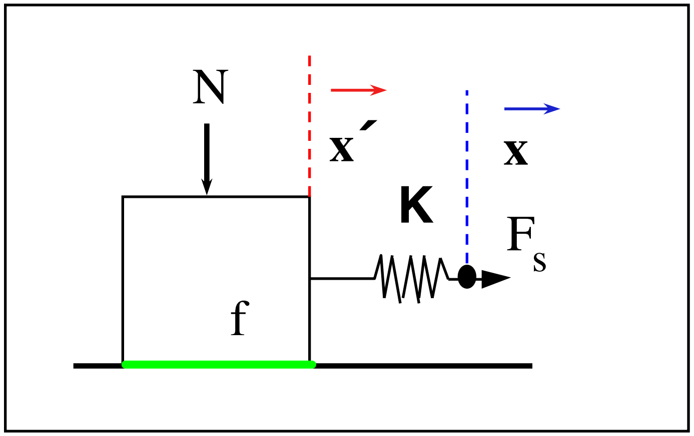






Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear

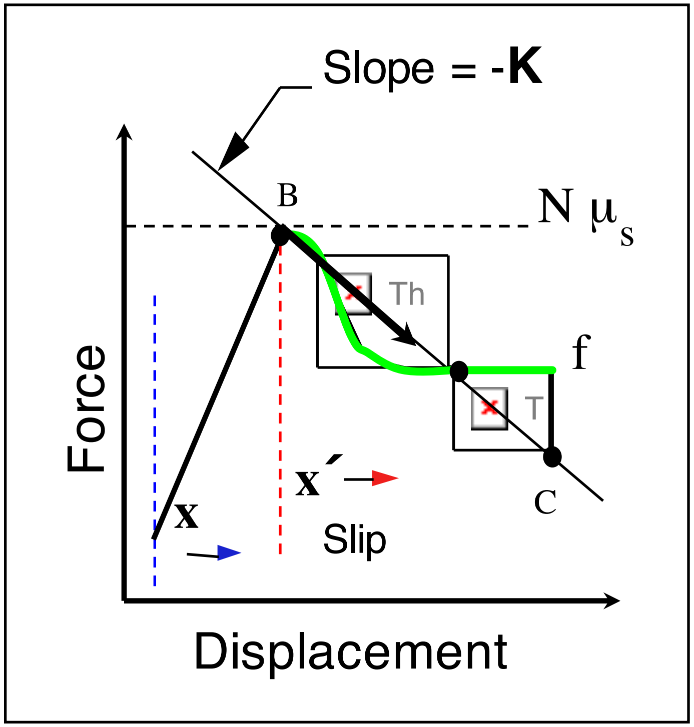


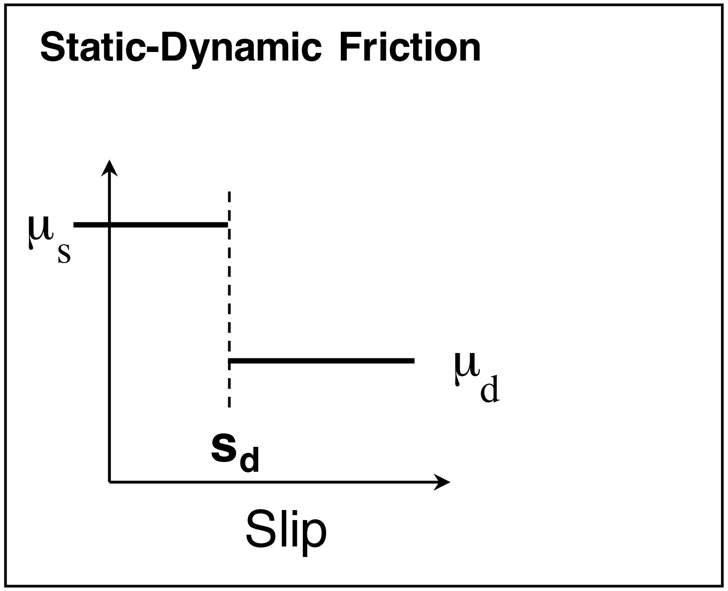
Stick-slip dynamics

$$m\ddot{x}' + \Gamma\dot{x}' + f(\dot{x}', x', t, \theta) = F_s$$

$$m\ddot{x}' + \Gamma\dot{x}' + f(\dot{x}', x't, \theta) = K(v_{lp} - v)t$$

$$m\ddot{x}' + Fx' = K(v_{lp} - v)t$$

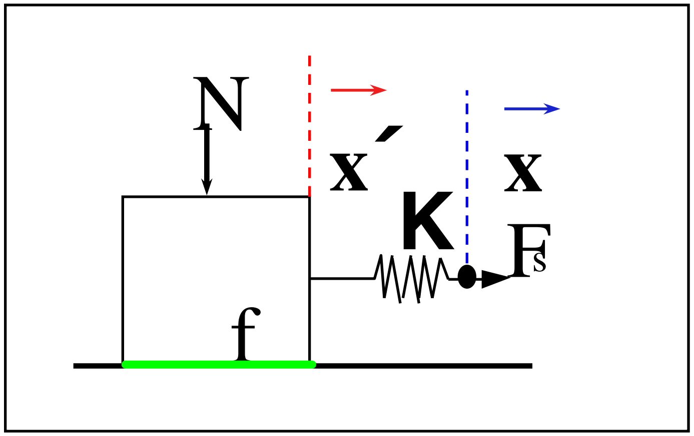


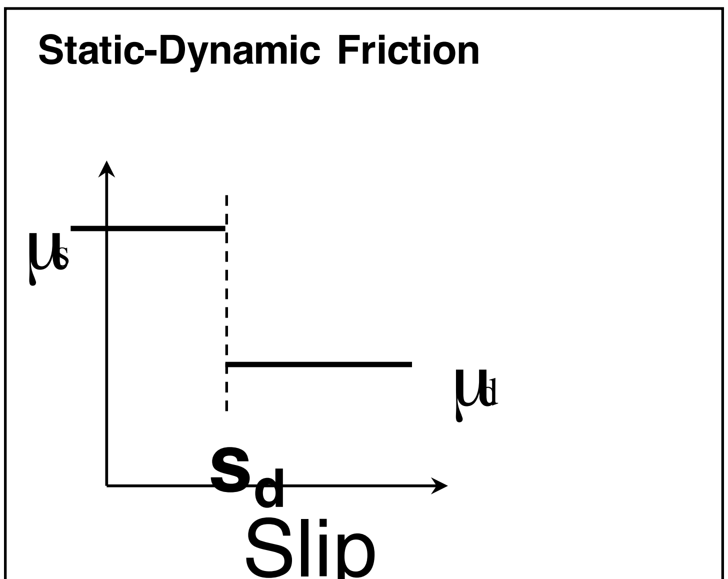


$$f = \Delta\mu N$$

Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear





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$$m\ddot{x}' + \Gamma\dot{x}' + f(\dot{x}', x't, \theta) = K(v_{lp} - v)t$$

$$m\ddot{x}' + f(x') = K(v_{lp} - v)t$$

$$m\ddot{x}' + Kx' = \Delta\mu N$$

$$x'(t) = \frac{\Delta\mu N}{K}(1 - cos\kappa t)$$

$$\kappa = \sqrt{\frac{K}{m}}$$

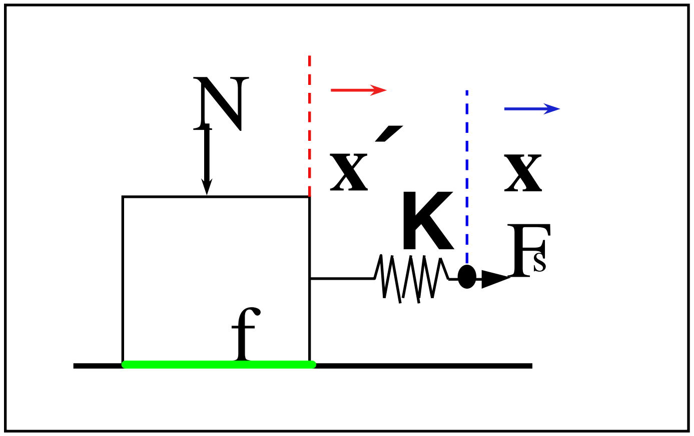
$$v(t) = \frac{\Delta\mu N}{\sqrt{Km}} sin\kappa t$$

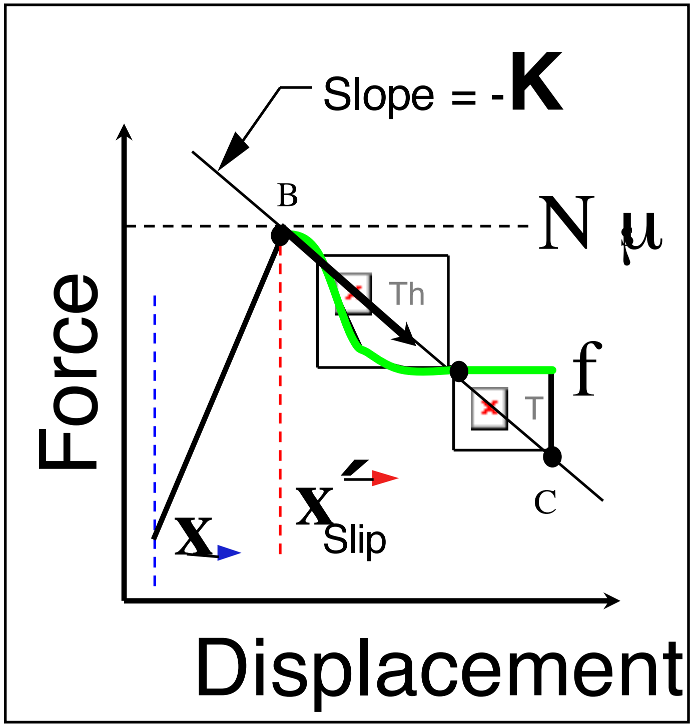
$$t_r = \pi\sqrt{\frac{m}{K}}$$

slip duration = rise time

Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear





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$$t_r = \pi\sqrt{\frac{m}{K}}$$

slip duration = rise time

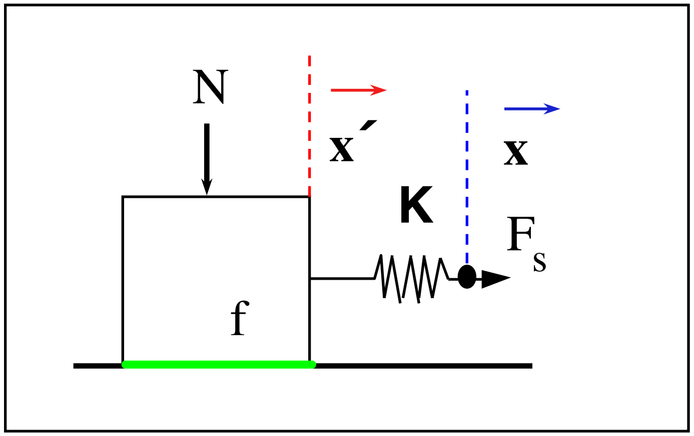
$$\Delta x' = \frac{2\Delta\mu N}{K}$$

total slip, particle velocity, and accel. all depend on friction drop (stress drop)

$$\Delta\sigma = 2(\mu_s - \mu_d)\sigma_n$$

Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear



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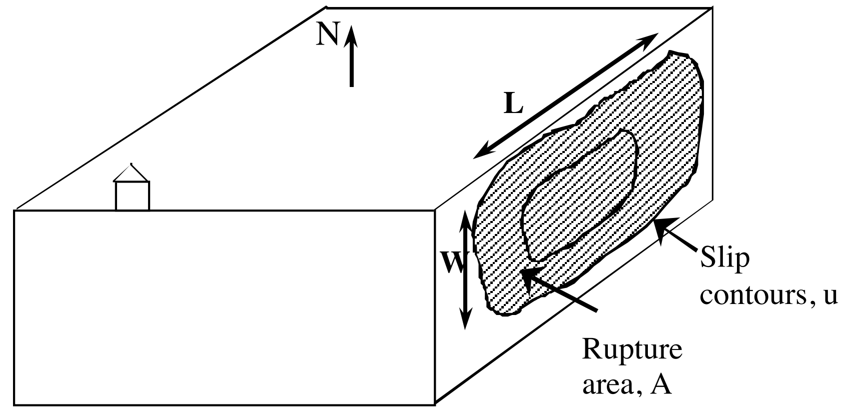
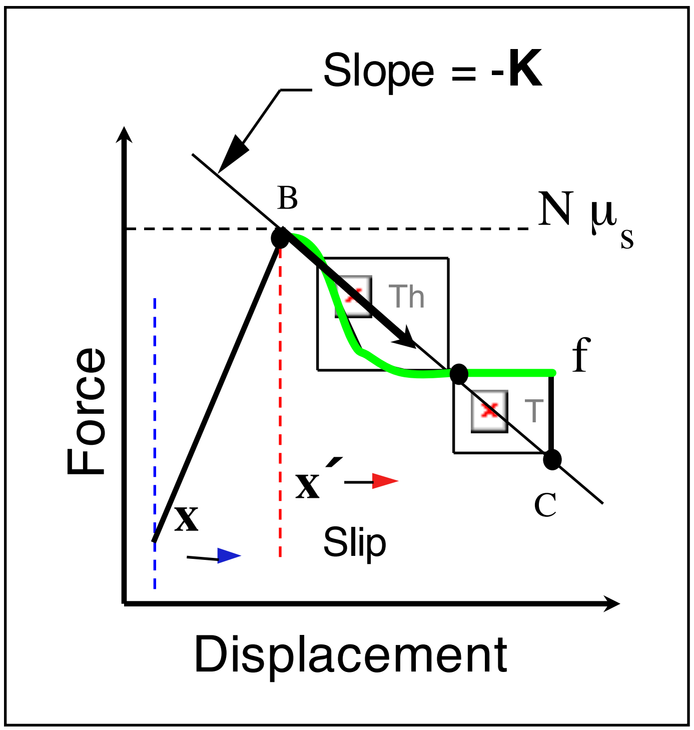




Frictional stability is determined by the combination of

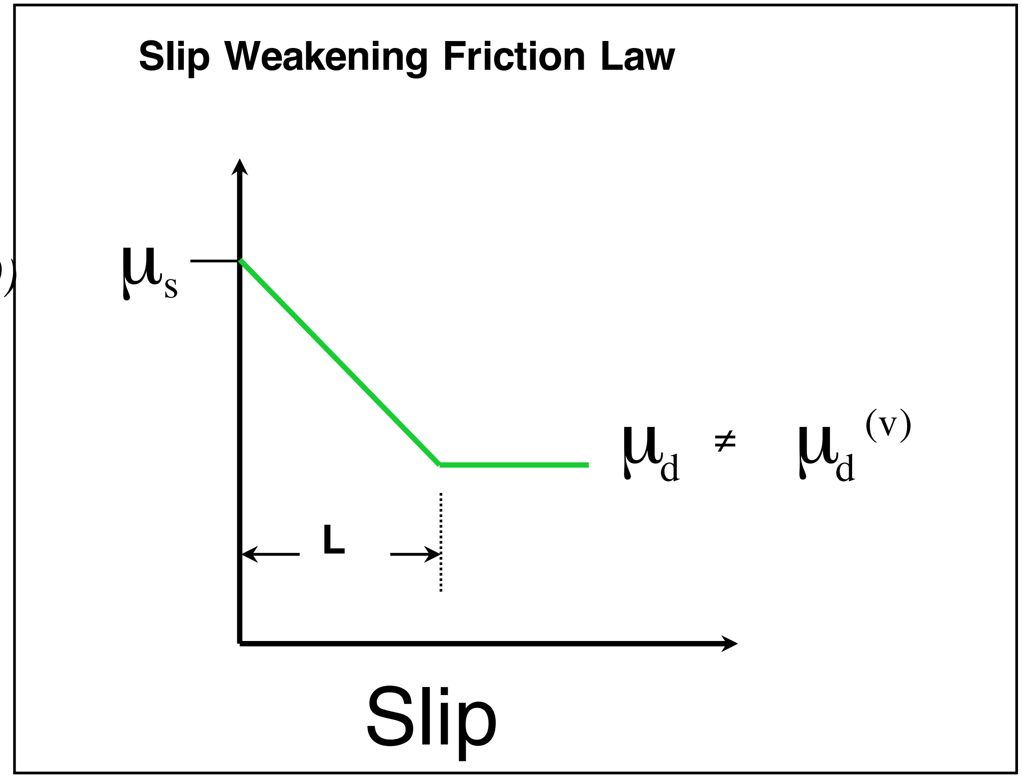
1) fault zone frictional properties and

2) elastic properties of the surrounding material

$$\mu(x) = \mu_s - \frac{x}{L}\Delta\mu \qquad (for\ L > x > 0)$$

$$\mu(x) = \mu_s - \Delta\mu \qquad (for\ x > L)$$

Palmer and Rice, 1973; Ide, 1972; Rice, 1980



Quasistatic Stability Criterion

$$K_c = \frac{\sigma_n(\mu_s - \mu_d)}{L}$$

$K < K_c$; Unstable, stick-slip

$K > K_c$; Stable sliding

Rate (v) and State (θ) Friction Constitutive Laws

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

state variable, characterizes physical state of surface or shearing region

critical slip distance

reference velocity

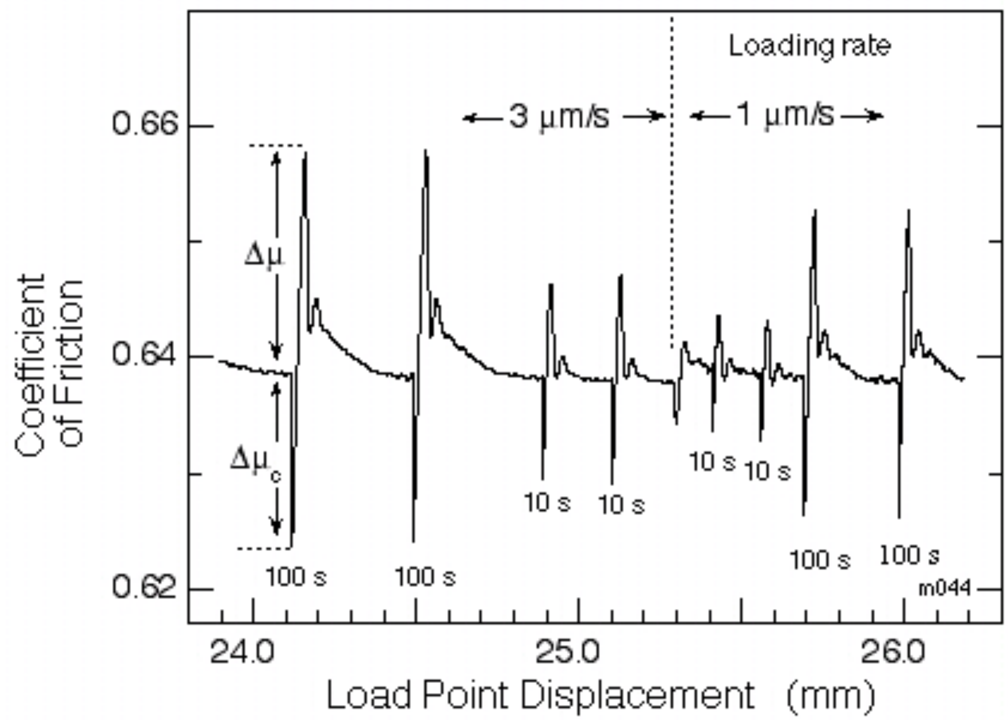
reference value of base friction

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Dieterich, aging law

$$\frac{d\theta}{dt} = -\frac{V\theta}{D_c} ln(\frac{V\theta}{D_c})$$

Ruina, slip law



Rate (v) and State (θ) Friction Constitutive Laws

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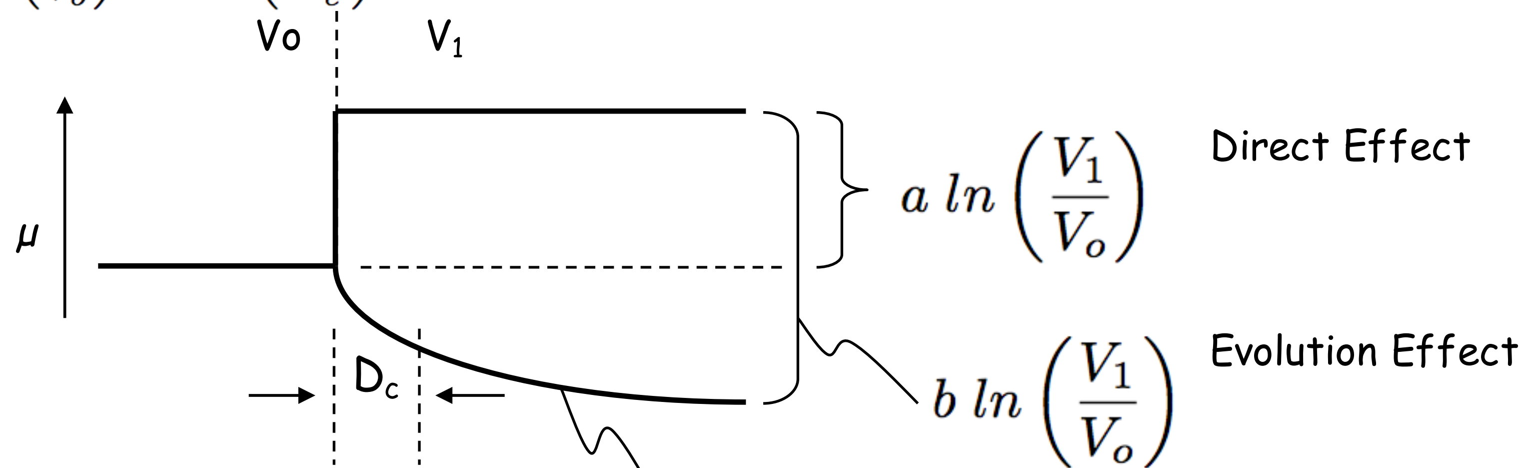
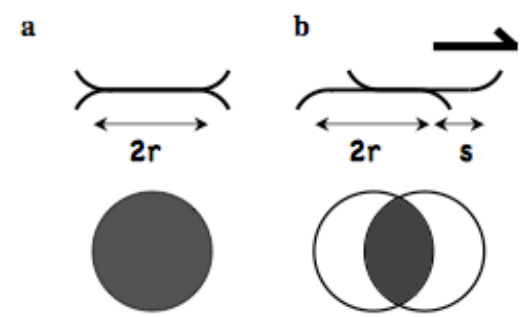


$\mu\ \alpha\ exp(-\frac{u}{D_c})$ Fading memory of past state

Adhesive Theory of Fricton (Bowden and Tabor)

- Real contact area << nominal area
- Contact junctions at inelastic (plastic) yield strength
- Contacts grow with “age”
- Add: Rabinowicz’s observations of static/dynamic friction
- “Static” friction is higher than “Dynamic” friction because contacts are older (larger)
- -> implies that contact size decreases as velocfiy increases



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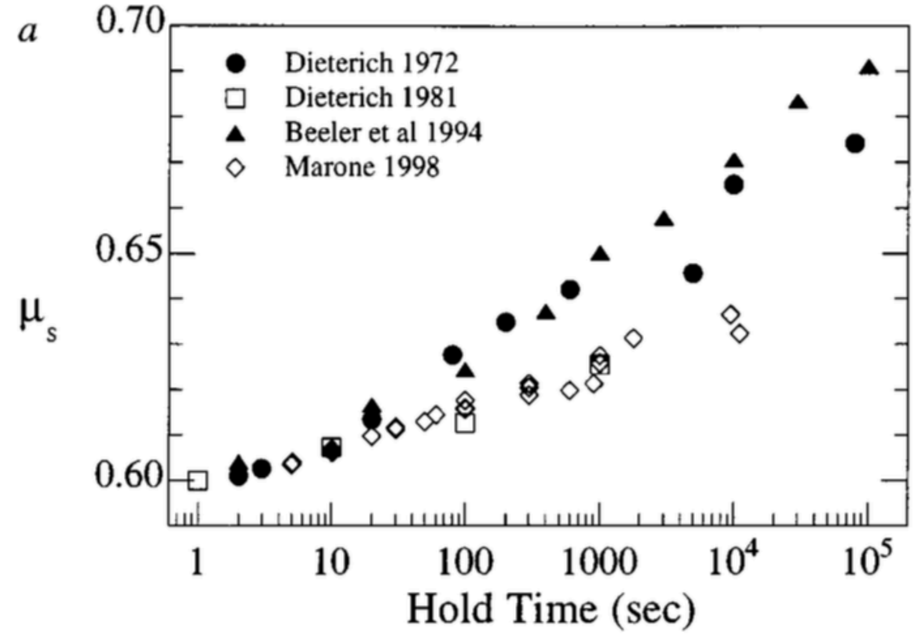
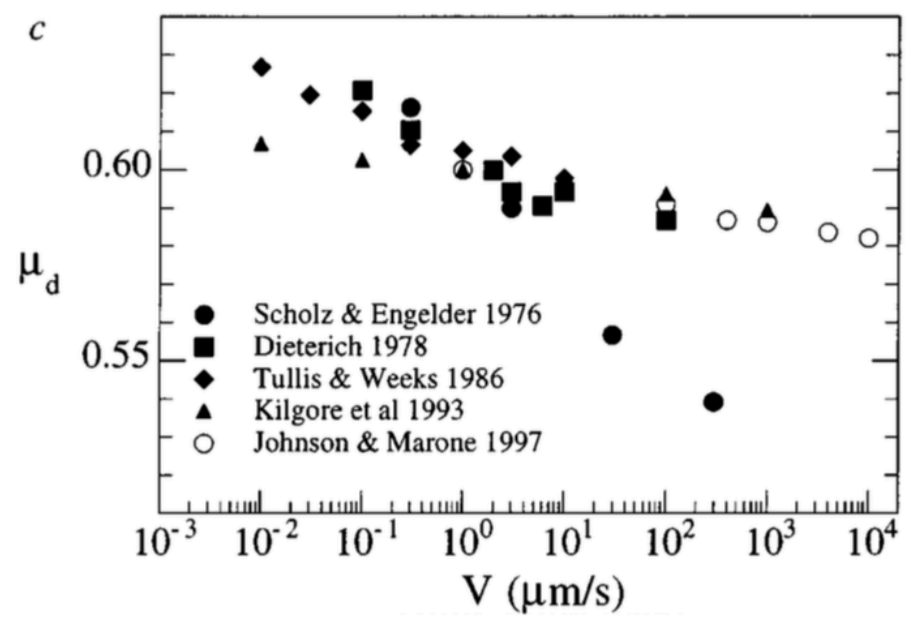
This "explains" healing and velocity dependence, but what about Dc?





