Mechanics of Earthquakes and Faulting

Lecture 8, 23 Feb. 2021

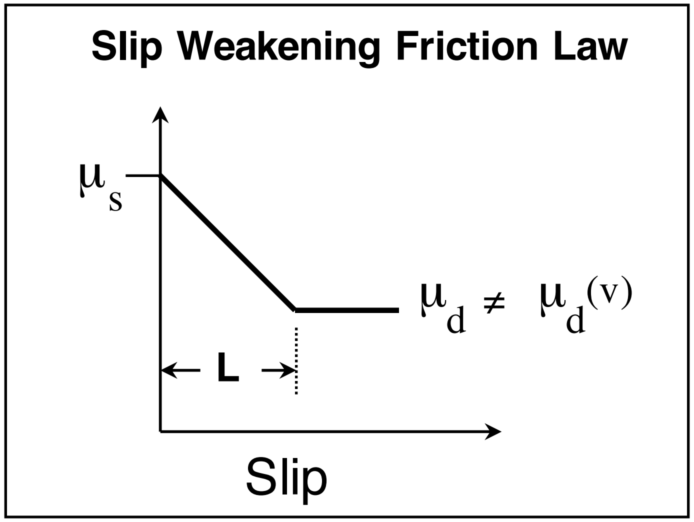
www.geosc.psu.edu/Courses/Geosc508

- Friction
- Minimum requirements for understanding the stability transition from stable to unstable sliding.
- Duality of static and kinetic friction

- See: **A microscopic model of rate and state friction evolution** **(Li and Rubin; 2017)** http://onlinelibrary.wiley.com/doi/10.1002/2017JB013970/abstract;jsessionid=4D088F82D4C43974D71FC36D7C15DED3.f03t02

Friction: 2nd order variations, slick-slip and stability of sliding

Rabinowicz' s work solved a major problem with friction theory: he introduced a way to deal with the singularity in going from μ_s to μ_d



$$\mu(x) = \mu_s - \frac{x}{L}\Delta\mu \qquad (for\ L > x > 0)$$

$$\mu(x) = \mu_s - \Delta\mu \qquad (for\ x > L)$$

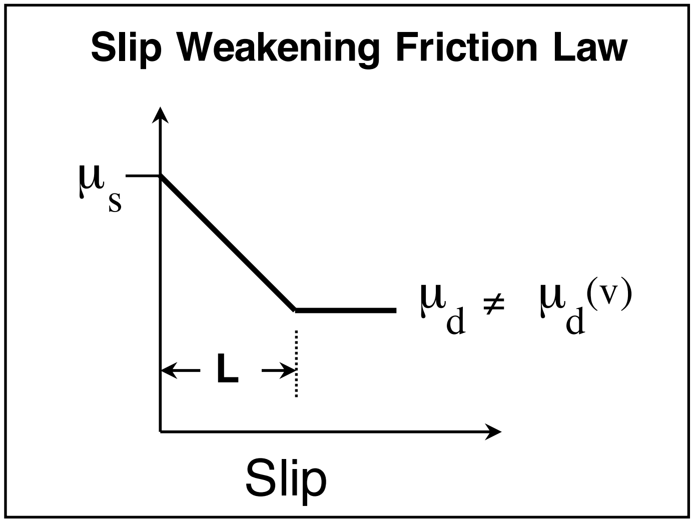
Palmer and Rice, 1973; Ide, 1972; Rice, 1980

For solid surfaces in contact (without wear materials), the slip distance L represents the slip necessary to break down adhesive contact junctions formed during 'static' contact.

The slip weakening distance is also known as the critical slip or the breakdown slip

This slip distance helps with the stress singularity at propagating crack tips, because the stress concentration is smeared out over the region with slip < L.

Friction: 2nd order variations, slick-slip and stability of sliding



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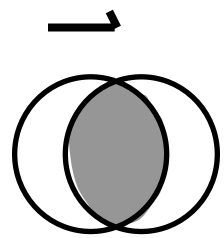
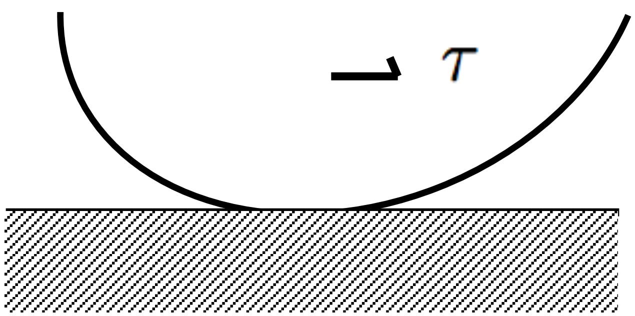
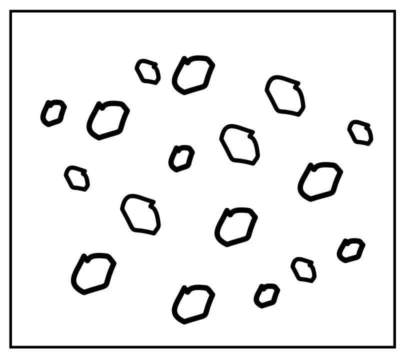