Rate (v) and State (θ) Friction Constitutive Laws

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

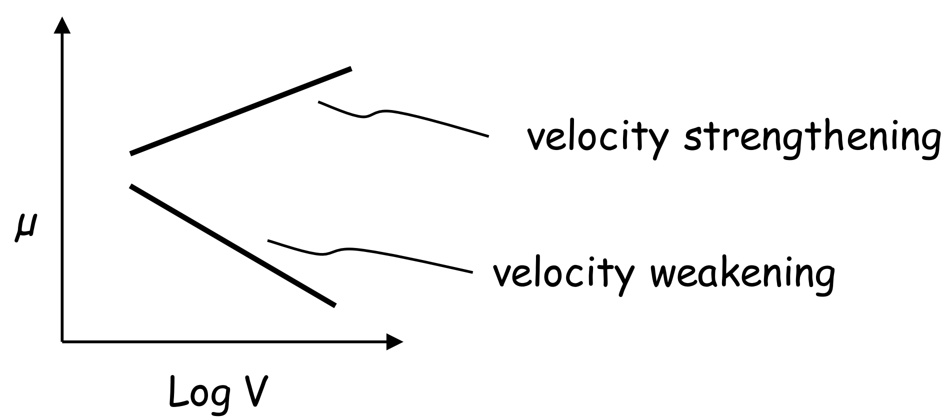
2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Convention is to use a, b for friction and A, B for Stress

$$\tau(\theta, v) = \tau_o + A\, ln\left(\frac{V}{V_o}\right) + B\, ln\left(\frac{V_o\theta}{D_c}\right)$$

$$A - B = \frac{\Delta\tau}{\Delta lnV}$$

Steady-state velocity strengthening if a-b > 0,
velocity weakening if a-b < 0



Rate (v) and State (θ) Friction Constitutive Laws

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Modeling experimental data

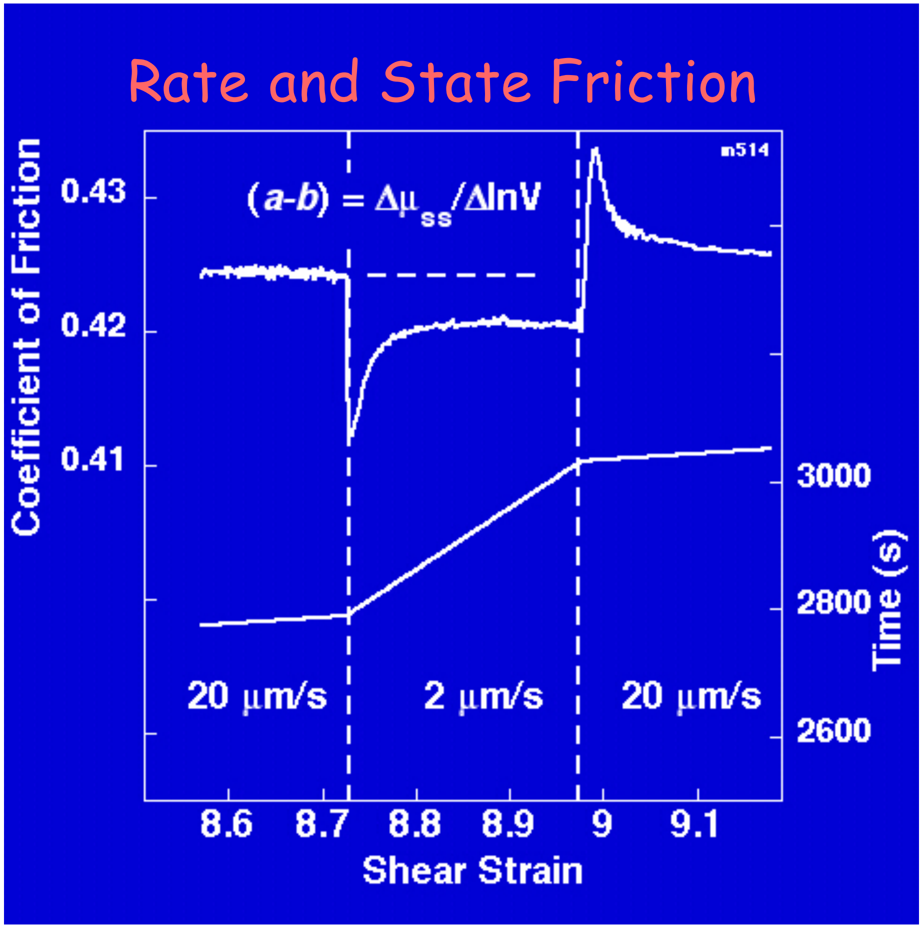
3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling

$$V = V_o\, exp\left[\frac{\mu - \mu_o - b\, ln(\frac{V_o\theta}{D_c})}{a}\right]$$

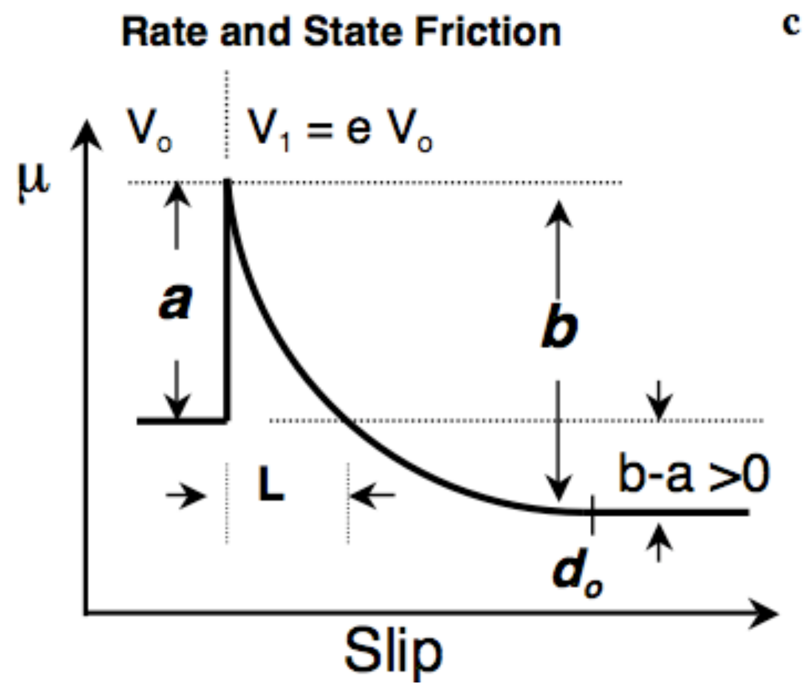
Solve:

$$\frac{d\mu}{dt} = k\left(V_{lp} - V_o\, exp\left[\frac{\mu - \mu_o - b\, ln(\frac{V_o\theta}{D_c})}{a}\right]\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$



Marone, 1998



$$\mu(\theta,v,\sigma) = \mu_0 + a\ln\left(\frac{v}{v_o}\right) + b\ln\left(\frac{v_o\theta}{D_c}\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c}$$

Dieterich State Evolution

$$\theta = \theta_o\left(\frac{\sigma_{initial}}{\sigma_{final}}\right)^{\frac{\alpha}{b}}$$

Modeling the effect of normal force vibration 1.

Rate and State Friction Theory

$$\mu(\theta,v,\sigma) = \mu_0 + a\ln\left(\frac{v}{v_o}\right) + b\ln\left(\frac{v_o\theta}{D_c}\right)$$

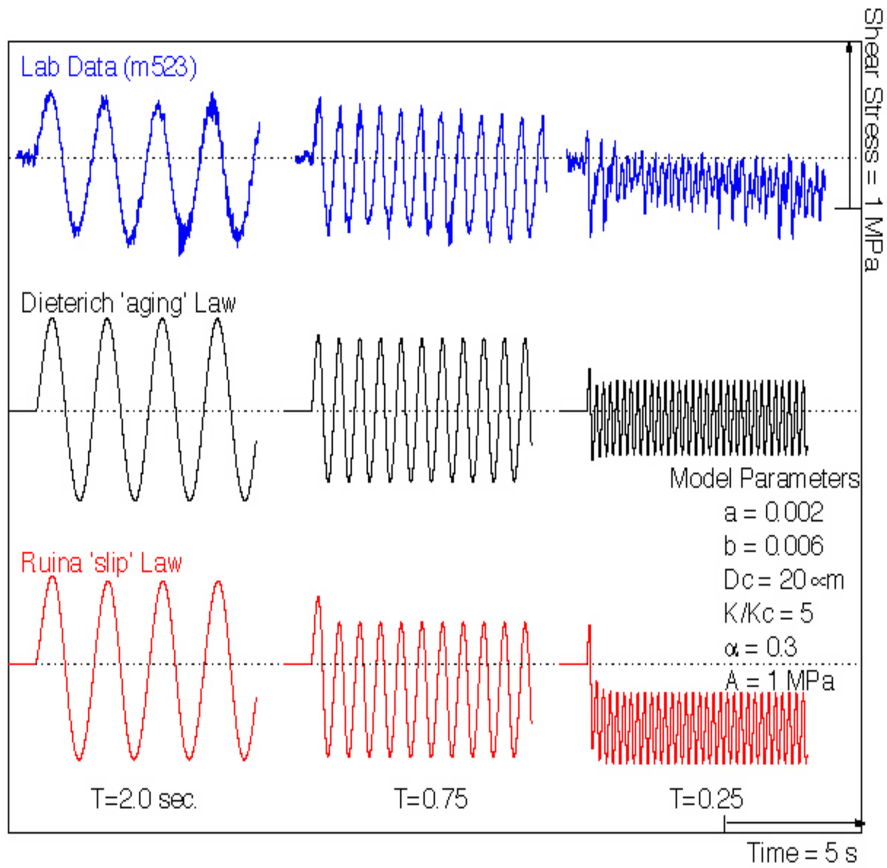
$$\frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c}$$ *Deiterich Law*

$$\theta = \theta_o\left(\frac{\sigma_{initial}}{\sigma_{final}}\right)^{\frac{\alpha}{b}}$$ *Normal Stress*

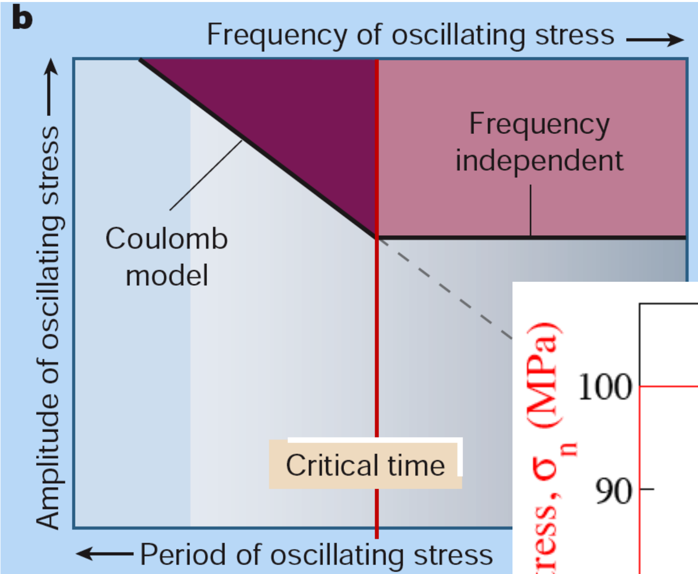
$$\frac{d\mu}{dt} = k'\left(v_{lp} - v\right)$$ *Elastic Coupling*

$$T_c = 2\pi\frac{D_c}{V}\sqrt{\frac{a}{b-a}}$$ *Critical Vibration Period*

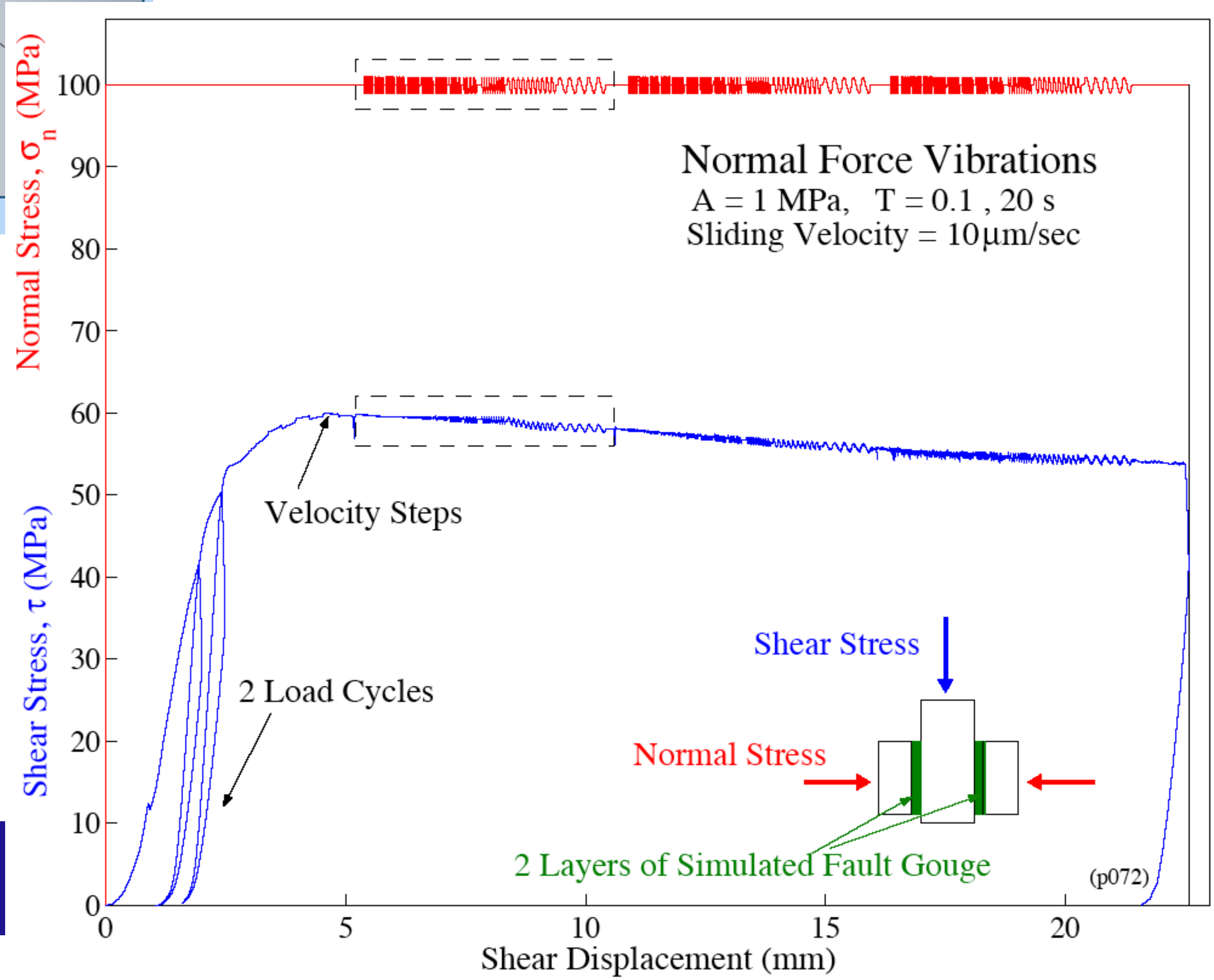
$$K_c = \sigma\frac{(b-a)}{D_c} + \frac{m v_0^2 (b-a)}{D_0^2}$$ *Critical Stiffness*



Lab: Normal Stress Vibrations
Critical period observed



Boettcher & Marone, JGR, 2004



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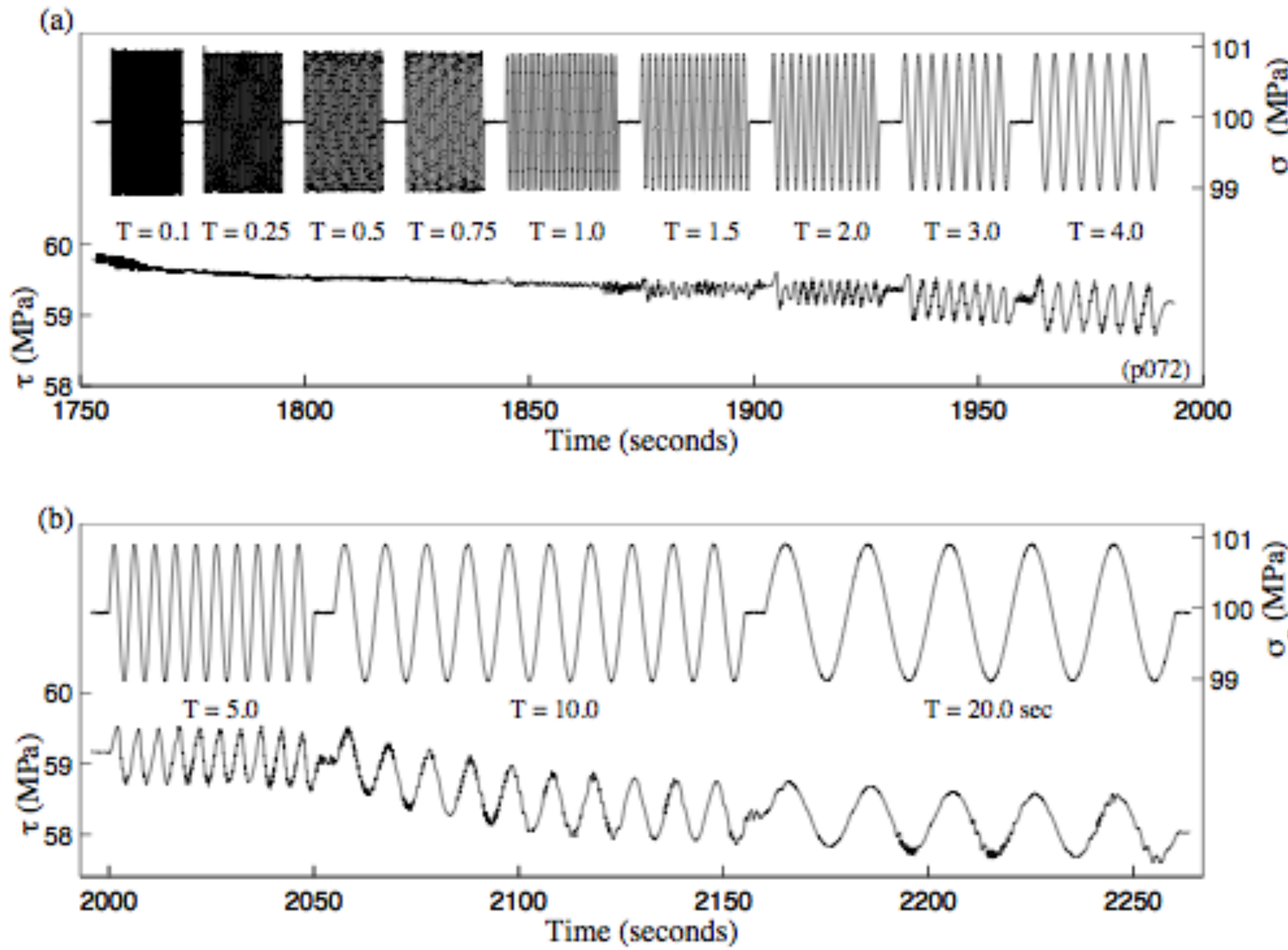
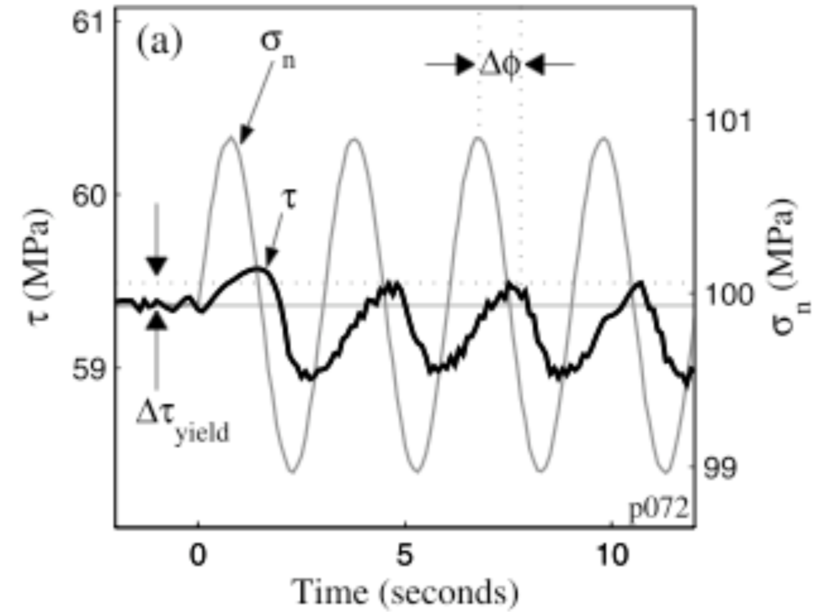


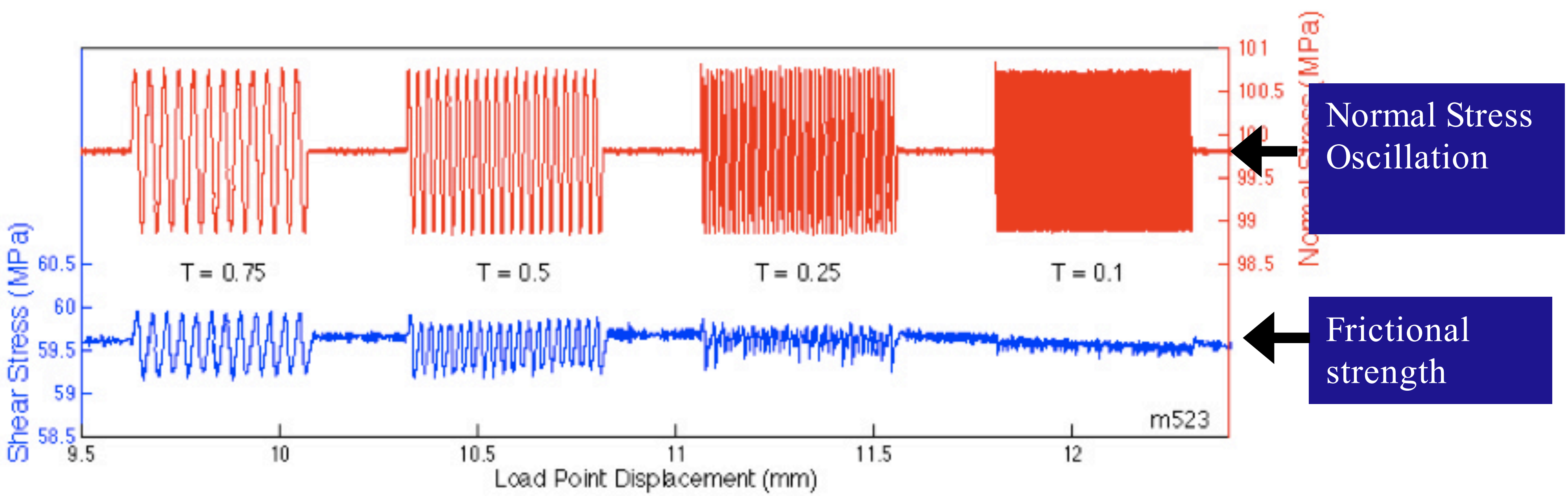
Critical period is 1 to 2 sec.

Boettcher & Marone, JGR, 2004

Also, Phase lag.
Friction response lags stressing.
Could explain delayed triggering



No frictional response to high frequency oscillations



Rate and State Friction

Dieterich, Scholz, Ruina, Rice

$$\mu(\theta,v,\sigma) = \mu_0 + a\ln\left(\frac{v}{v_o}\right) + b\ln\left(\frac{v_o\theta}{D_c}\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c}$$ Dieterich State Evolution

$$\theta = \theta_o \left(\frac{\sigma_{initial}}{\sigma_{final}}\right)^{\frac{\alpha}{b}}$$

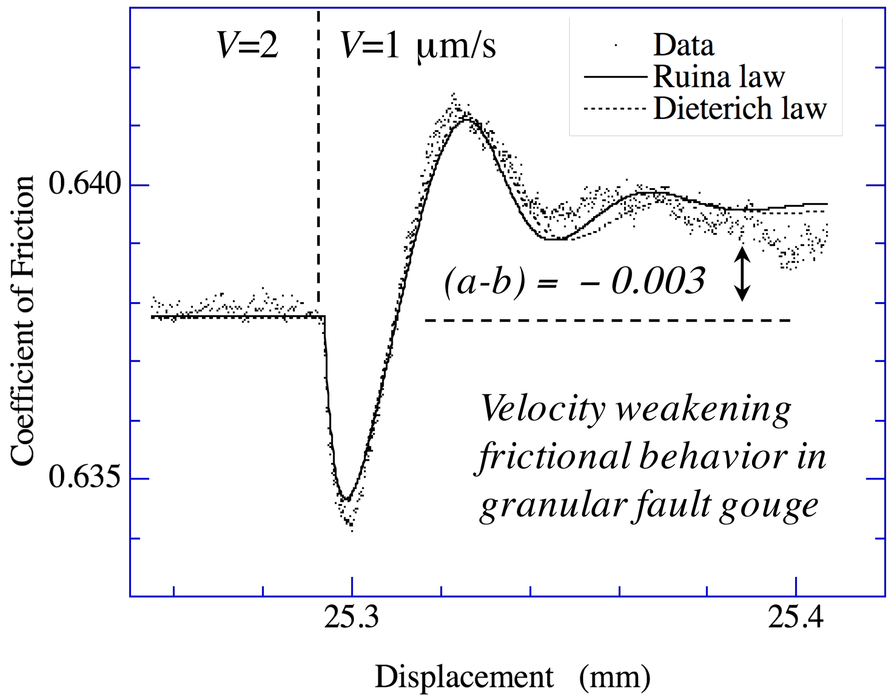
$$\theta_{ss} = \frac{D_c}{v}$$

$$\Delta\mu_{ss} = (a-b)\ln\left(\frac{v}{v_o}\right)$$

$$\frac{d\mu}{dt} = k'\,(v_{lp} - v)$$

$$K_c = \sigma\frac{(b-a)}{D_c} + \frac{m v_0^2 (b-a)}{D_c^{\ 2}}$$

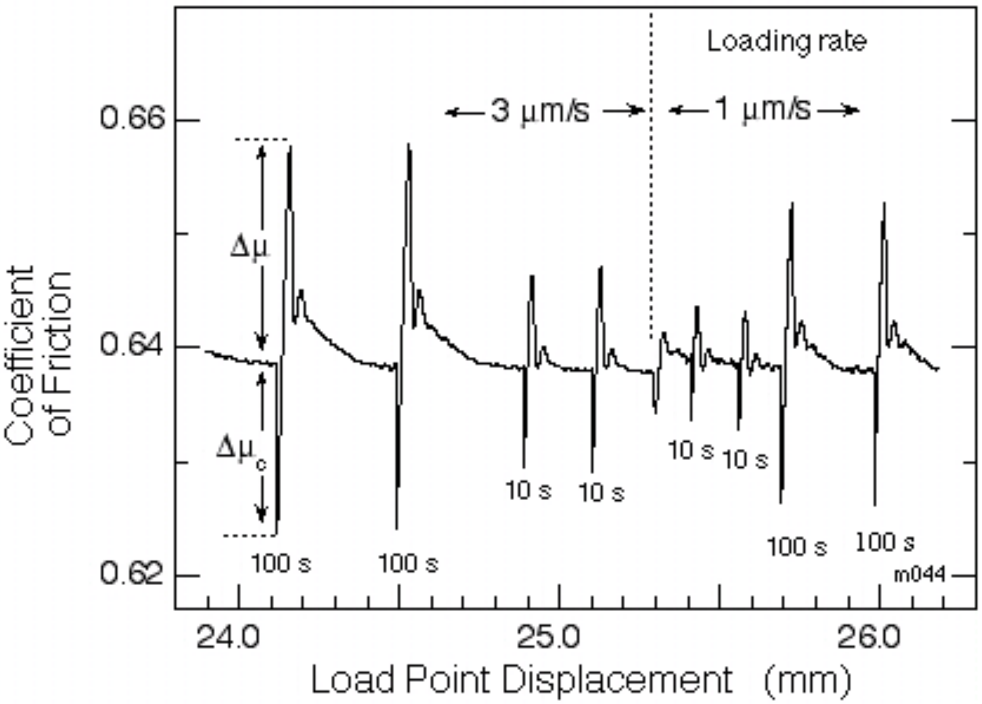
Empirical laws, based on laboratory friction data

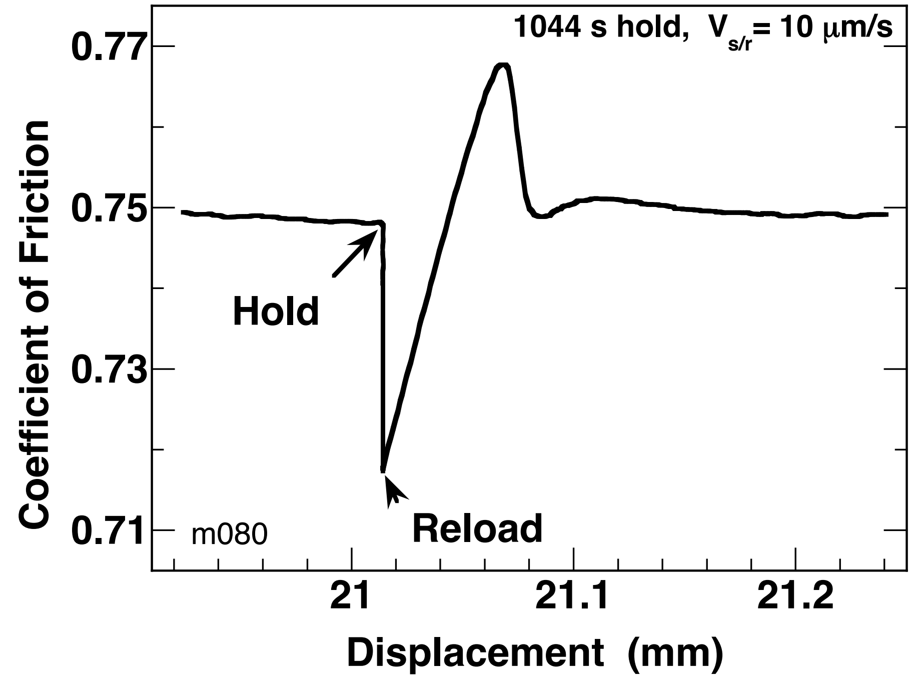


Thermally-activated process

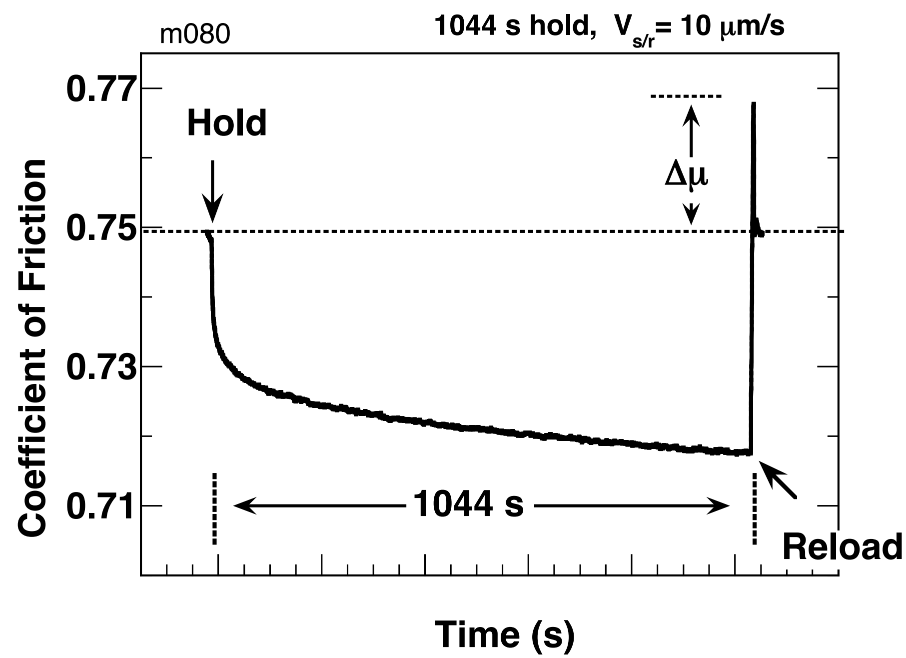
$$v = v_o \exp\left(\frac{\mu - \mu_o - b\varphi}{a}\right)$$

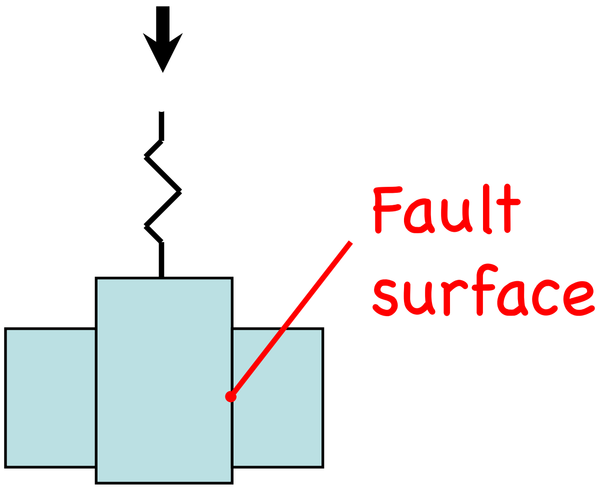
$$\dot{\varepsilon} = \dot{\varepsilon}_o \exp\left[-\frac{(Q - \tau_c\Omega)}{kT}\right]$$

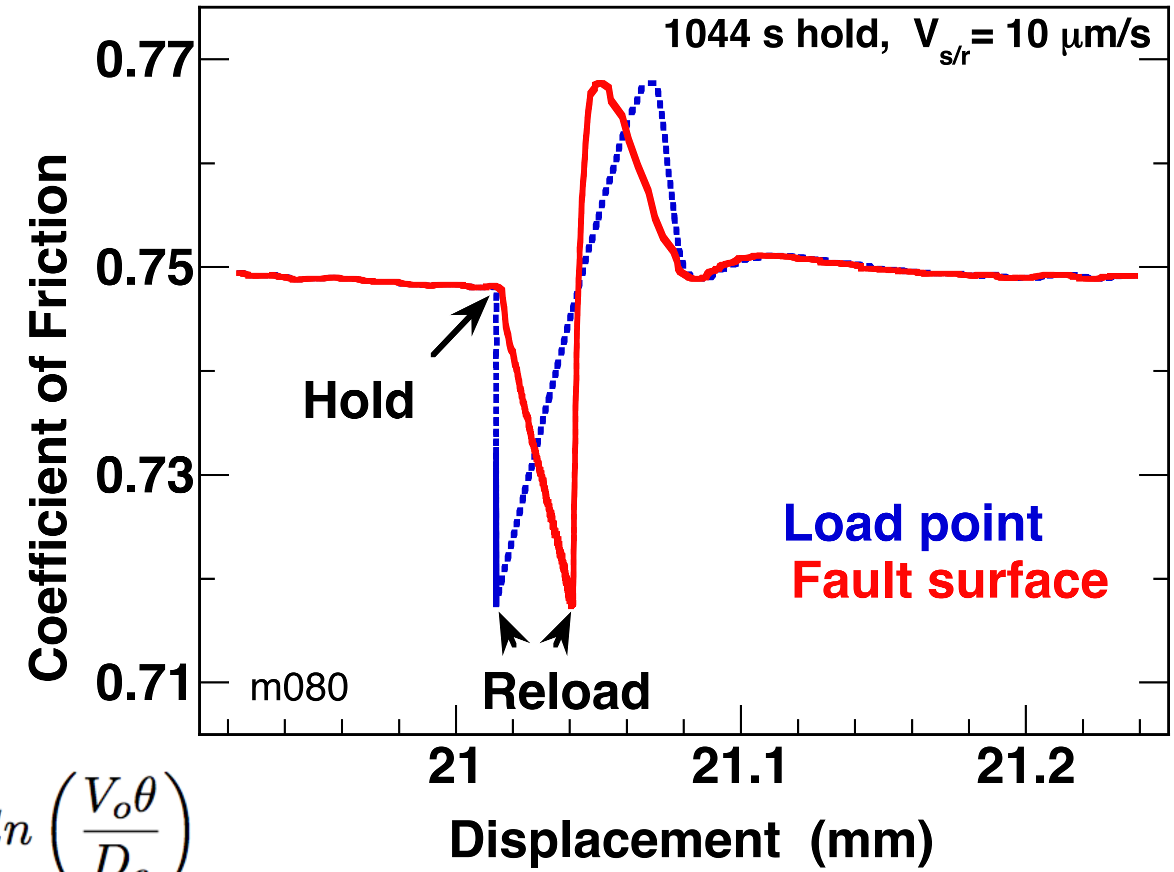




Sheared layer of quartz particles
(100-150 μm), 25 MPa normal stress .
Marone, 1998







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2) $\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$

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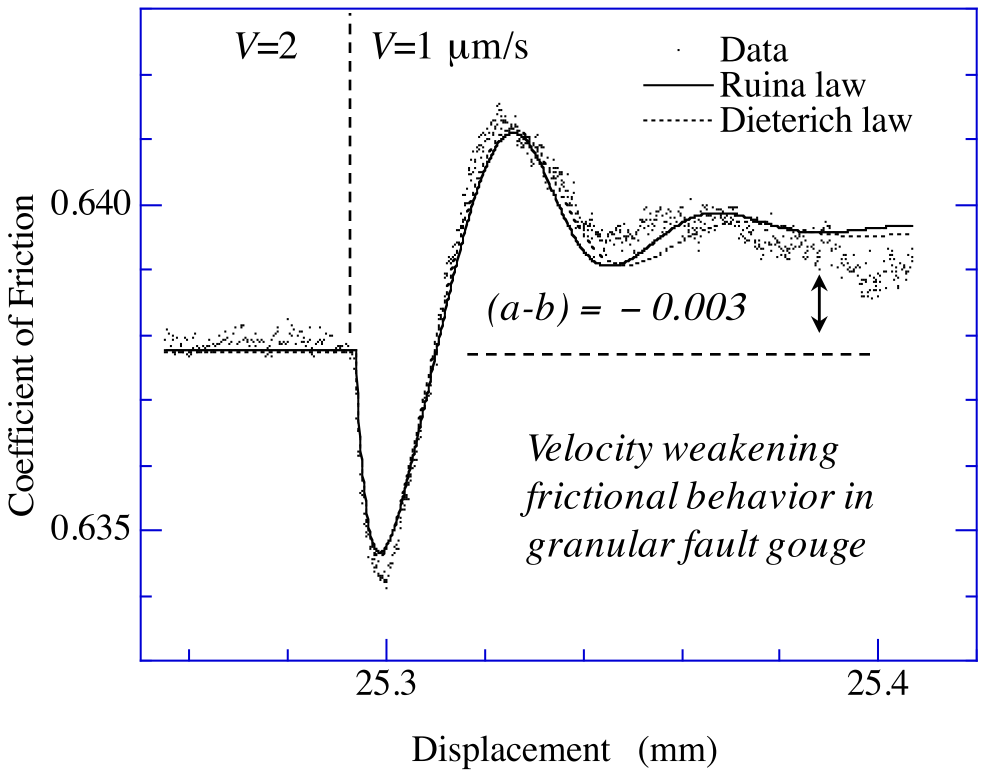
Rate/State Friction
Measuring the friction constitutive parameters

Empirical laws, based on laboratory friction data



Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

$$\mu(\theta, v) = \mu_0 + a \ln\left(\frac{v}{v_o}\right) + b \ln\left(\frac{v_o \theta}{D_c}\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c}$$

$$\theta_{ss} = \frac{D_c}{v}$$

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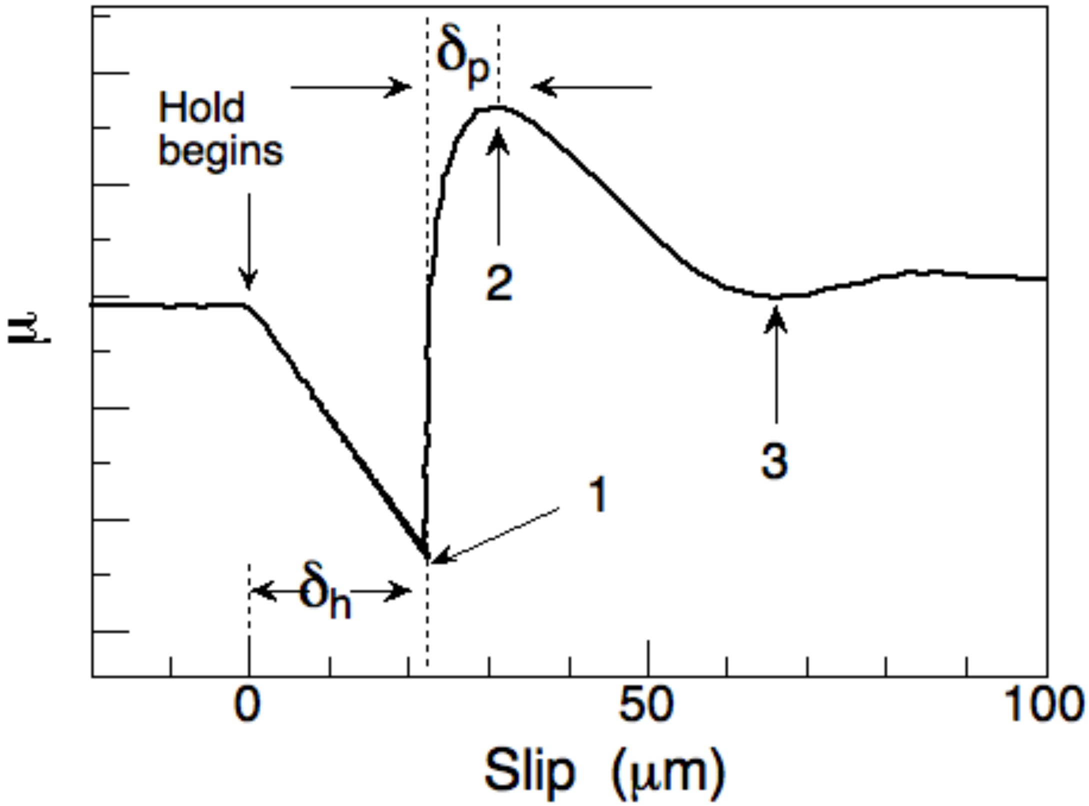
Rate/State Friction
Measuring the friction constitutive parameters



Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

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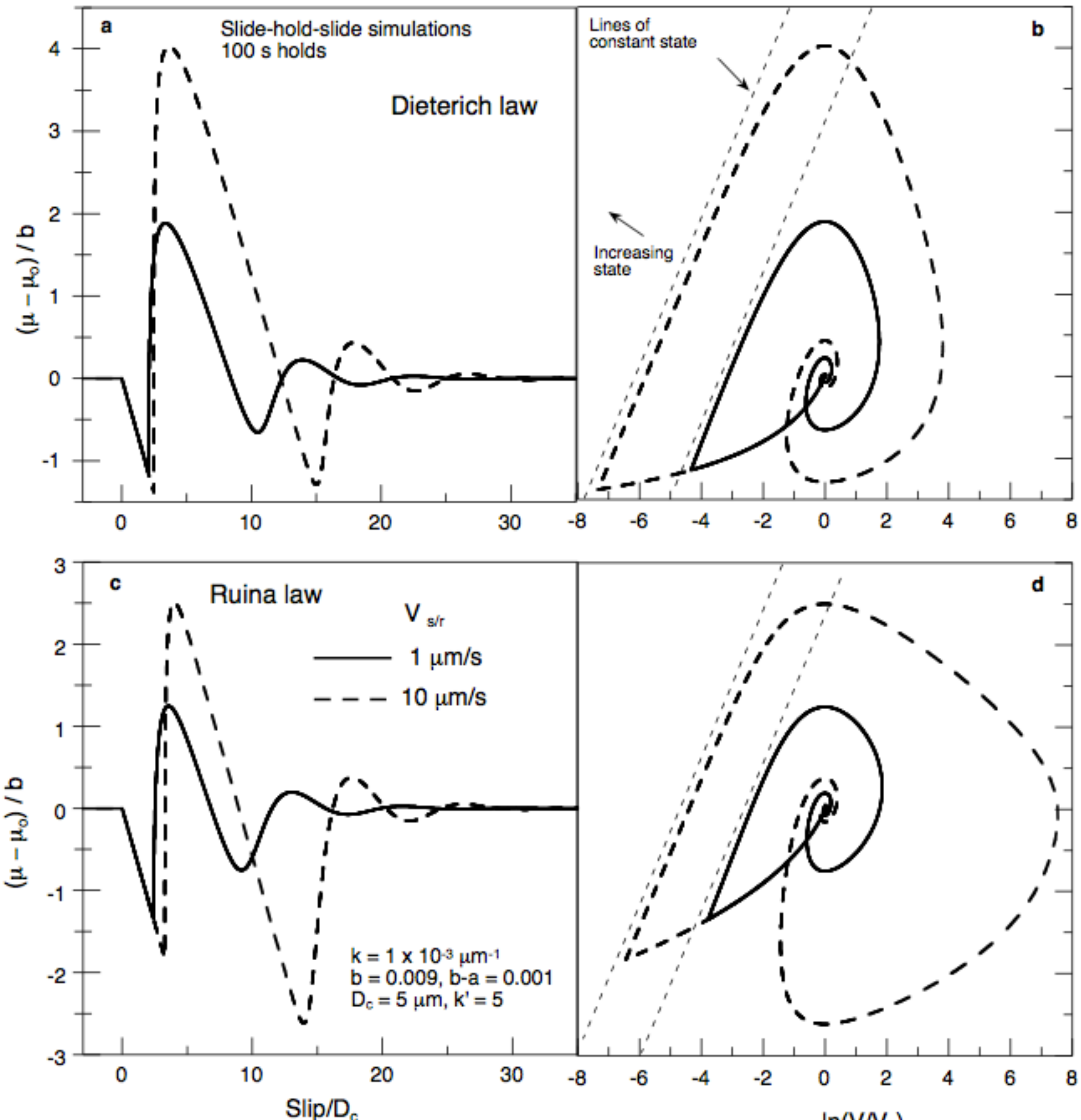
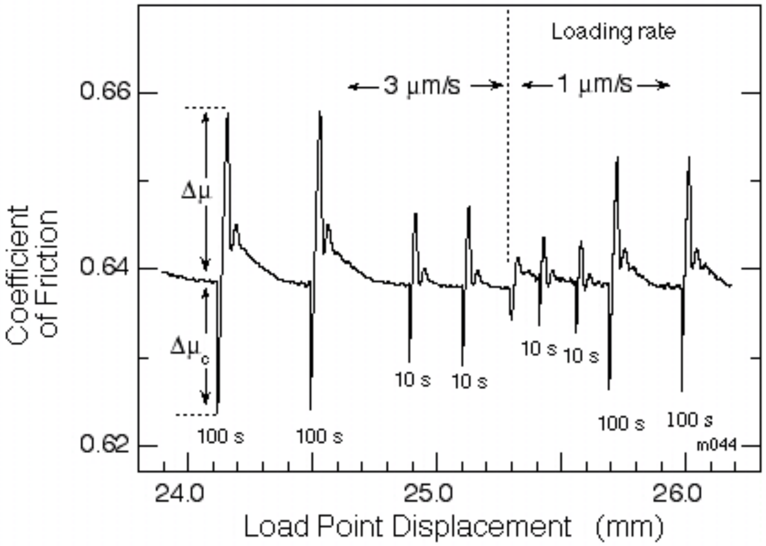
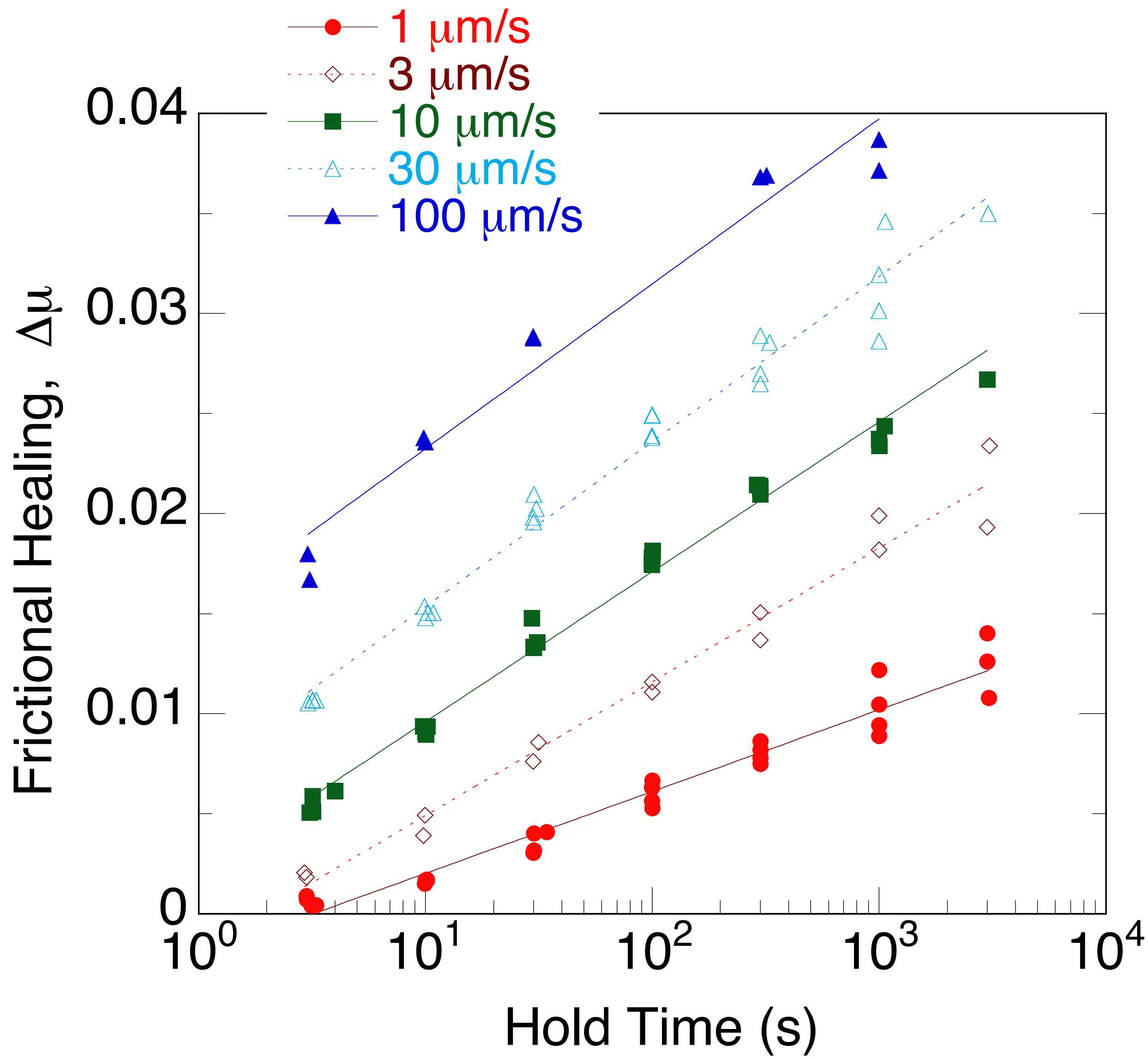