Critical friction distance represents slip necessary to erase existing contact

Adhesive Theory of Friction



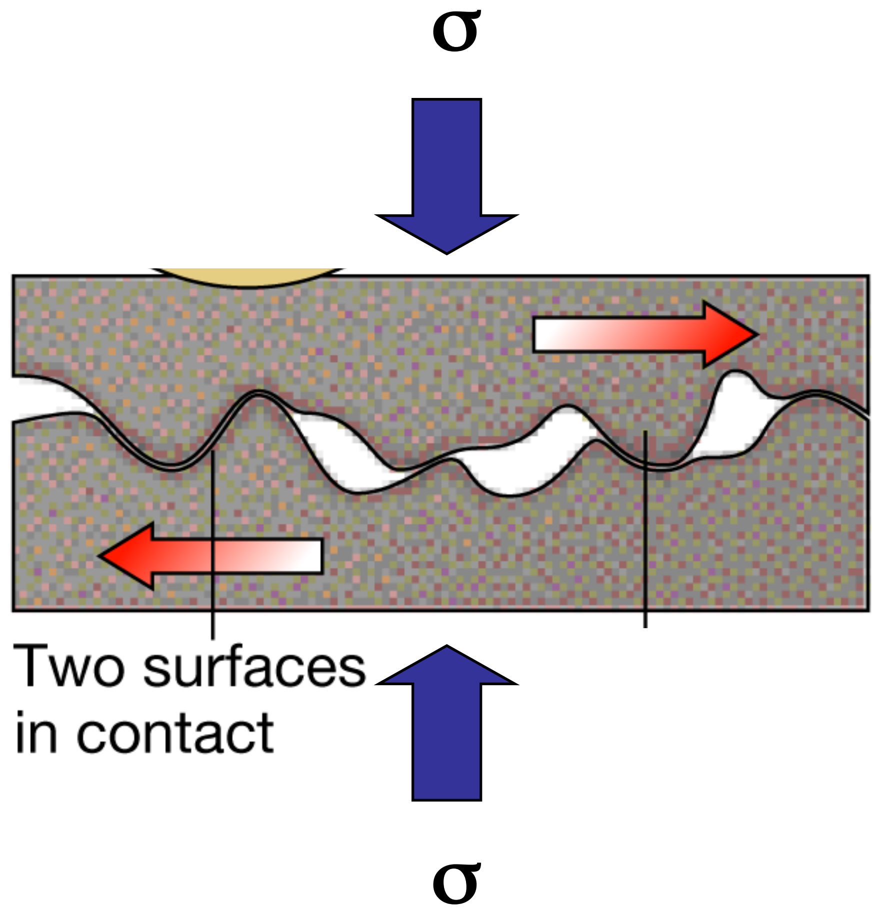


For a surface with a distribution of contact junction sizes, L, will be proportional to the average contact dimension.