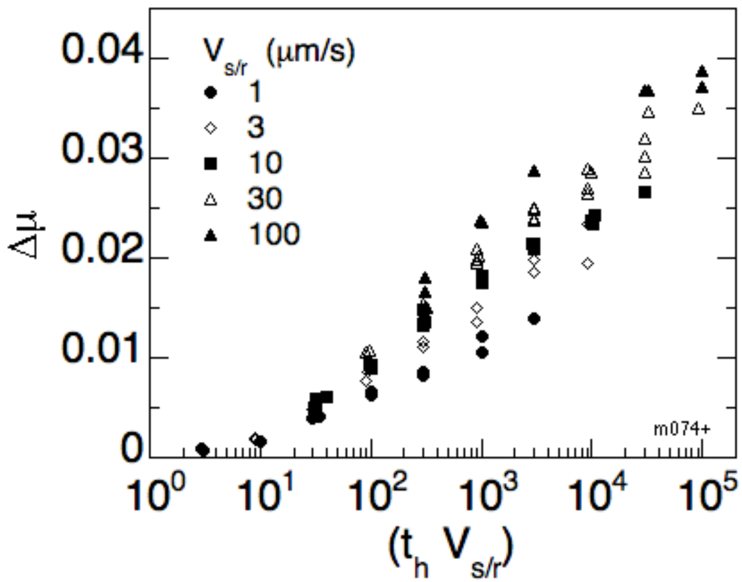


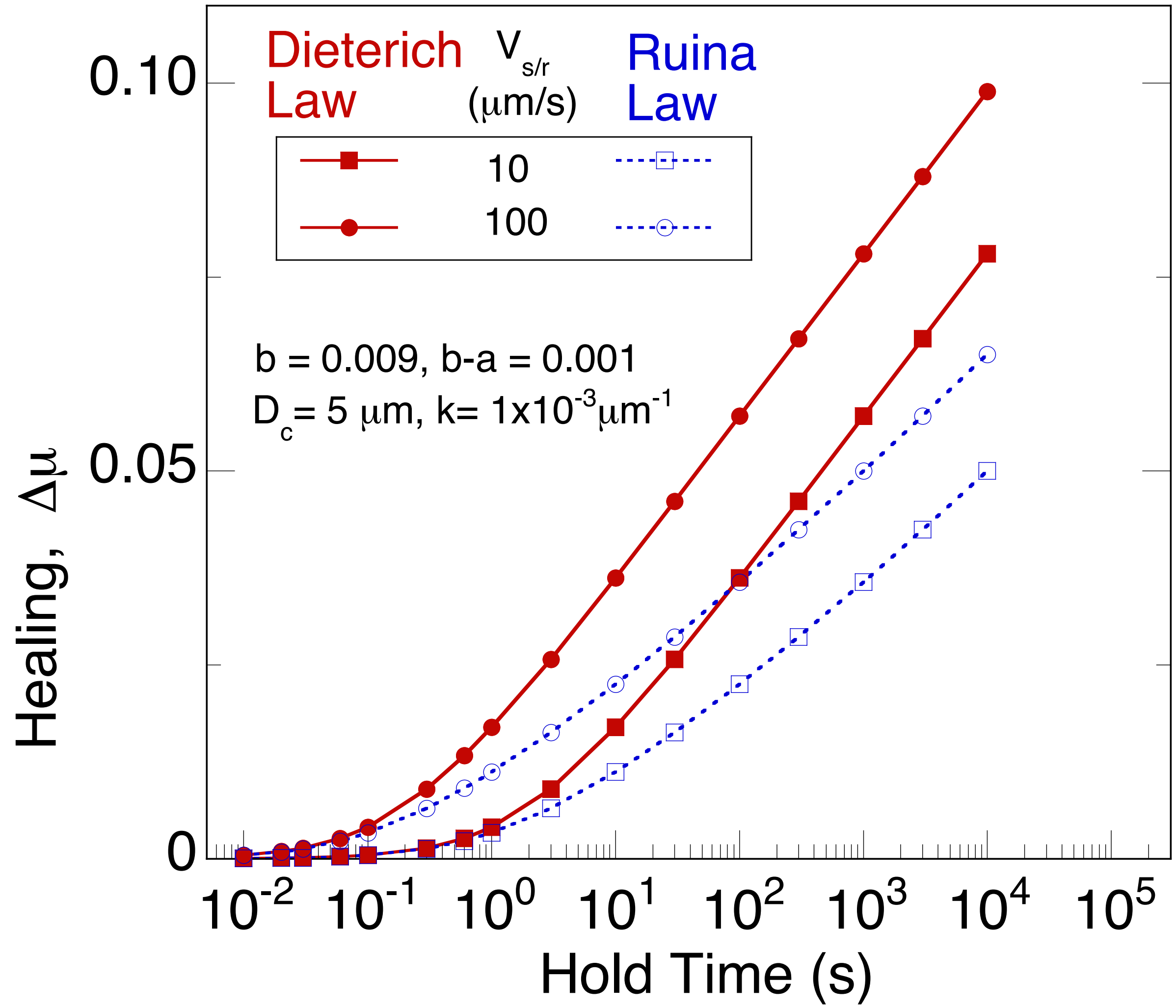


Stressed Aging

Aging rate depends on the rate of shearing

(Marone, 1998, *Nature*)





Friction Law

$\mu = \mu_o + a \ln(V/V_o) + b \ln(V_o \theta / D_c)$

State Evolution

$d\theta/dt = 1 - V\theta/D_c$

$d\theta/dt = - V\theta/D_c \ln(V\theta/D_c)$

Elastic Coupling

$d\mu/dt = k(V_{lp} - V)$

The rate of frictional healing depends on the rate of shearing (Marone, 1998, *Nature*)

Rate State Friction Laws predict this behavior

Friction Law

$$\mu = \mu_o + a \ln(V/V_o) + b \ln(V_o\theta/D_c)$$

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$$d\mu/dt = k(V_{lp} - V)$$

The rate of frictional healing depends on the rate of shearing (Marone & Saffer, 2015)

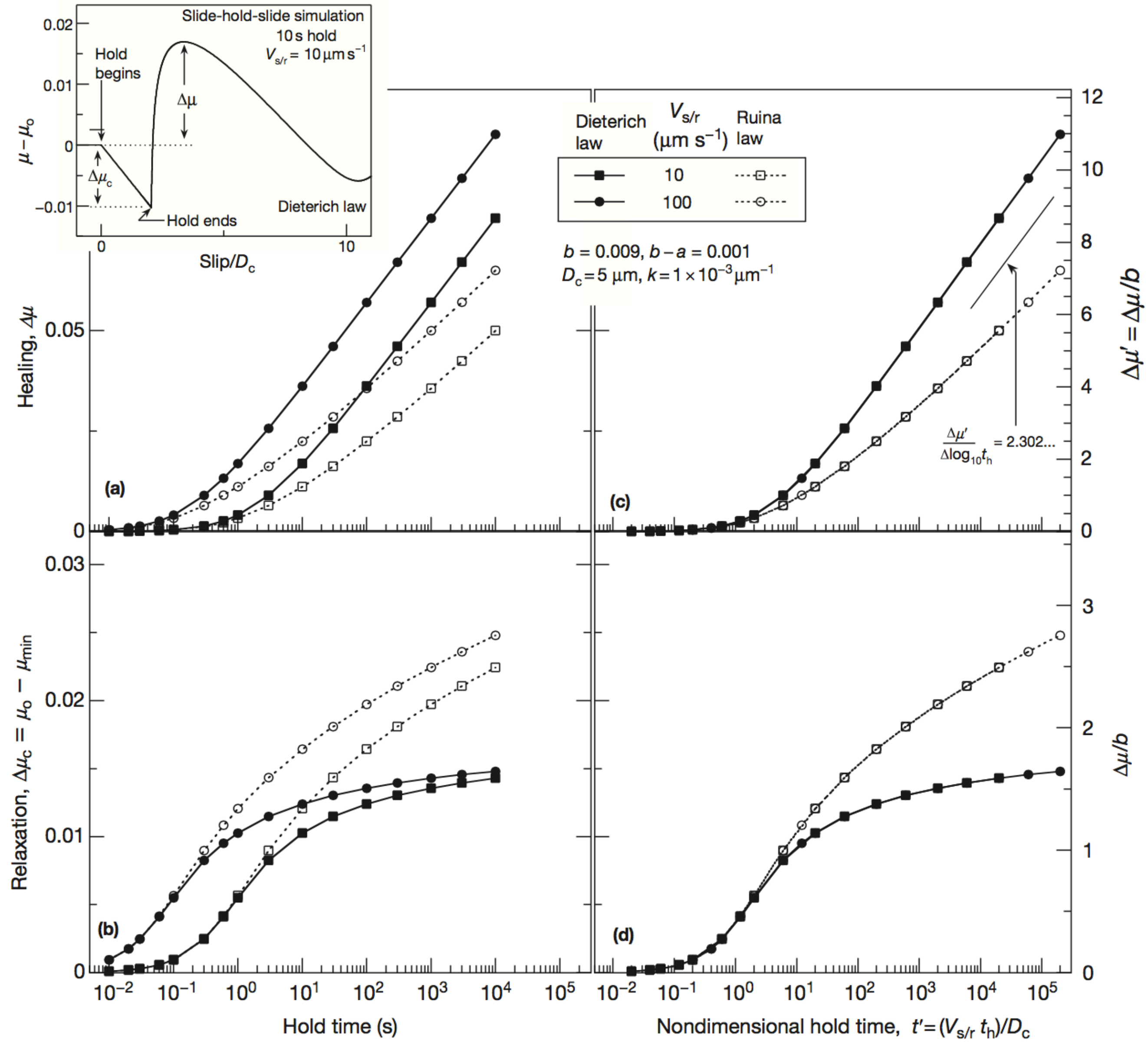
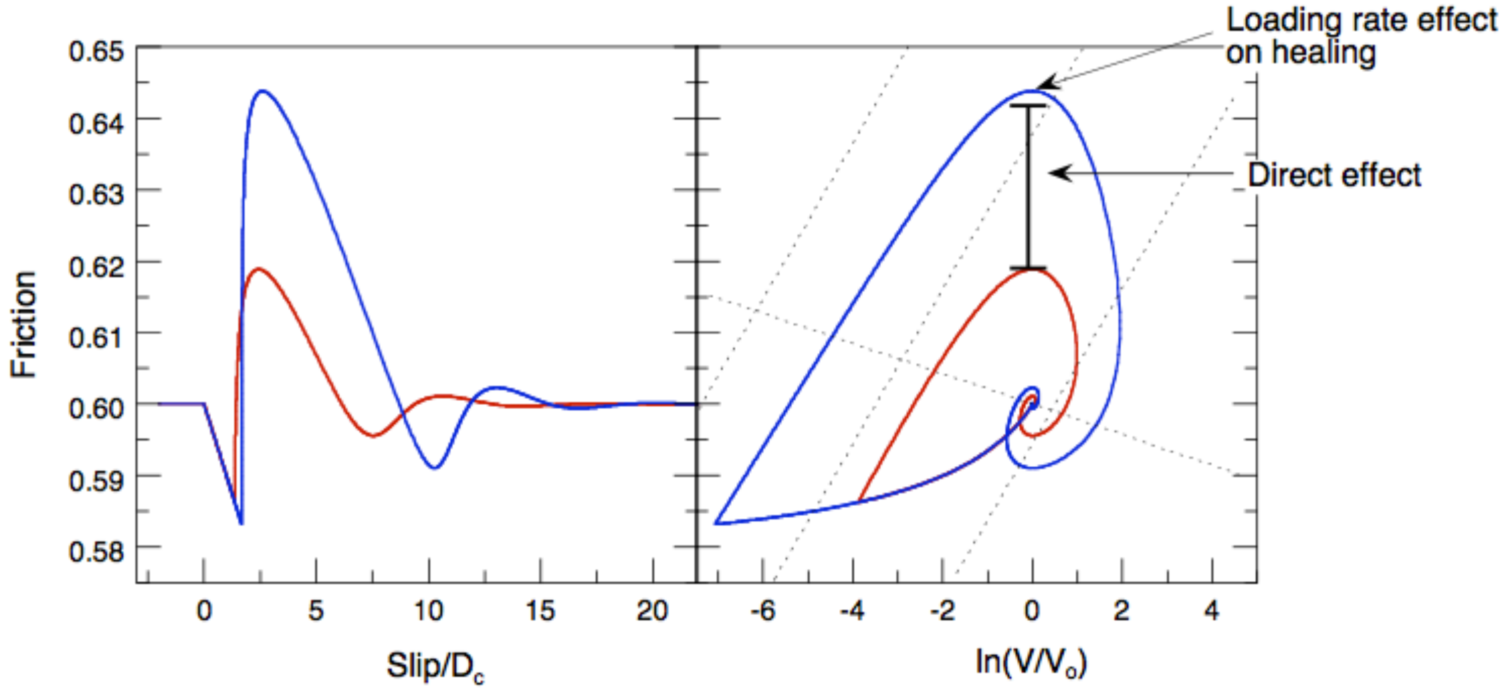


Figure 13 Numerical simulations of SHS tests showing healing and relaxation predicted by the RSF laws and elastic interaction. Inset to (a) shows details of a single simulation for which steady sliding is prescribed prior to a hold that begins at a normalized slip of 0 (the friction level is arbitrary and chosen as 0). The RSF laws predict that friction decreases during the hold due to creep and elastic interaction. Each point in the main panels shows results from a simulation. Four cases are shown corresponding to two velocities for each of the friction laws. Panels (a) and (b) show variations in the coefficient of friction. The same results are shown in nondimensional form in panels (c) and (d). Both laws show that healing and relaxation scale with loading rate and hold time, consistent with the experimental results. Simulations were carried out using the constitutive parameters given in the figure.

Phase Plane Plots



Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution

$$\mu(\theta, V) = \mu_o + a\,ln\left(\frac{V}{V_o}\right) + b\,ln\left(\frac{V_o\theta}{D_c}\right)$$

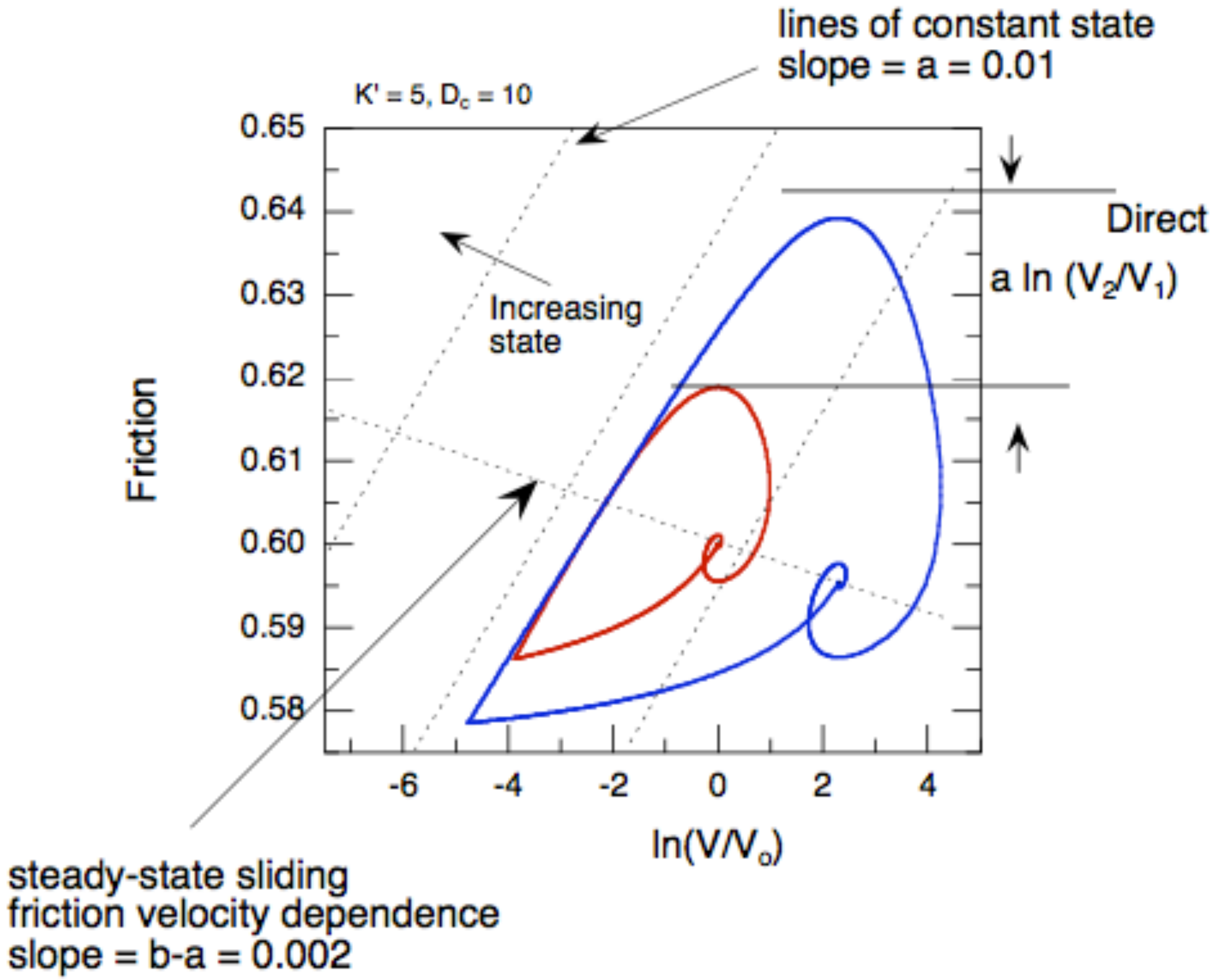
$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

$$\frac{d\mu}{dt} = k(V_{lp} - V)$$

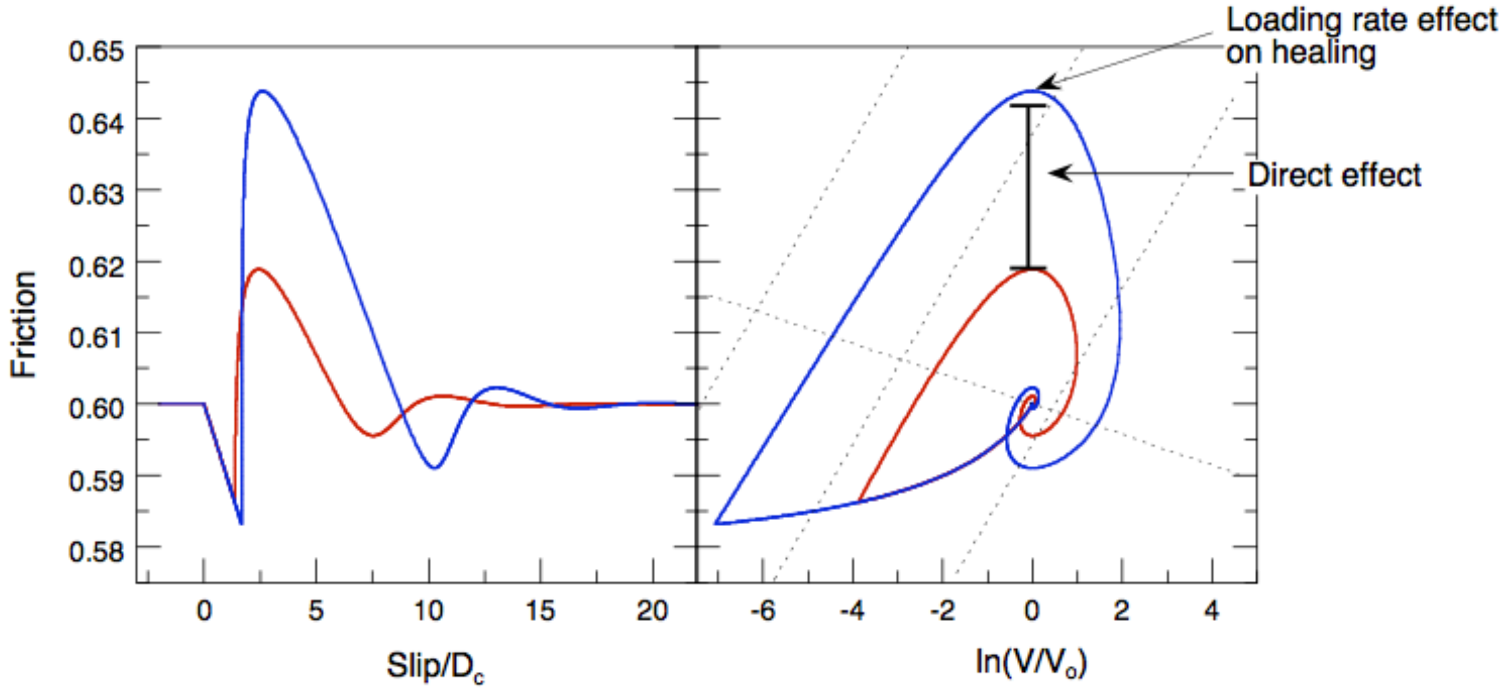
shs test:
1 μm/s
10 μm/s

Phase Plane Plots

shs test:
1 μm/s
10 μm/s



Derivation of the healing rate



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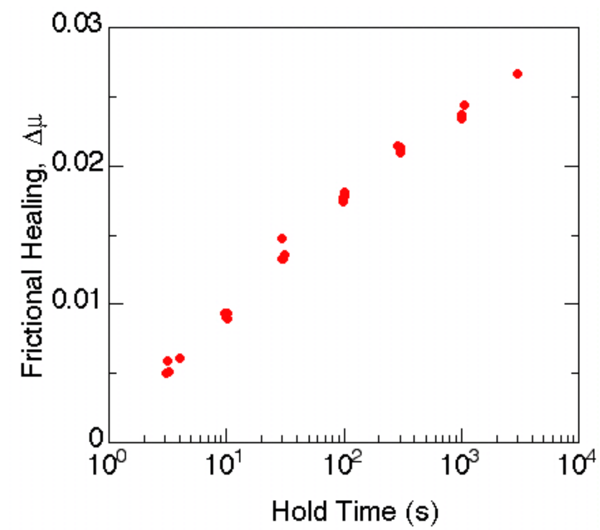
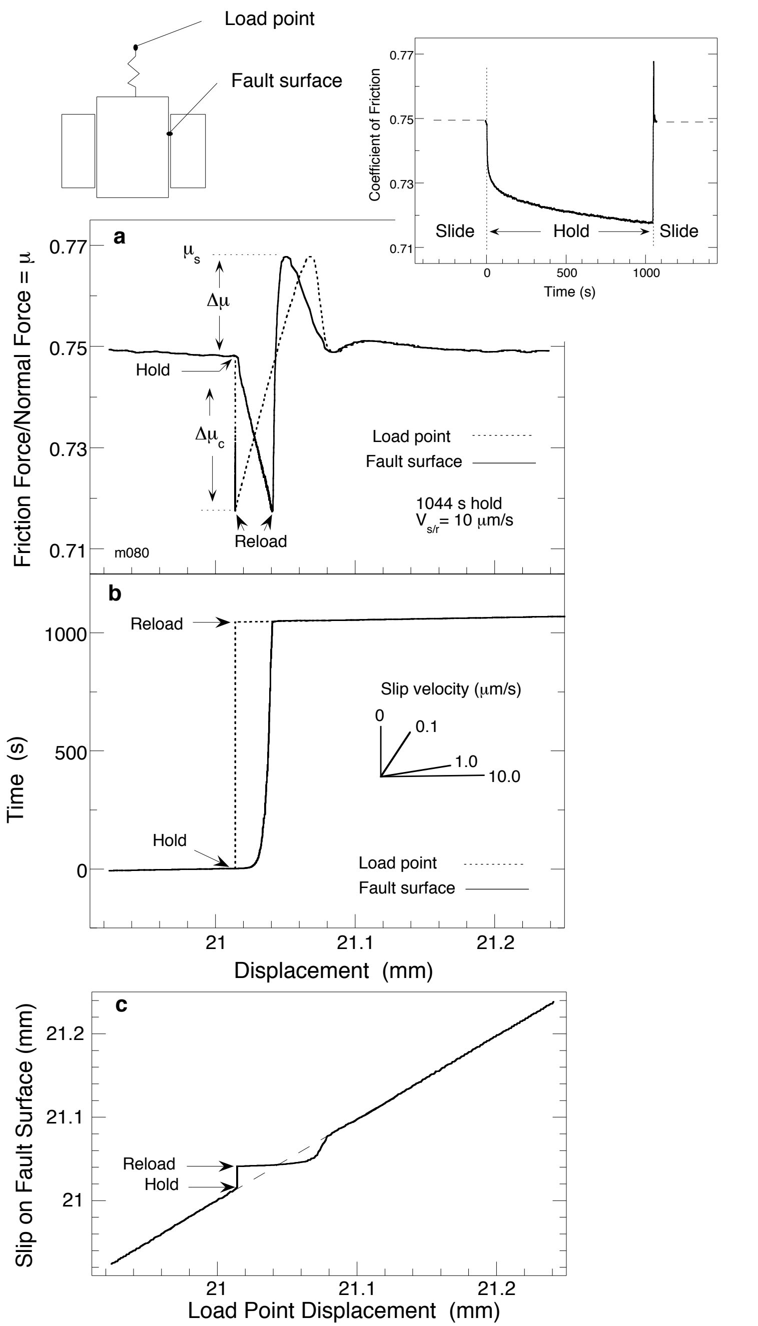


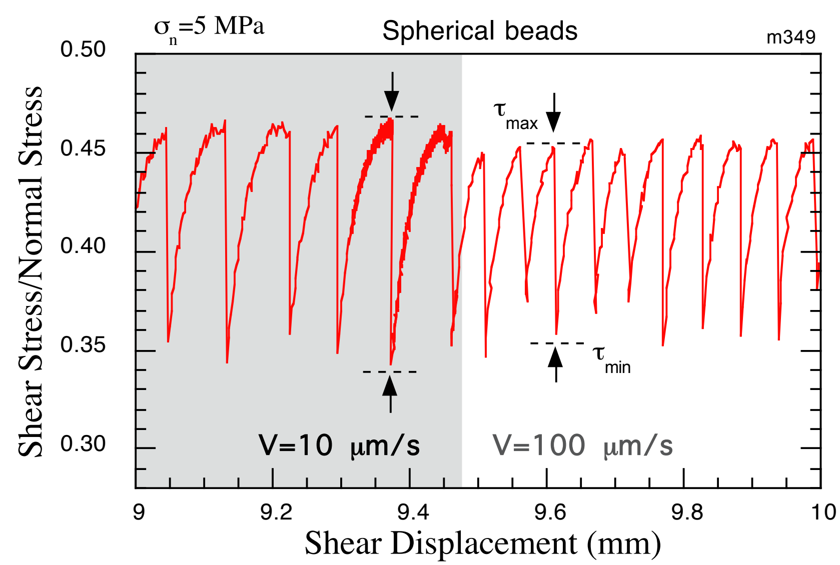
Slide-hold-slide

Slip-reload-slip

Earthquake-interseismic healing and reloading-earthquake

The full seismic cycle of stick-slip, frictional restrengthening, and interseismic reloading





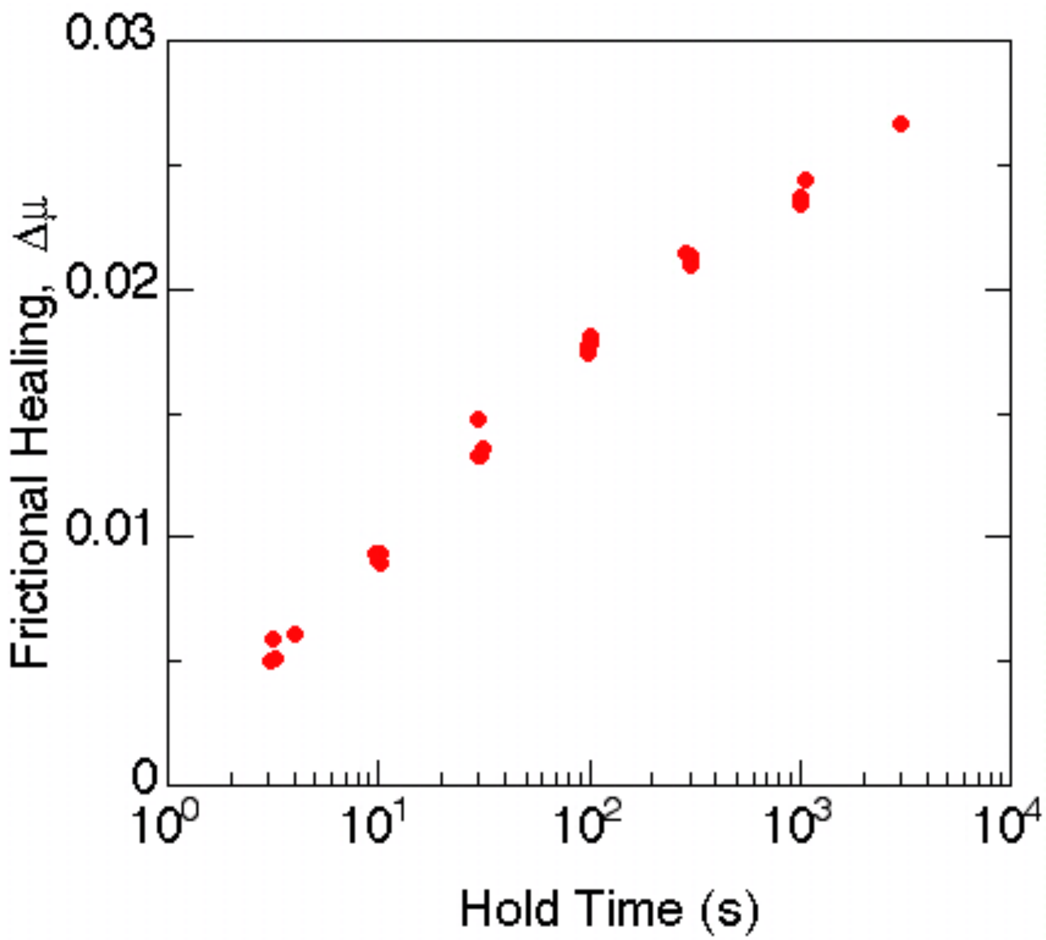
Marone and Saffer, 2013 Figure 4

Rate (v) and State (θ) Friction Constitutive Laws

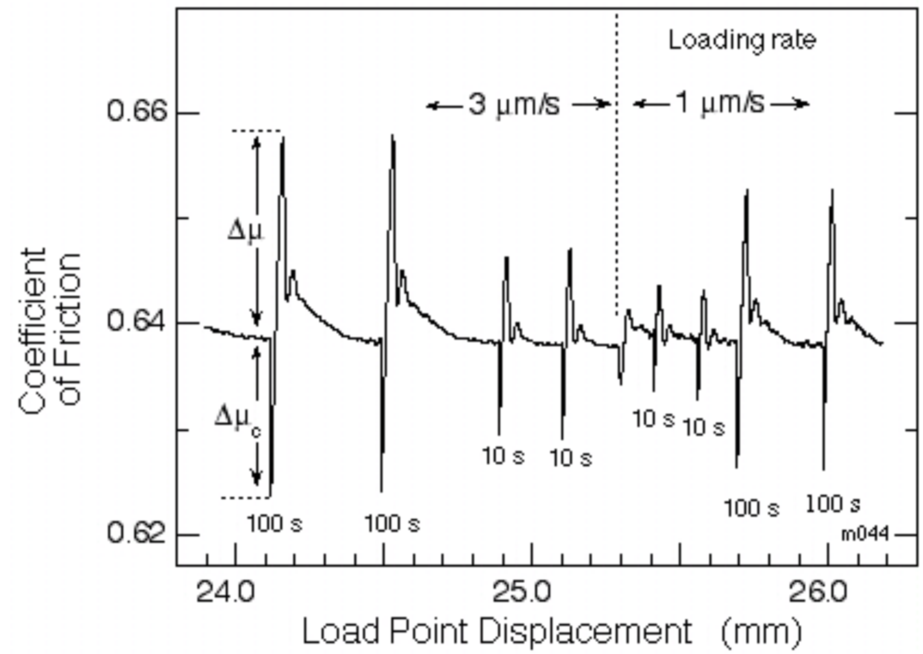
1) $$\mu(\theta, V) = \mu_o + a\ ln\left(\frac{V}{V_o}\right) + b\ ln\left(\frac{V_o\theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling



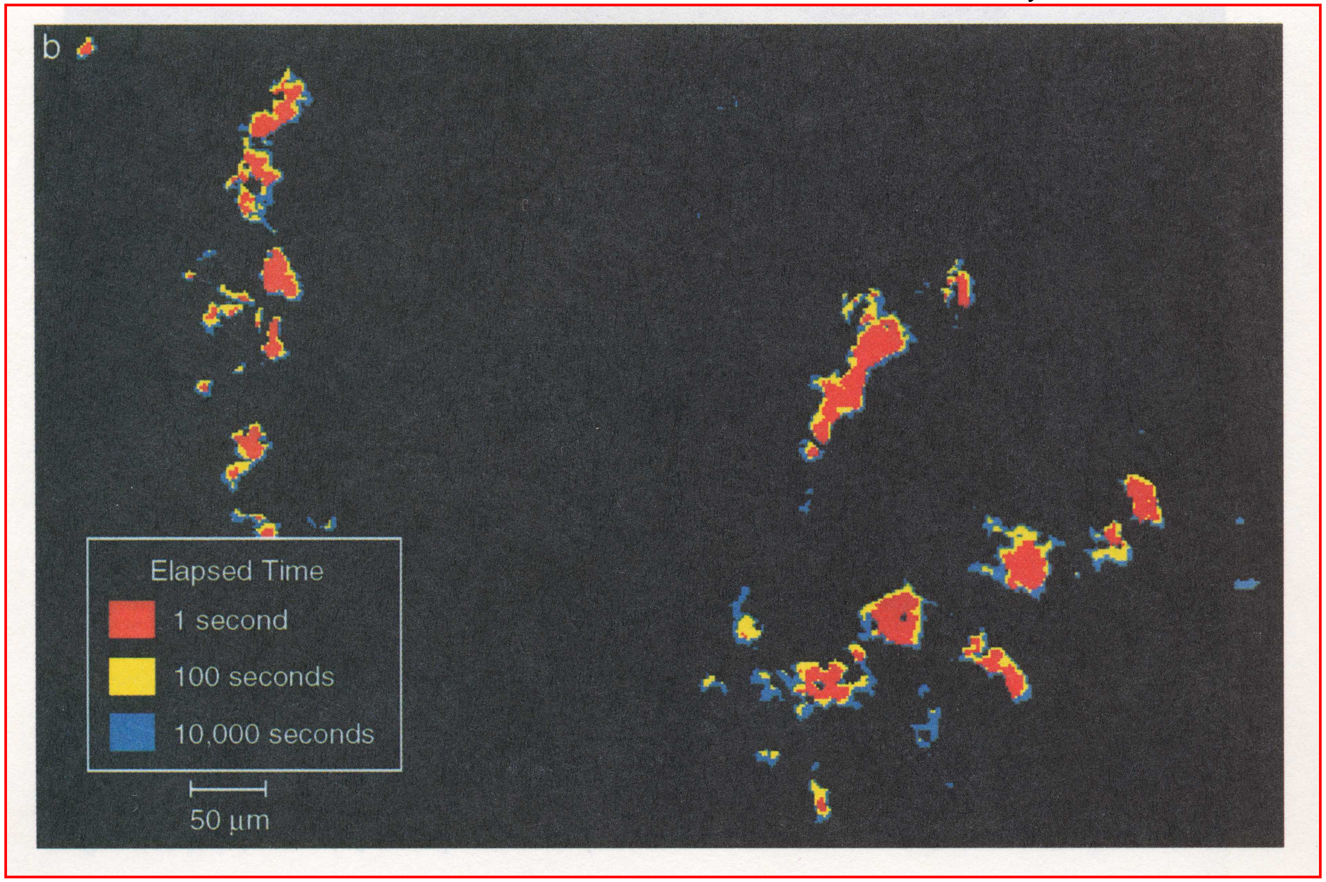
Modeling experimental data



$$\frac{d\mu}{dt} = k(V_{lp} - V)$$

Time dependent yield strength:

$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$



Dieterich and Kilgore [1994]

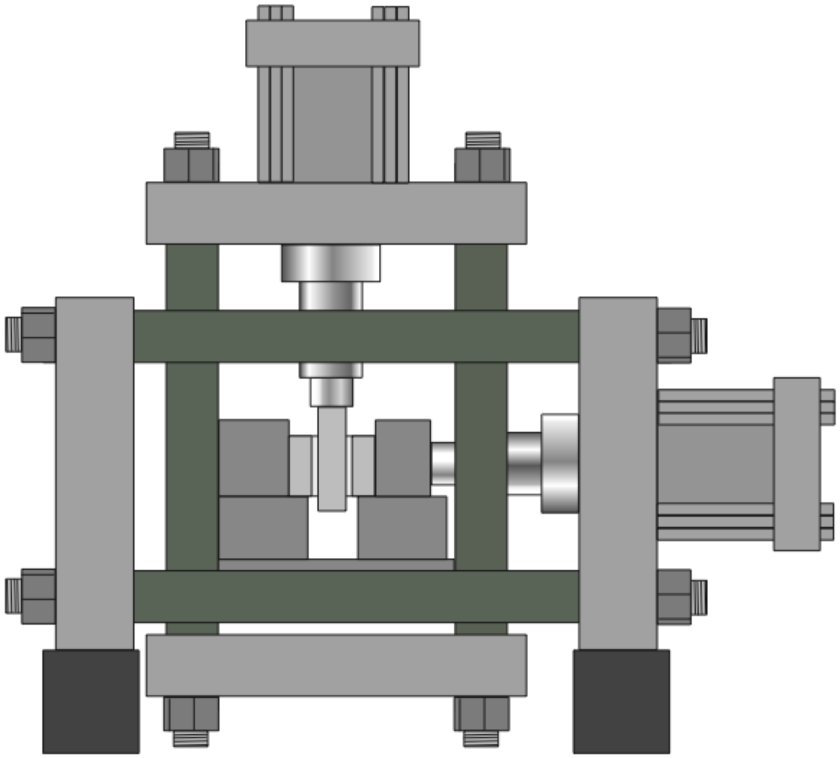
$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$

Time dependent growth of contact (acyrlic plastic)- true static contact

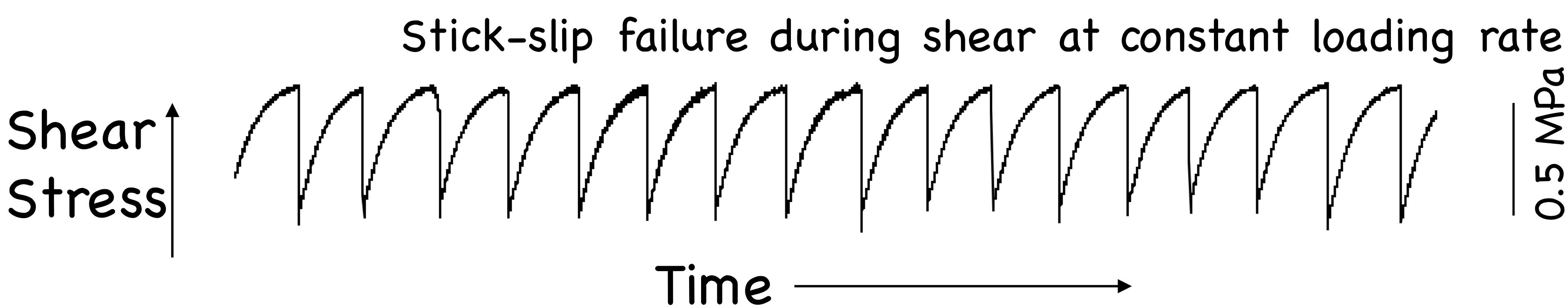
$$\sigma_y = \sigma_o + f(t)$$

Modified from Beeler, 2003

How do fault/frictional surfaces heal (regain strength) after failure?



Earthquakes & Fault Mechanics: seismic cycle, fault reactivation.
(friction and stick slip: doors, windows, machines, ships in dry dock, dancers…)



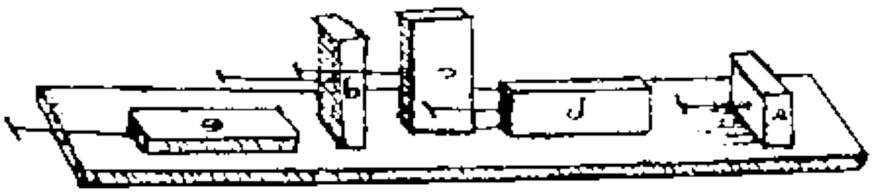
Time dependence of "static" friction
Aging of frictional contacts

C. A. Coulomb (1736-1806)

Table 9.1

	T (time of repose, min)	$A+mT^{\mu}$ (static friction force, lbf)
Ière observation	0	$A=502$
IIe	2	790
IIIe	4	866
IVe	9	925
Ve	26	1,036
VIe	60	1,186
VIIe	960	1,535

static friction of two pieces of well-worn oak lubricated with tallow.



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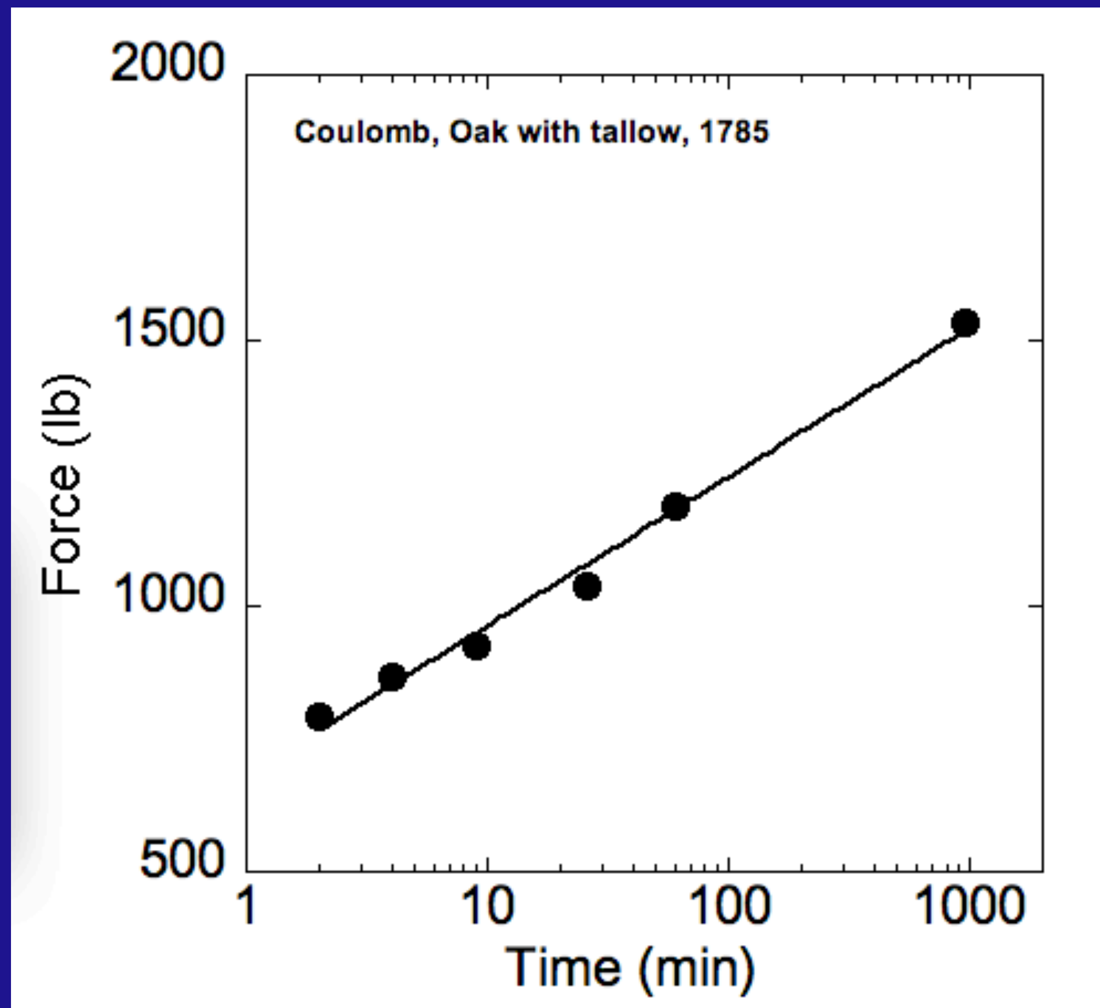
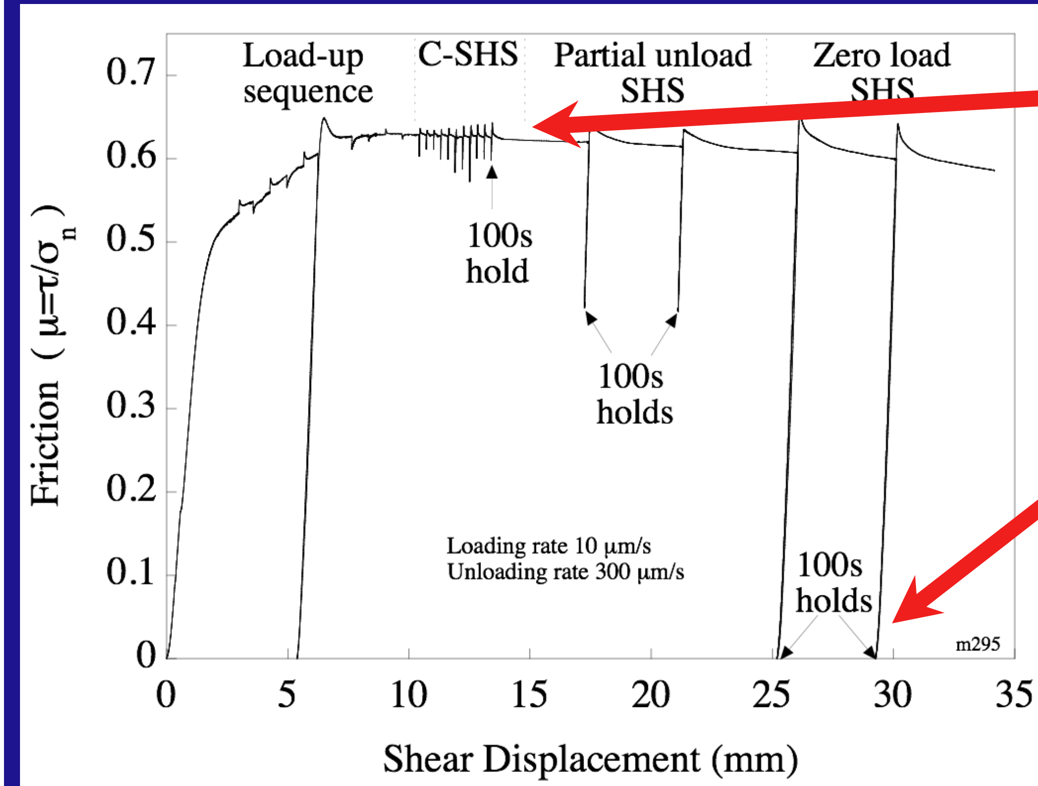
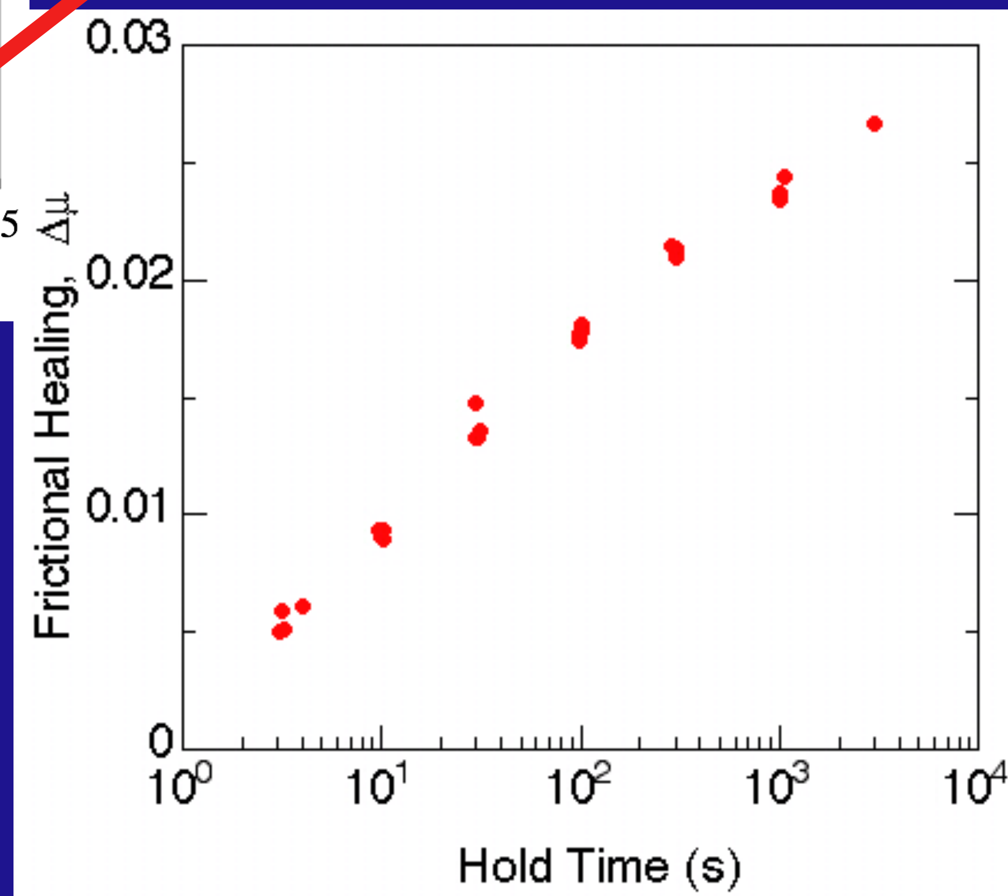