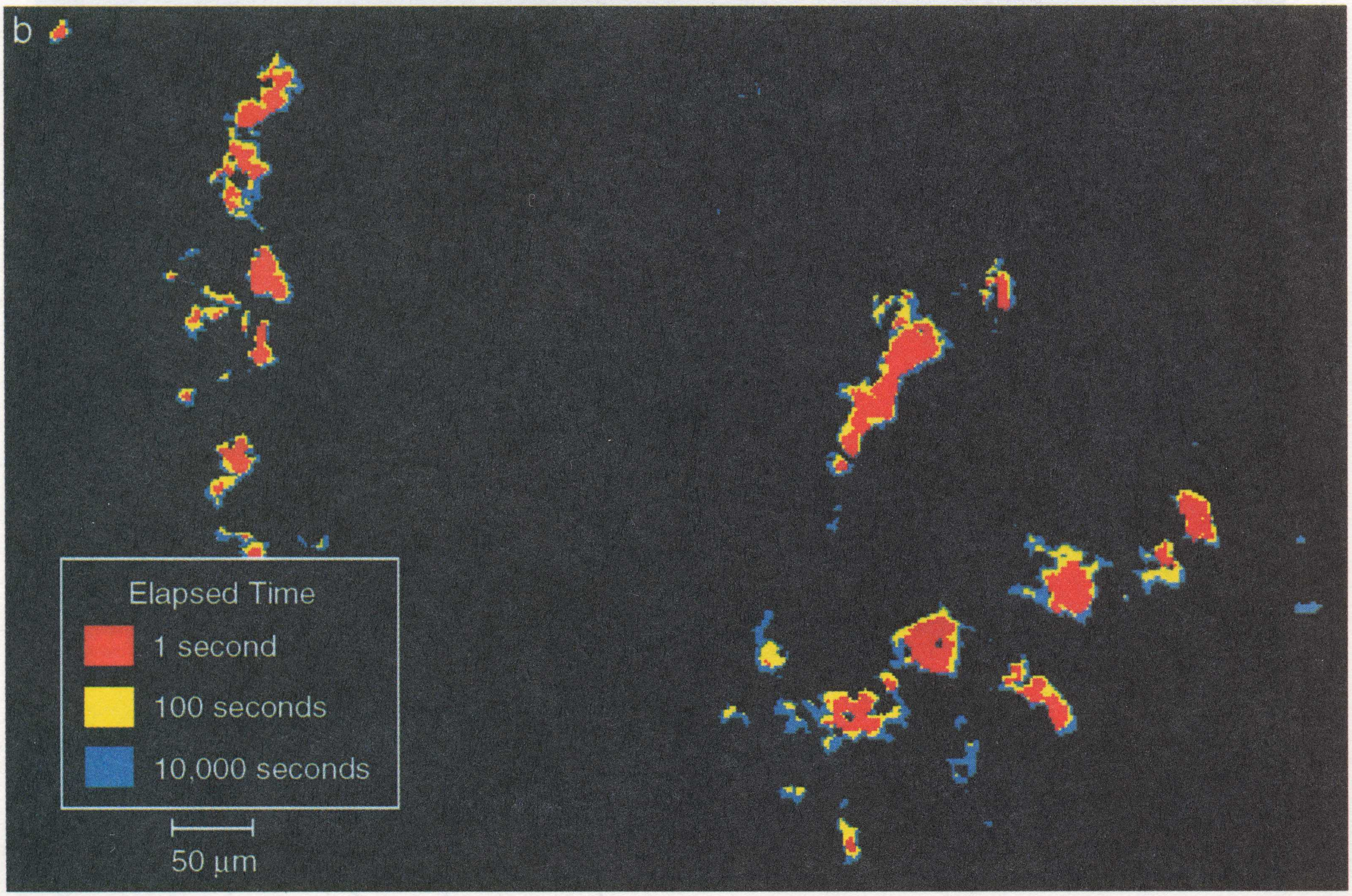
Critical friction distance scales with surface roughness

Friction

Base-level friction coefficient in terms of contact mechanics and hardness



Time dependent yield strength: $\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$

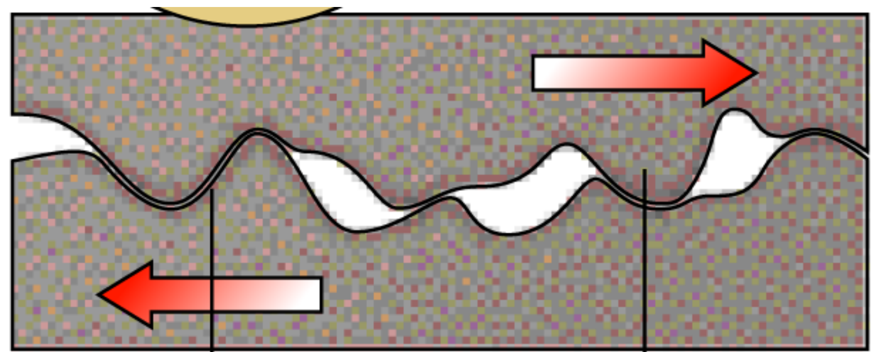


Dieterich and Kilgore [1994]

Time dependent growth of contact (acyrlic plastic)- true static contact

Friction

Base-level friction coefficient in terms of contact mechanics and hardness

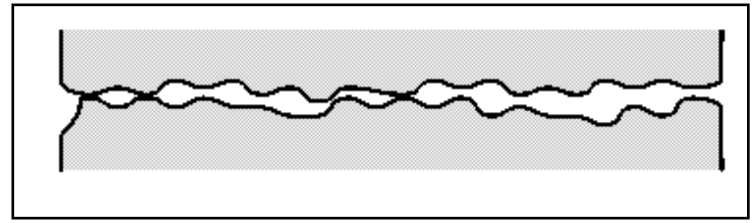


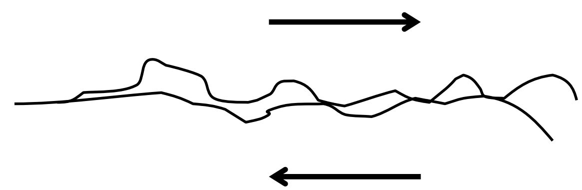
Adhesive Theory of Fricton (Bowden and Tabor)