Time dependence of friction in rocks; Macroscopic frictional aging

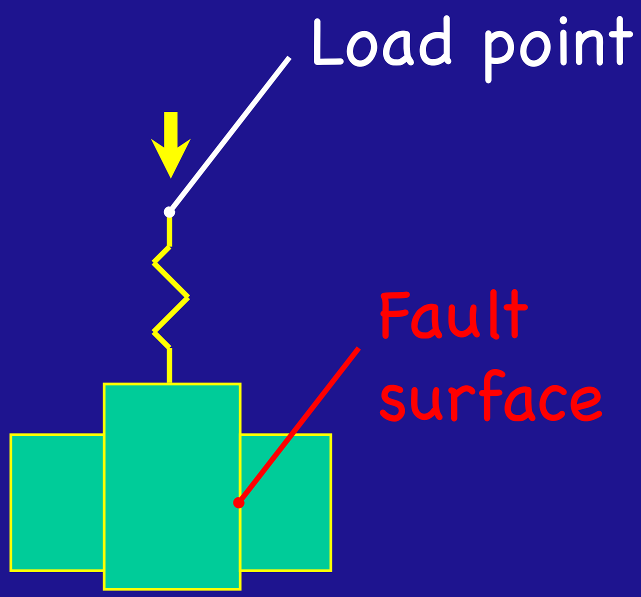


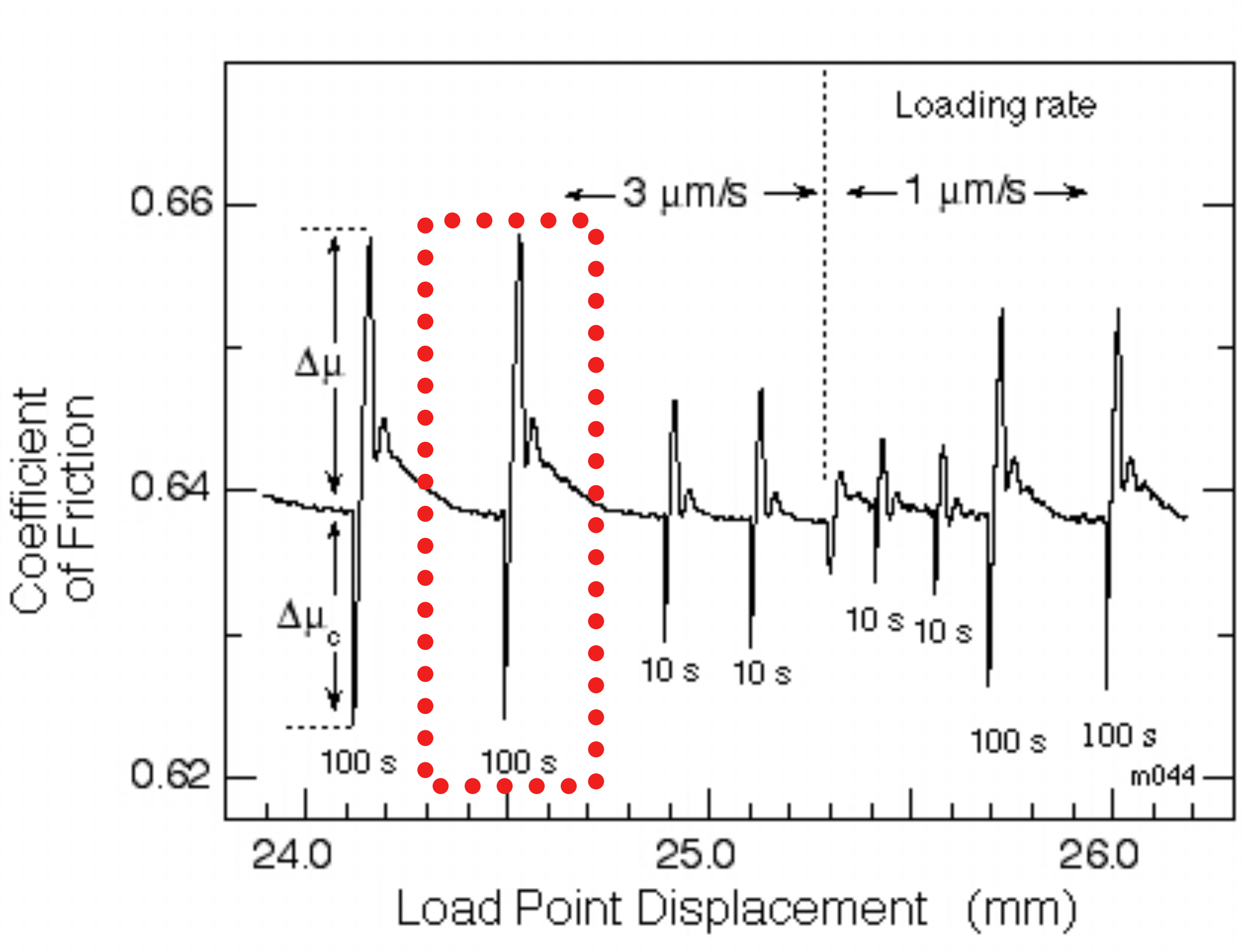
Karner and Marone, J.Geophys. Res, 2001.

Stressed vs. unstressed aging



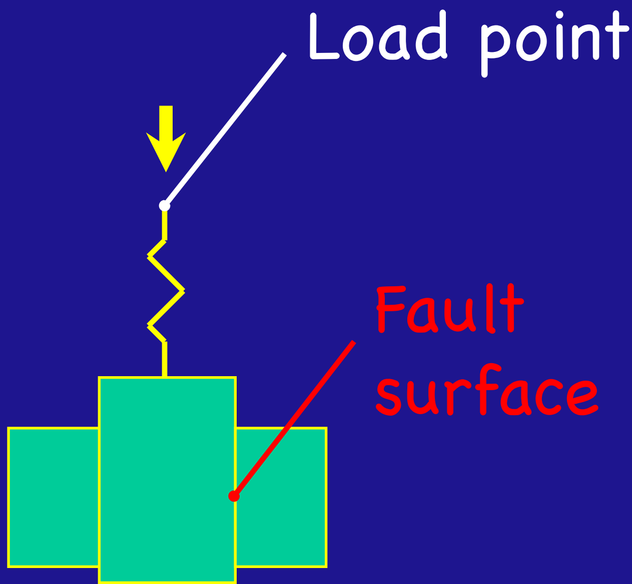
Frictional Healing
Stressed aging

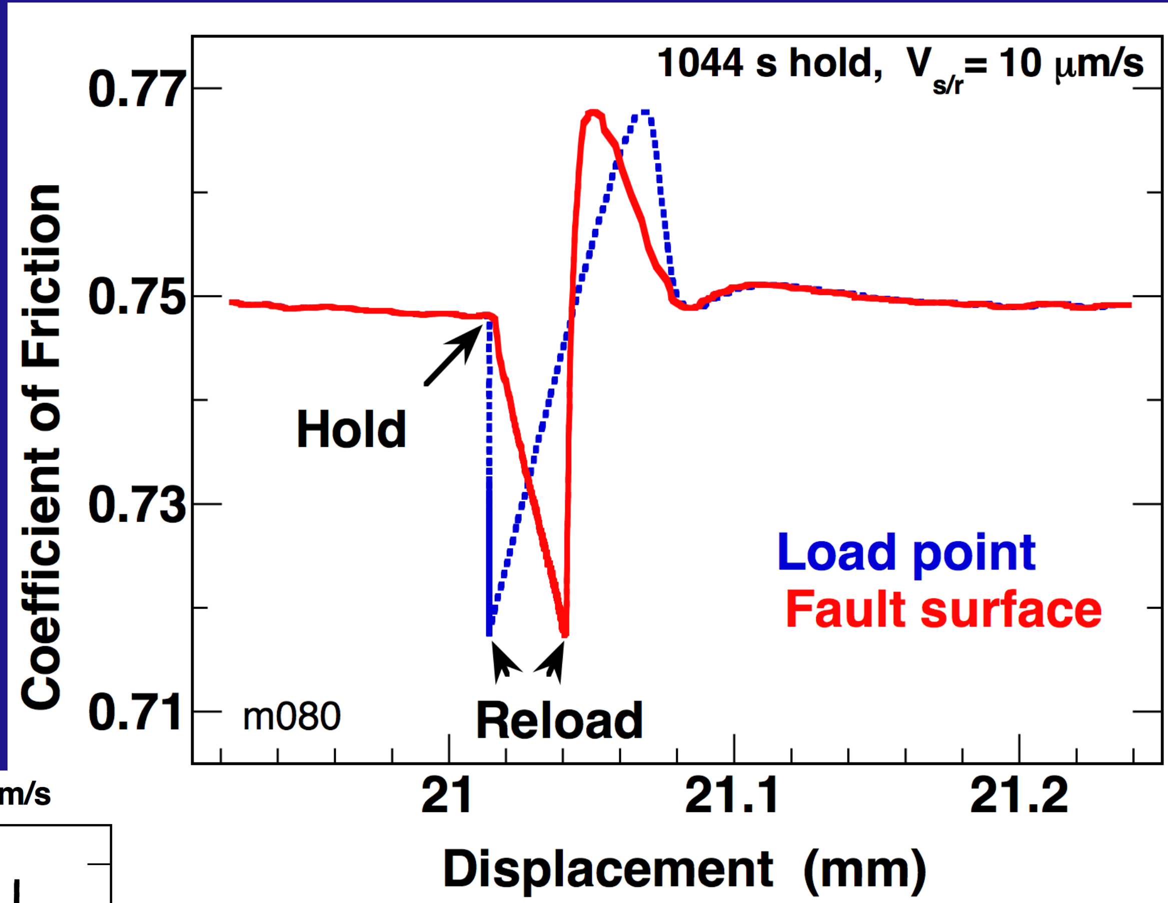


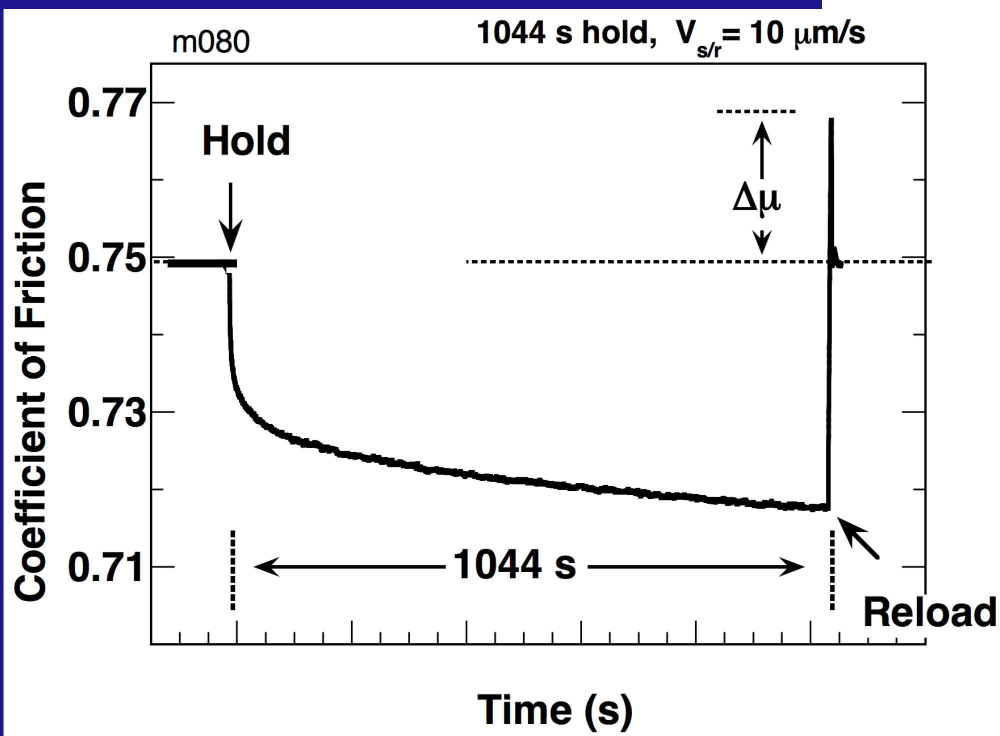


Steady state friction & the rate of healing vary with sliding velocity

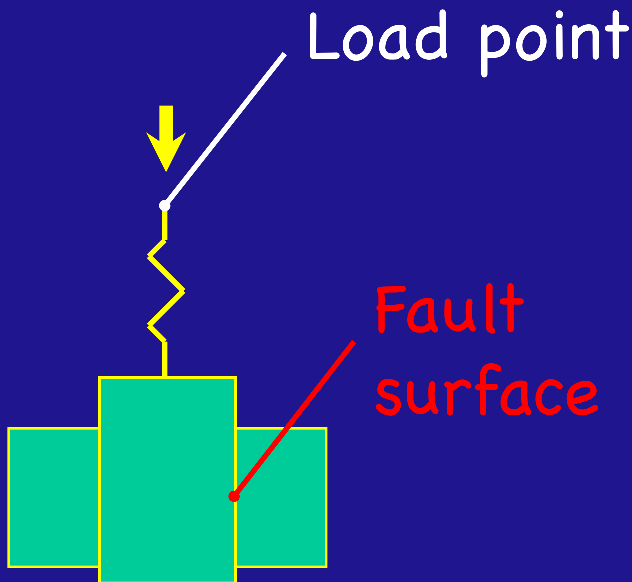
Angular quartz particles (100-150 µm), 3 mm thick, 25 MPa normal stress. Marone, Nature, 1998

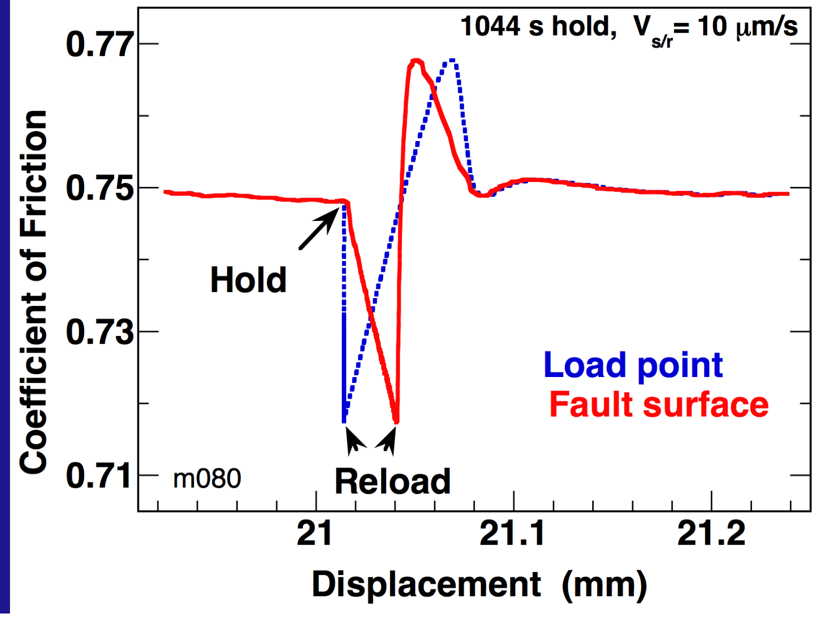


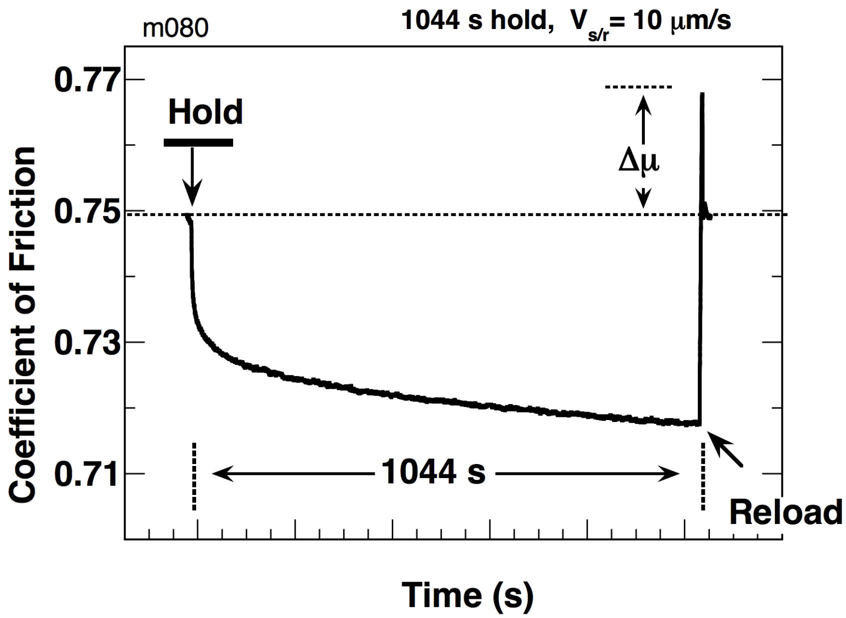


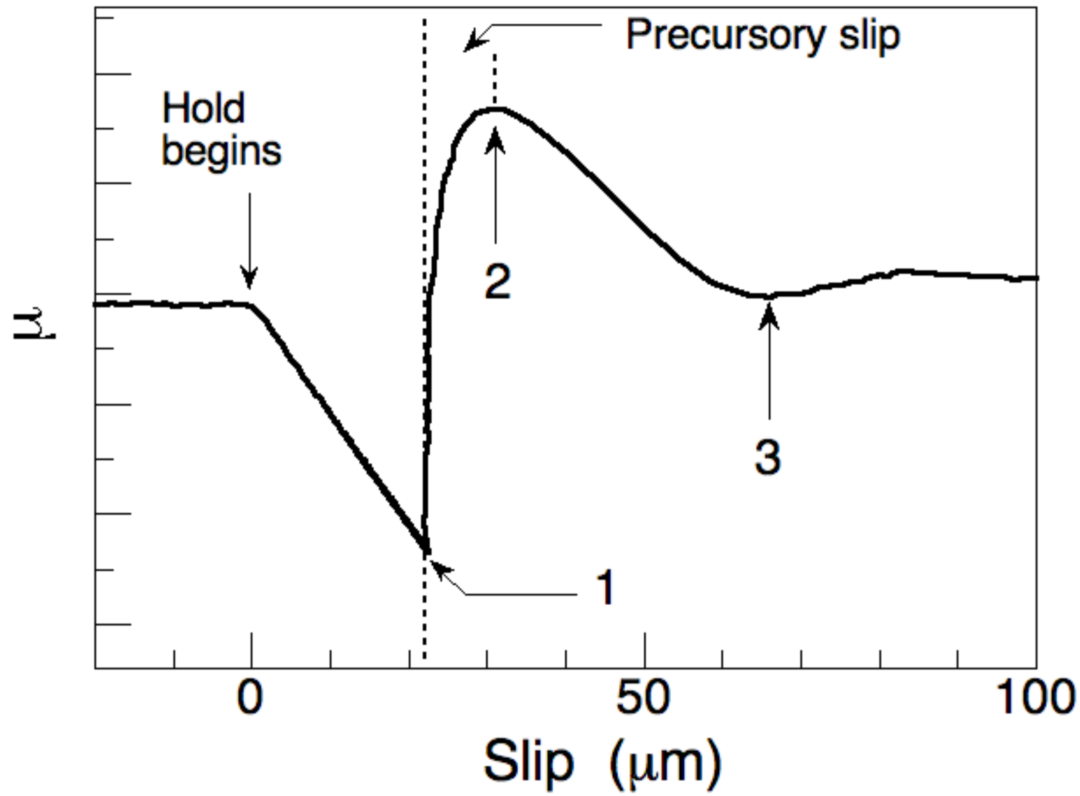


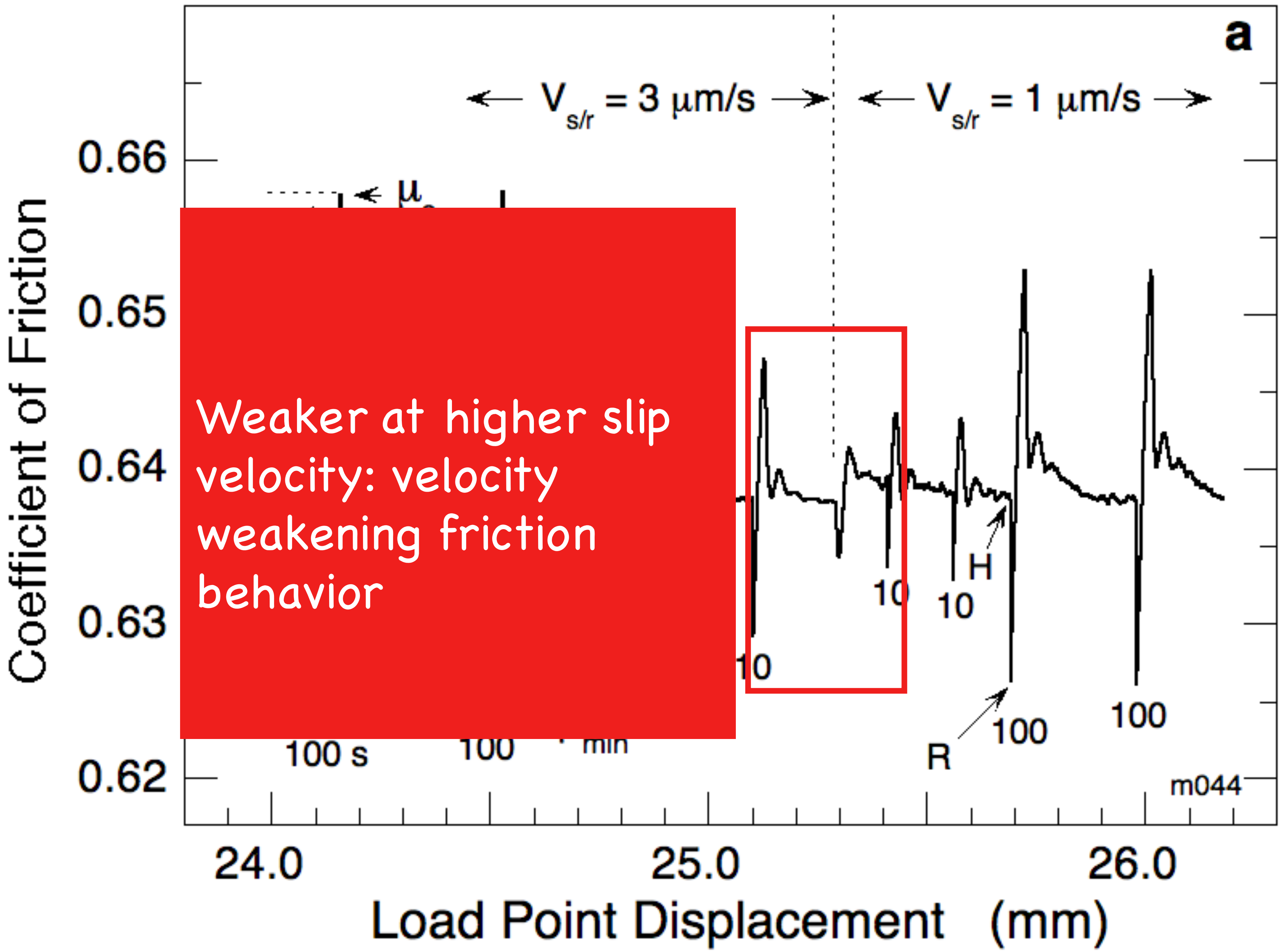
Stress relaxation via creep


Load point
Fault
surface


1044 s hold, $V_{s/r}$ = 10 μm/s
Coefficient of Friction
0.77
0.75
0.73
0.71
Hold
Load point
Fault surface
m080
Reload
21
21.1
21.2
Displacement (mm)

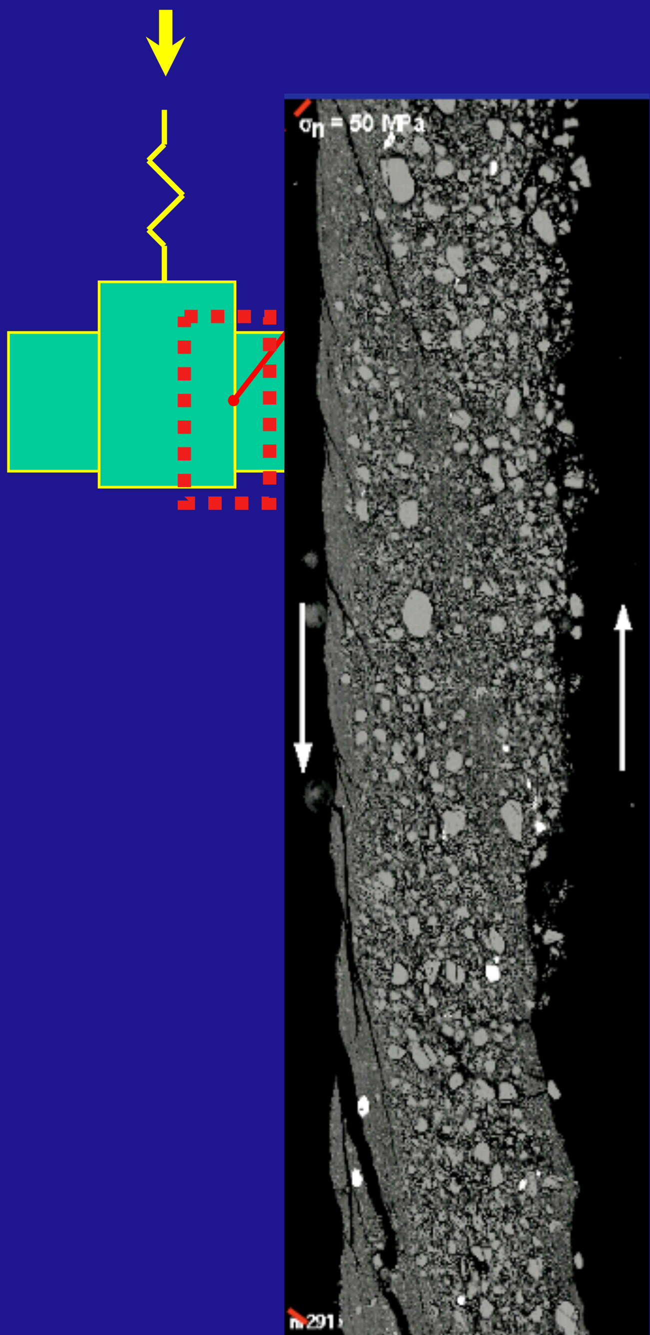

m080
1044 s hold, $V_{s/r}$ = 10 μm/s
Coefficient of Friction
0.77
0.75
0.73
0.71
Hold
Δμ
1044 s
Reload
Time (s)

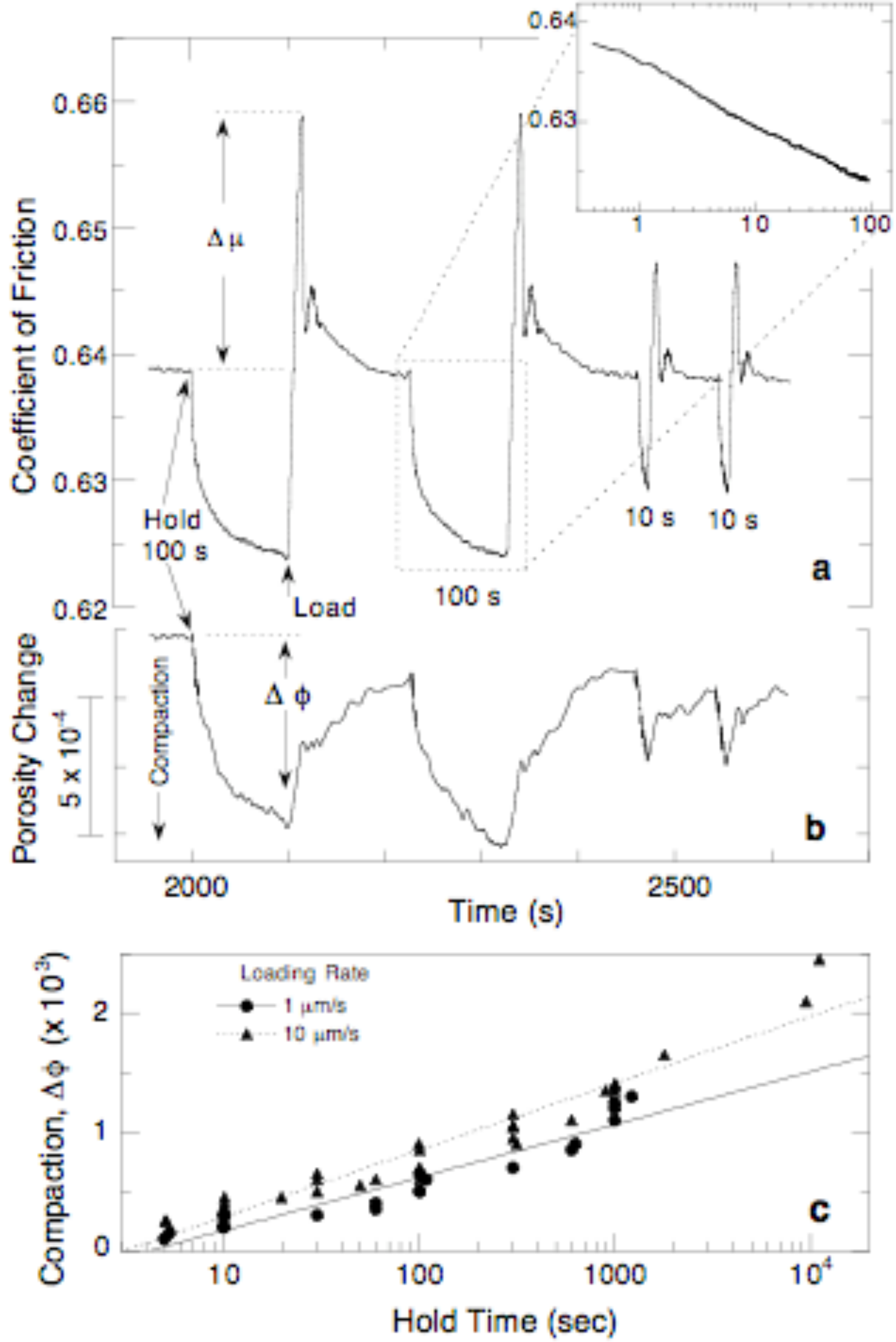

Precursory slip
Hold
begins
μ
2
3
1
0
50
100
Slip (μm)

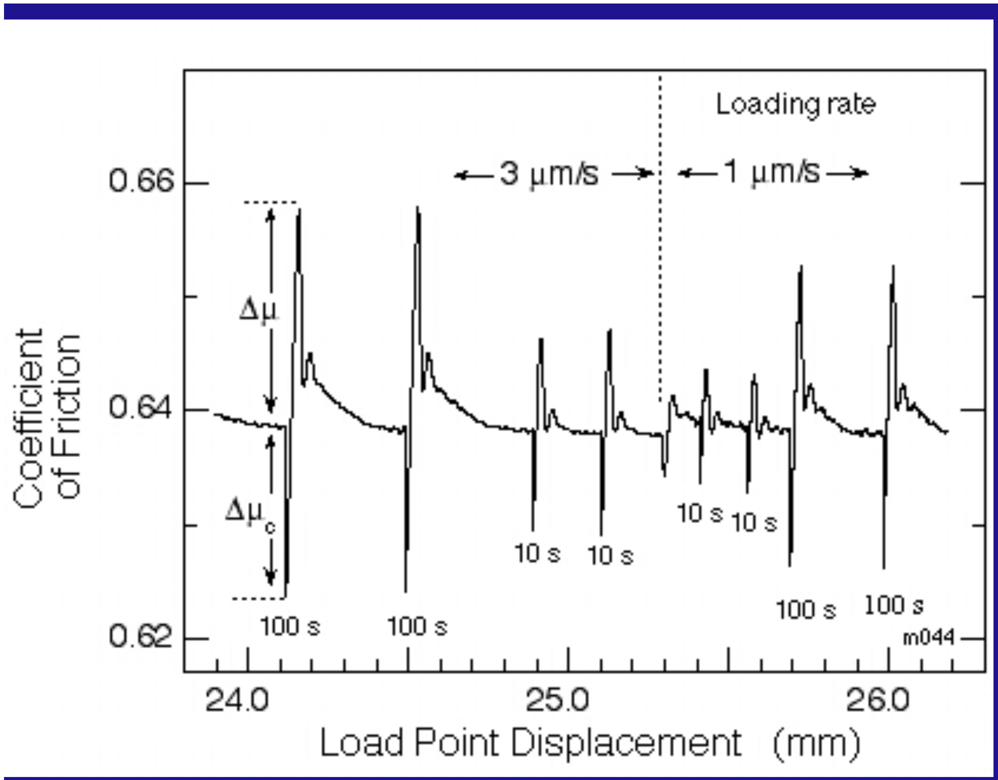


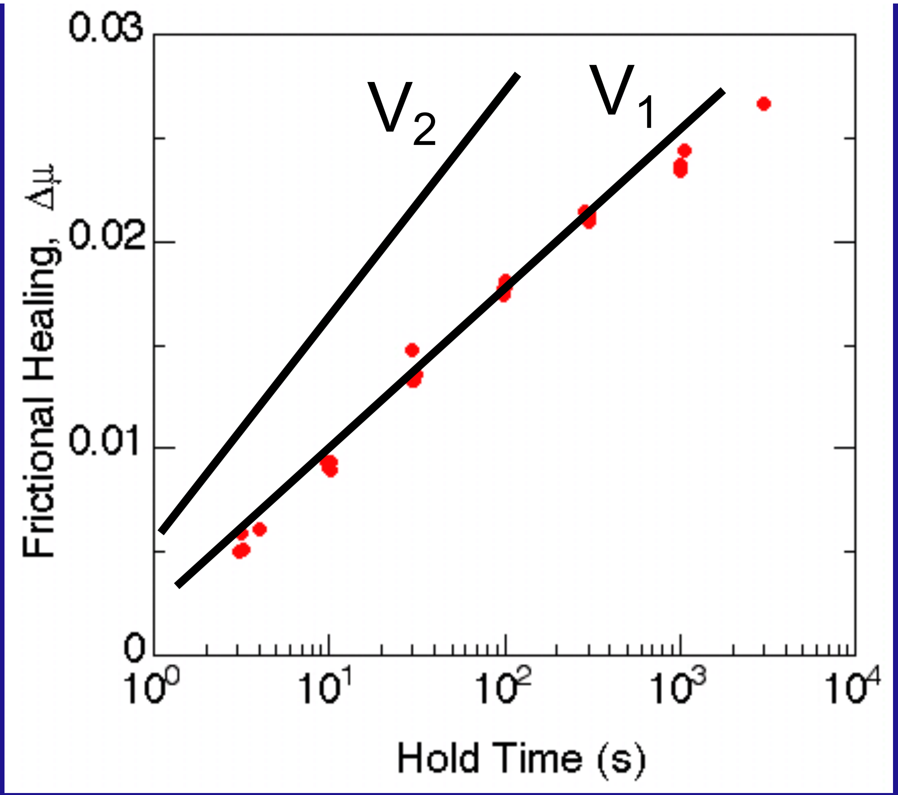
Weaker at higher slip velocity: velocity weakening friction behavior

Marone, Nature, 1998


σn = 50 MPa


Coefficient of Friction
0.66
0.65
0.64
0.63
0.62
Δ μ
Hold
100 s
Load
100 s
10 s
10 s
a
0.64
0.63
1
10
100
Porosity Change
5 x 10⁻⁴
Compaction
Δ ϕ
b
2000
2500
Time (s)
Compaction, Δϕ (x 10⁻³)
Loading Rate
1 μm/s
10 μm/s
c
0
1
2
10
100
1000
10⁴
Hold Time (sec)



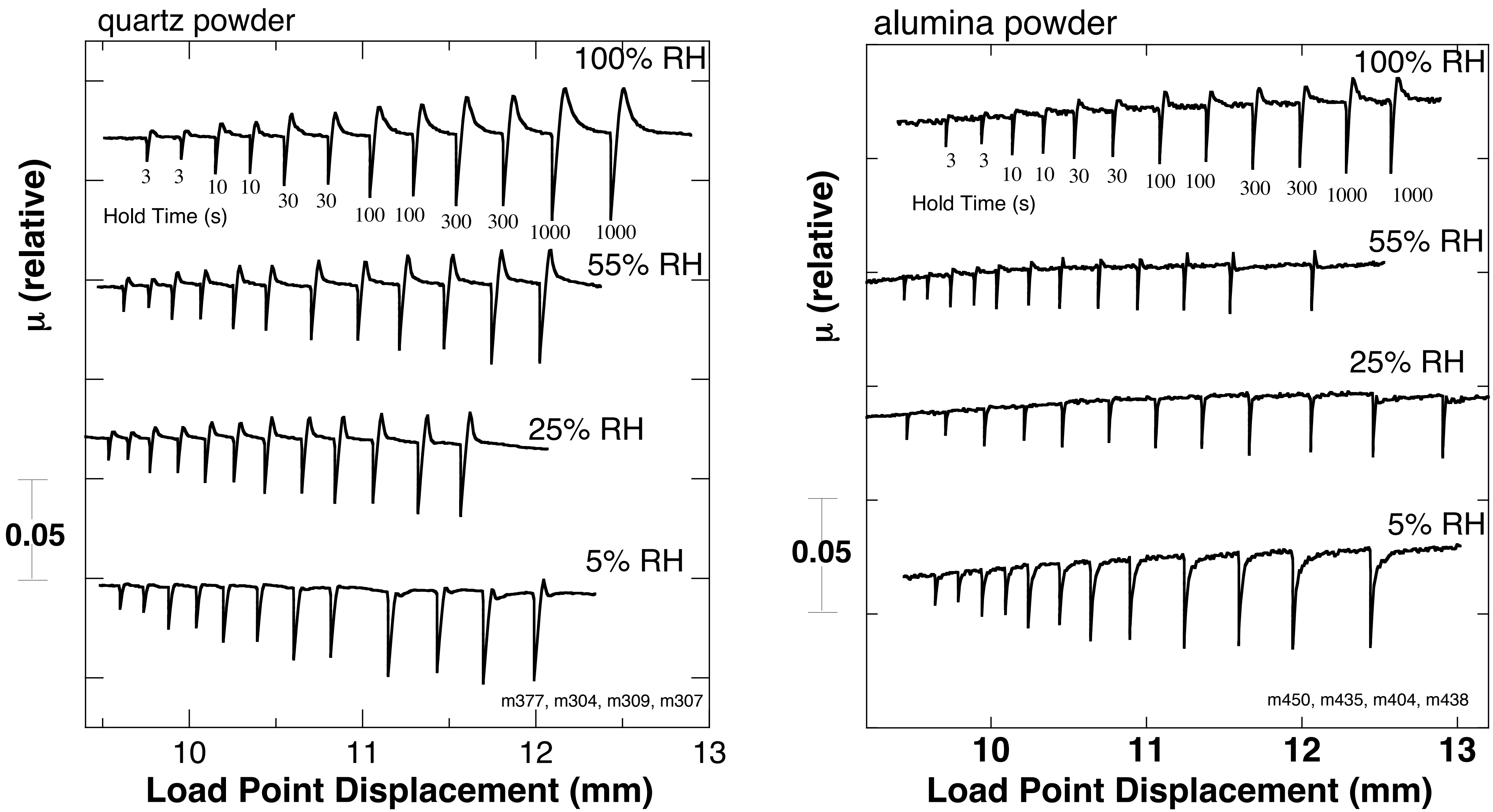


Frictional aging.

How does it work?

What is the role of shear velocity?

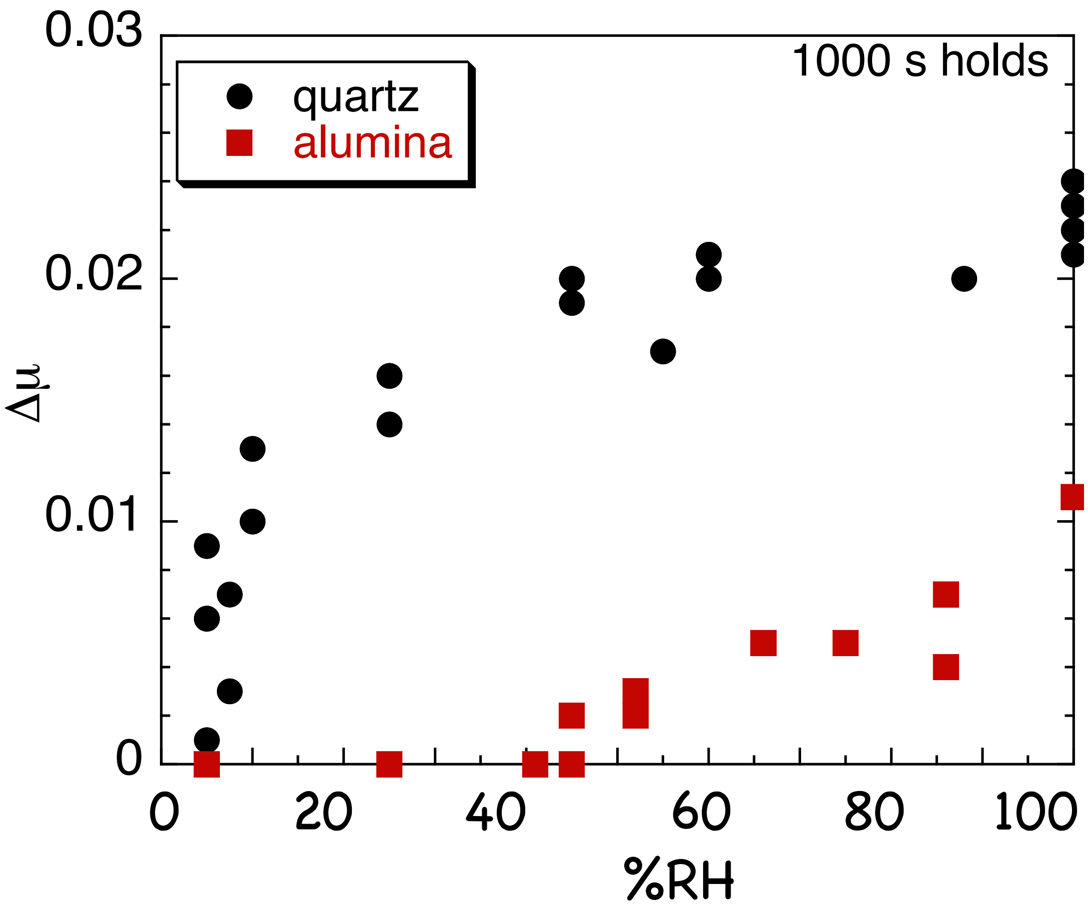
Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions



In-situ Particle Comminution; Production of Fresh Surface Area

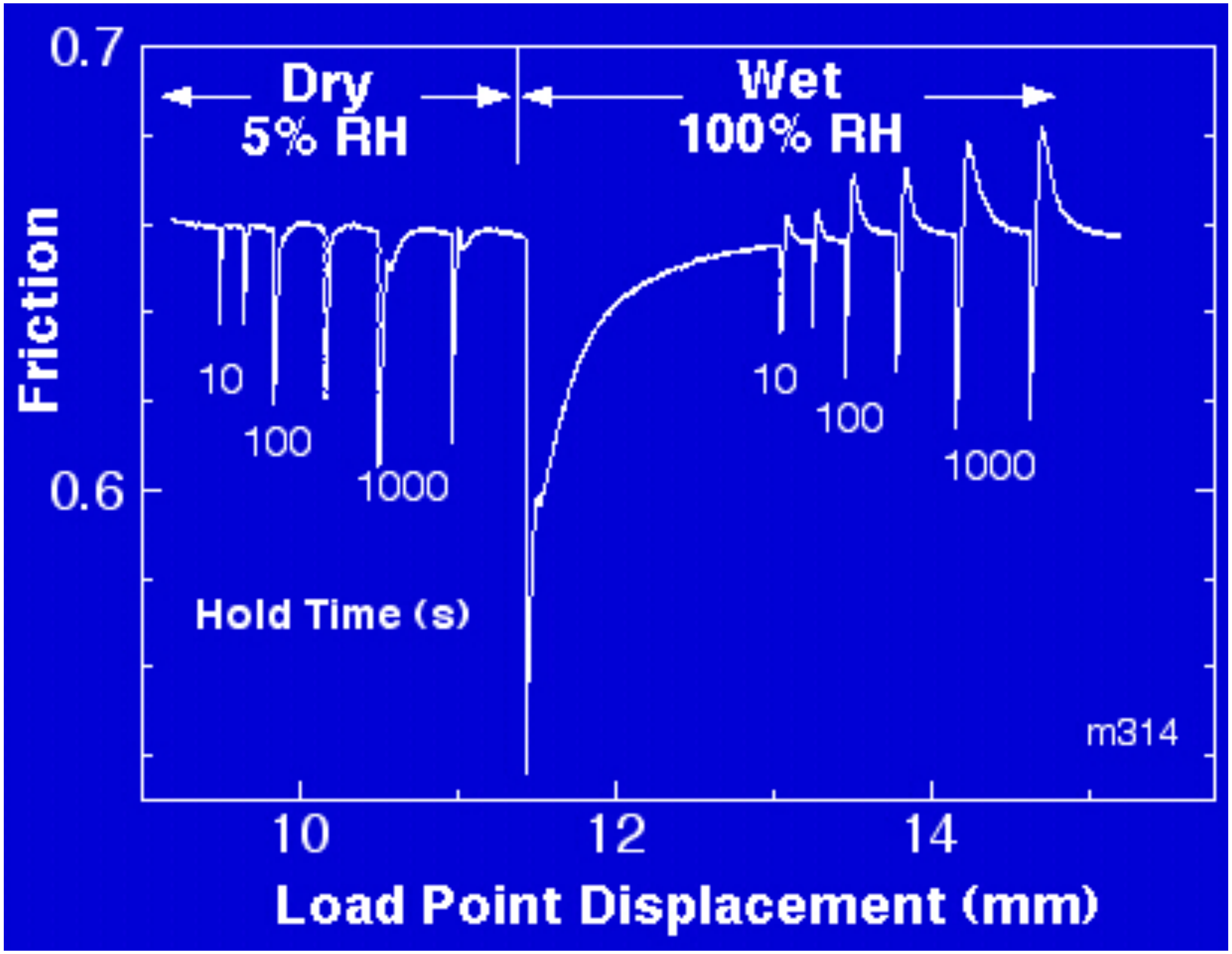
Frye and Marone, JGR 2002

Granular quartz



Hydrolytic Weakening causes enhanced rate of strengthening

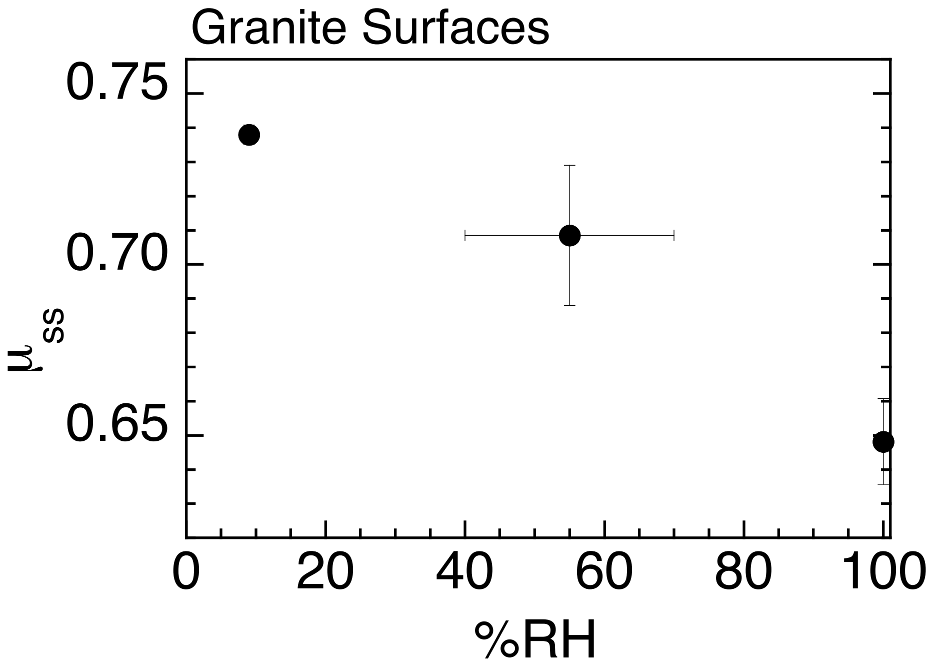
Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions



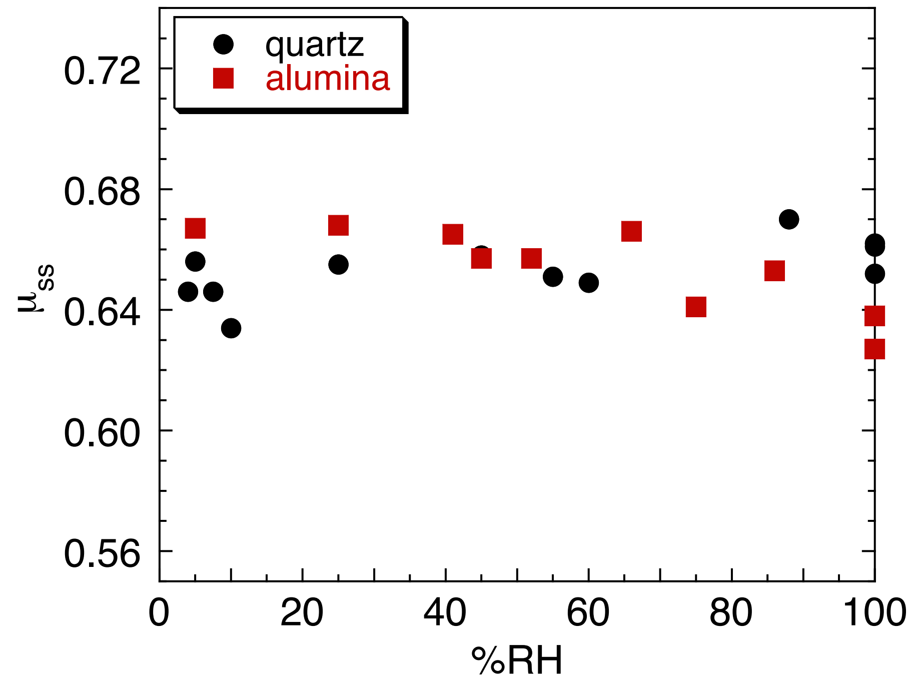
Frye and Marone, JGR 2002

Hydrolytic Weakening causes enhanced rate of strengthening, but base level frictional strength is unchanged

Granite Surfaces



Granular Materials



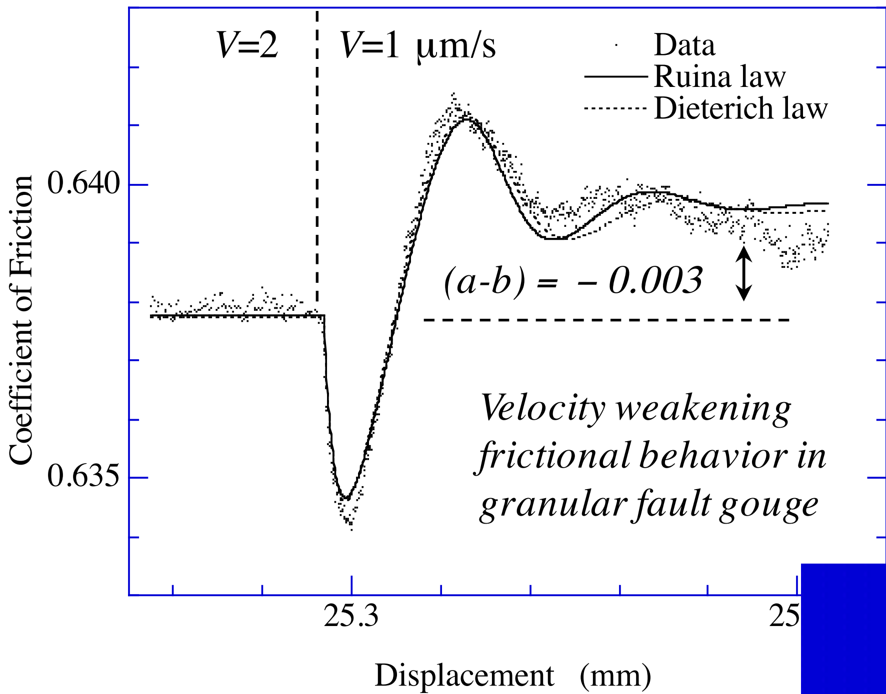
Solid Surfaces: Base level of frictional strength decreases with increasing water content (cf. Dieterich & Conrad, 1984)

Granular Materials: Frictional strength is independent of water content

Interpretation: Contact junctions subject to time dependent strengthening or growth, which inhibits sliding, but particle rolling is not affected by these factors.

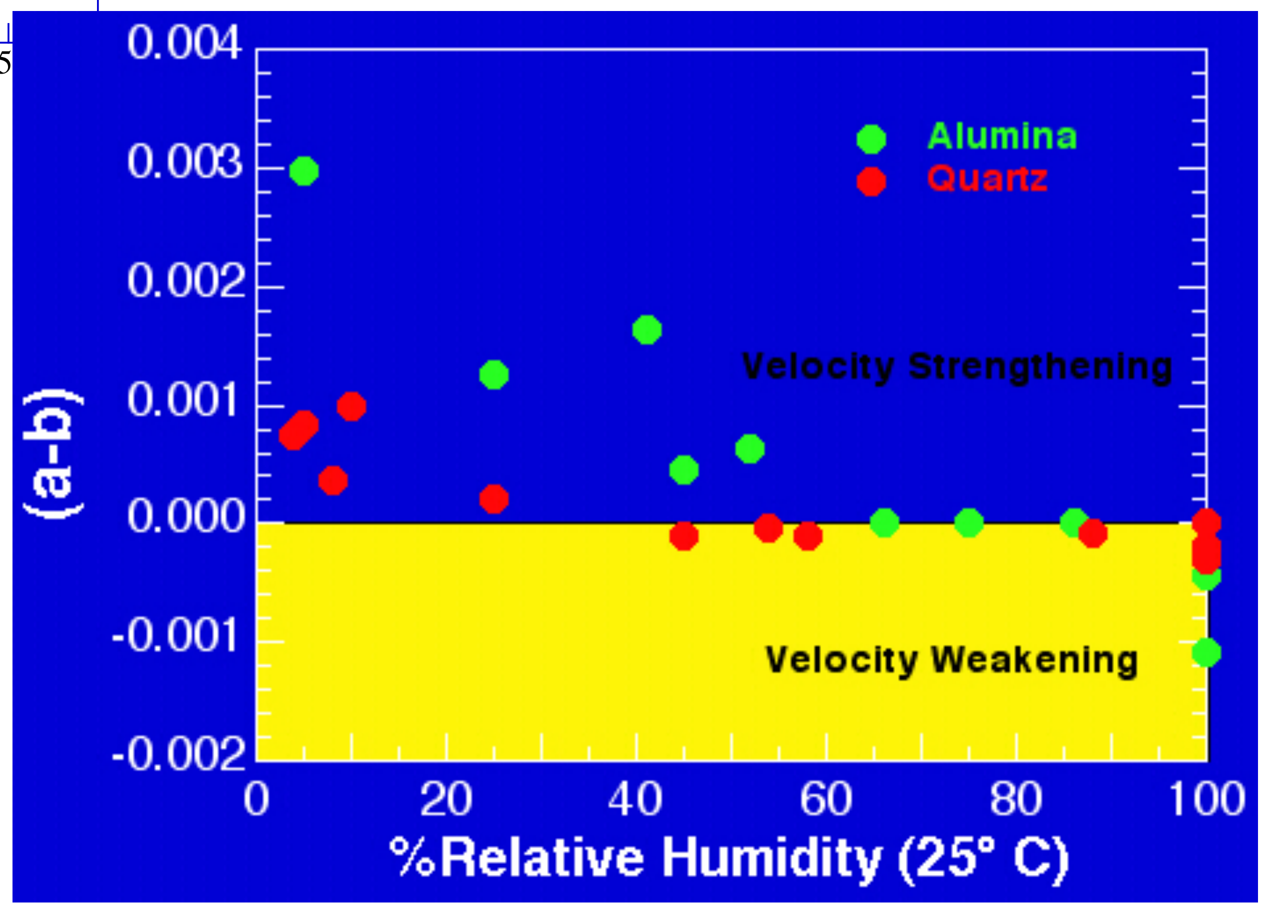
Frye and Marone, JGR 2002

Empirical laws, based on laboratory friction data



Velocity dependence of steady state friction varies changes from positive to negative. (cf. Tullis and co-workers)

Chemically-assisted creep at adhesive contact junctions

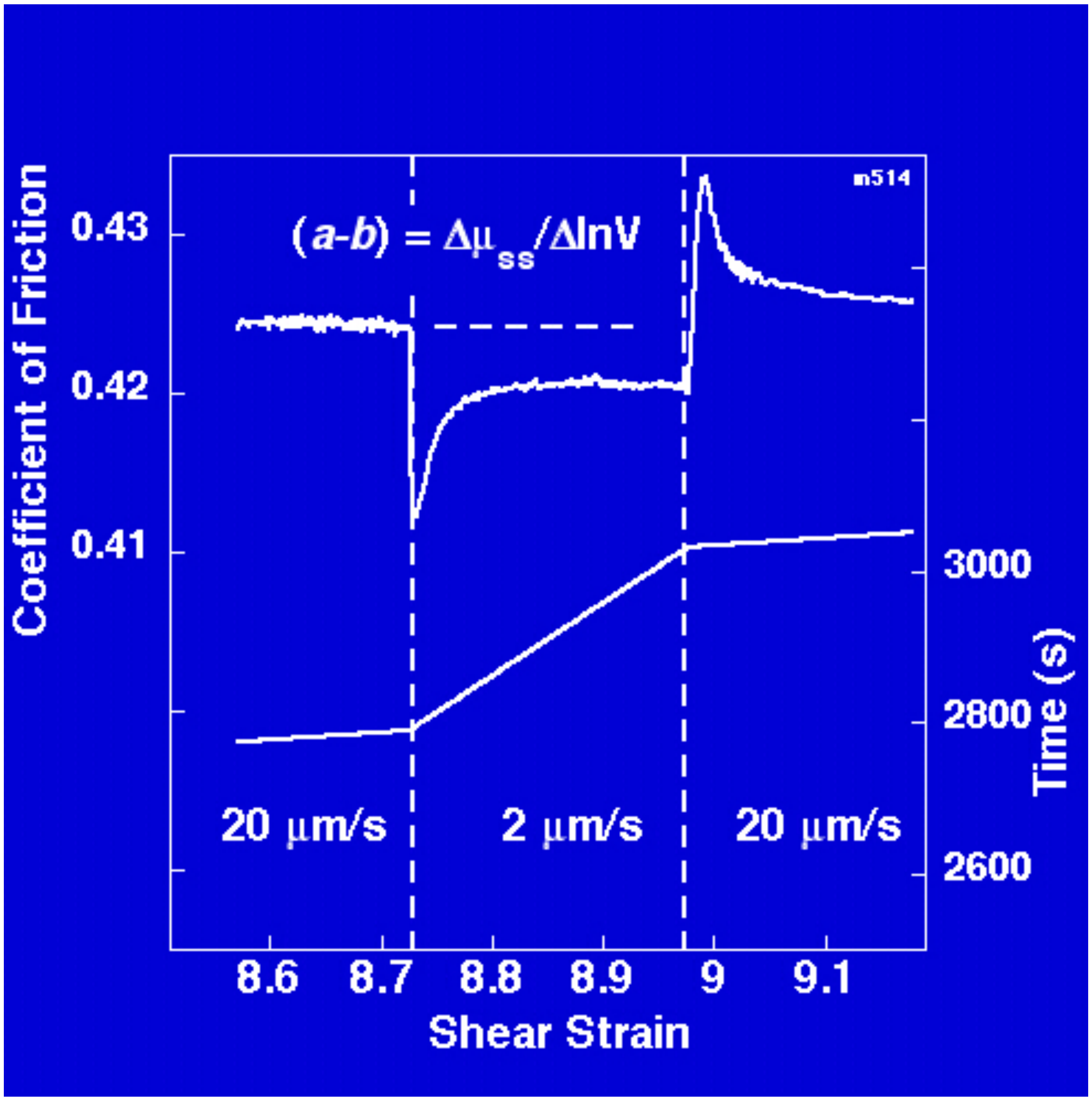


Frye and Marone, JGR 2002

Frictional Instability

Requires (a-b) < 0

Measuring the velocity dependence of friction



Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

$$\mu(\theta, v) = \mu_0 + a\ln\left(\frac{v}{v_o}\right) + b\ln\left(\frac{v_o\theta}{D_c}\right)$$

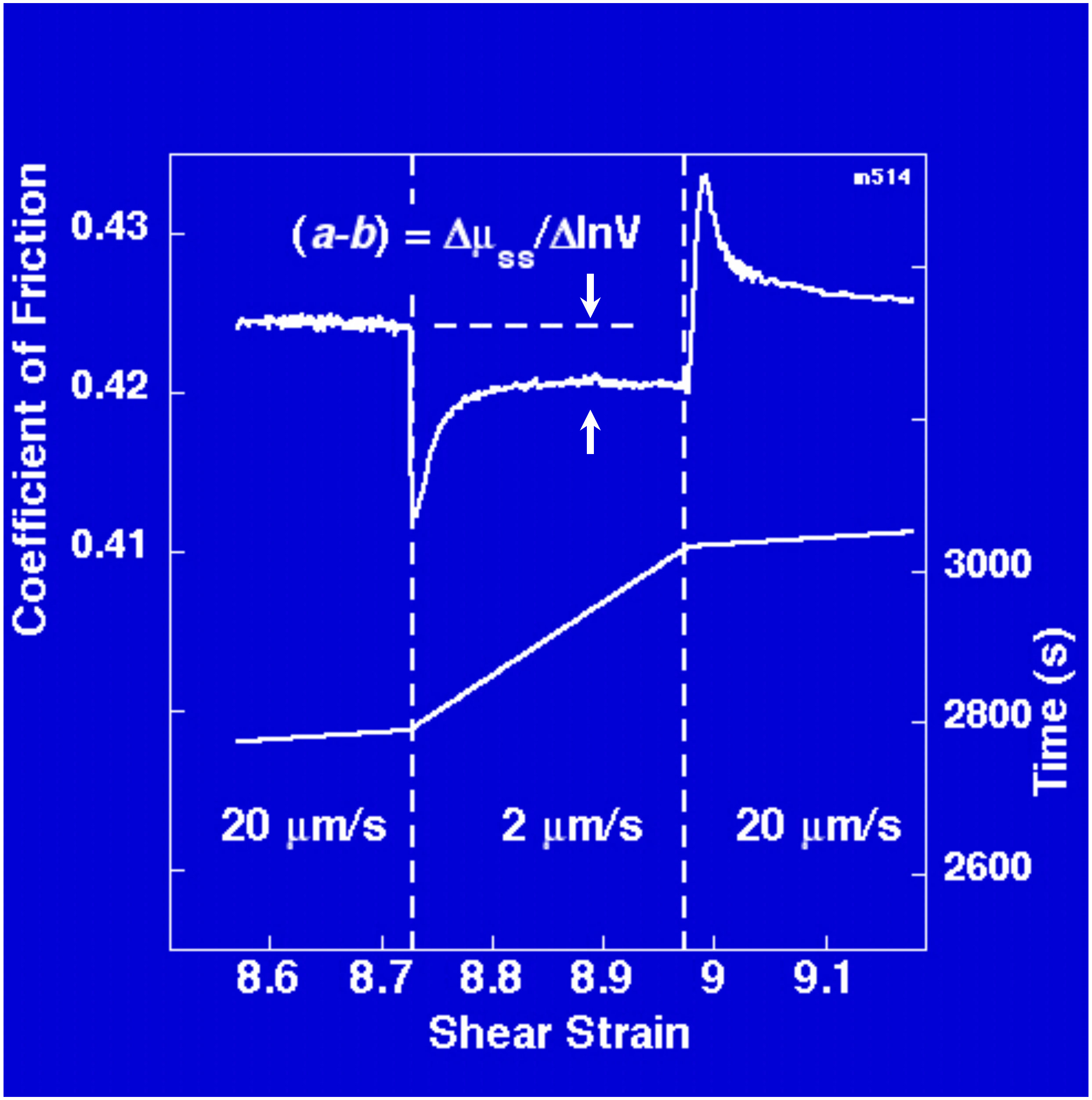
$$\frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c}$$

$$\theta_{ss} = \frac{D_c}{v}$$

$$\Delta\mu_{ss} = (a-b)\ln\left(\frac{v}{v_o}\right)$$

$$\frac{d\mu}{dt} = k'\left(v_{lp} - v\right)$$

Results: Velocity stepping
Measuring the velocity dependence of friction



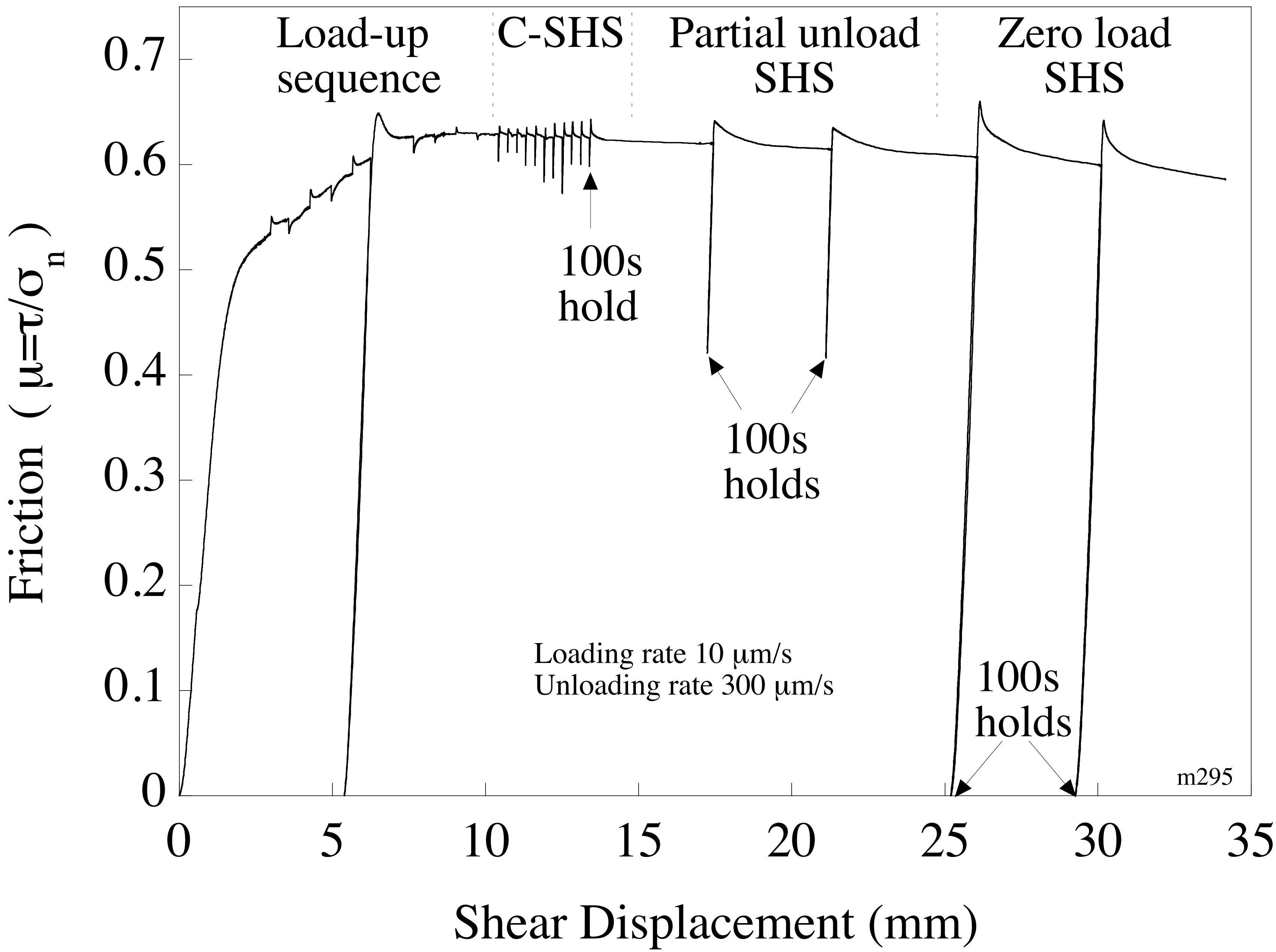
Frictional Instability

Requires K < K_c

$$K_c = \frac{\sigma_n (b-a)}{D_c}$$

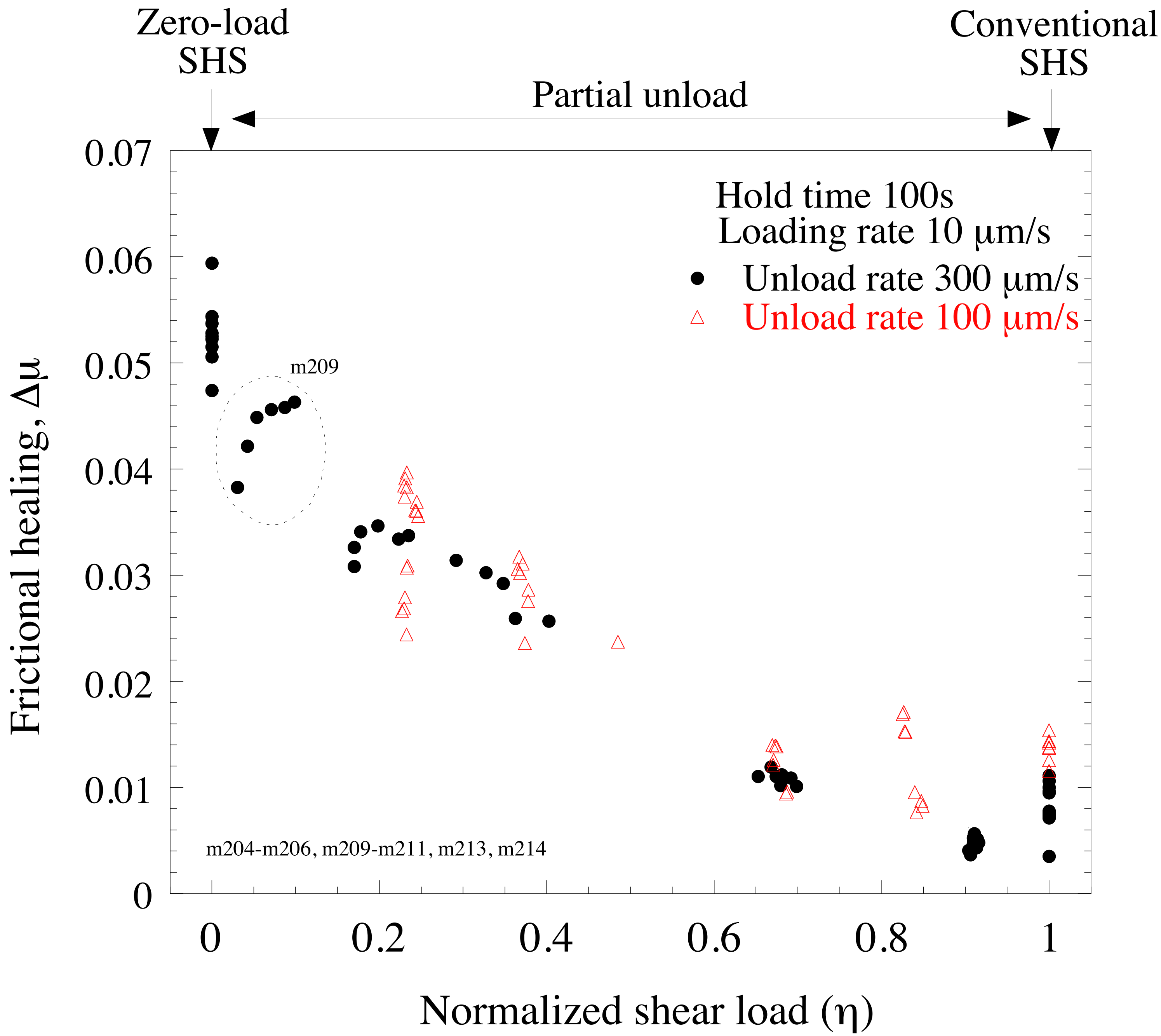
This example shows steady-state velocity strengthening: (a-b) > 0

Stresses v. Unstressed Aging



Karner & Marone (GRL 1998, JGR 2001)

100 s holds, Healing rate varies systematically with shear stress



Karner & Marone (GRL 1998 JGR 2001)