- Real contact area << nominal area
- Contact junctions at inelastic (plastic) yield strength
- Contacts grow with "age"
- Add: Rabinowicz's observations of static/dynamic friction
- "Static" friction is higher than "Dynamic" friction because contacts are older (larger)
- -> implies that contact size decreases as velocity increases

Friction

Base-level friction coefficient in terms of contact mechanics and hardness





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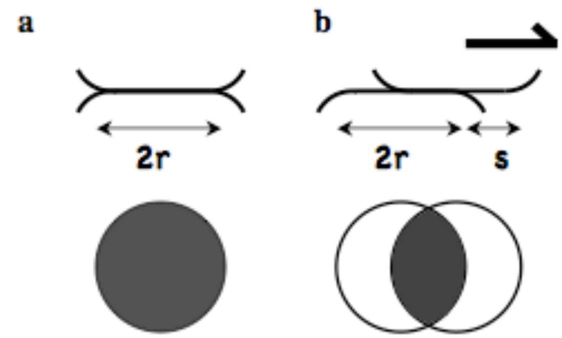
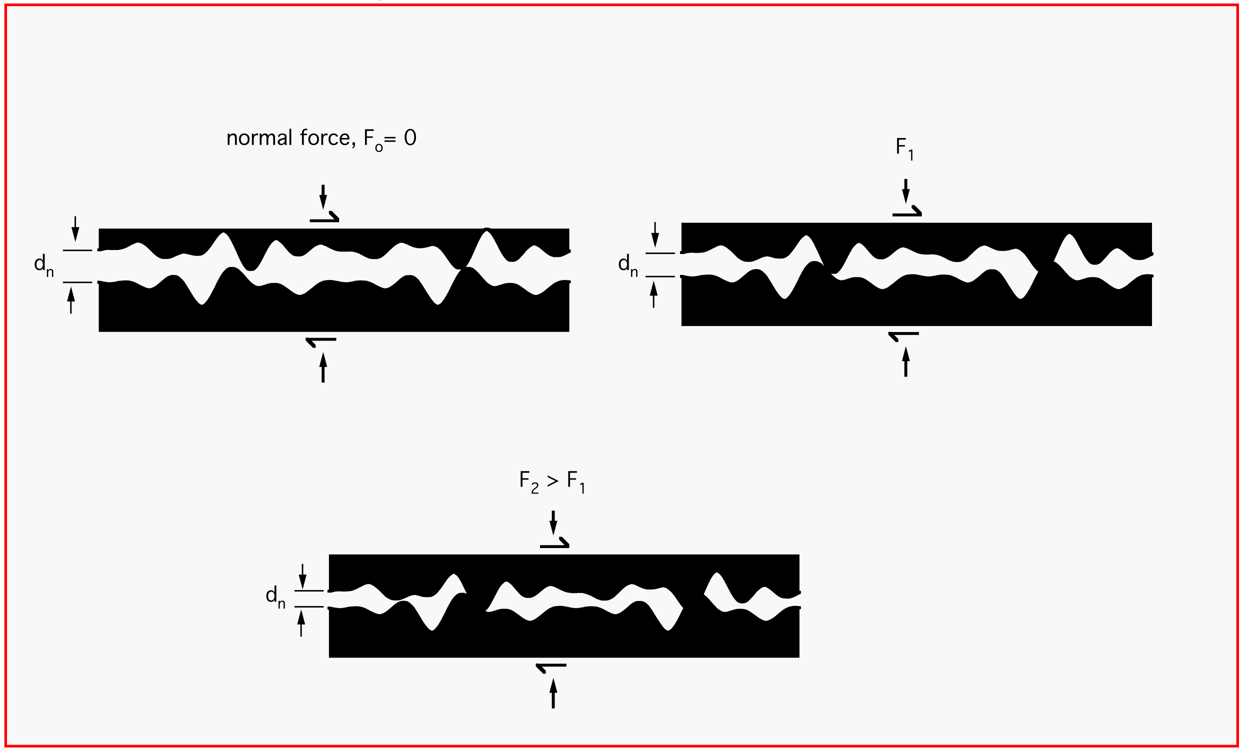


Classic theory of friction

Bowden and Tabor [1960]



τ - shear stress

σ_n - normal stress

F_n - normal force

F_s - shear force

A_T - total fault area

A_C- the real area of contact

S- contact shear strength

σ_y - yield strength or hardness

$$\sigma_n = \frac{\sigma_y A_c}{A_T}$$

$$\tau = \frac{S A_c}{A_T}$$

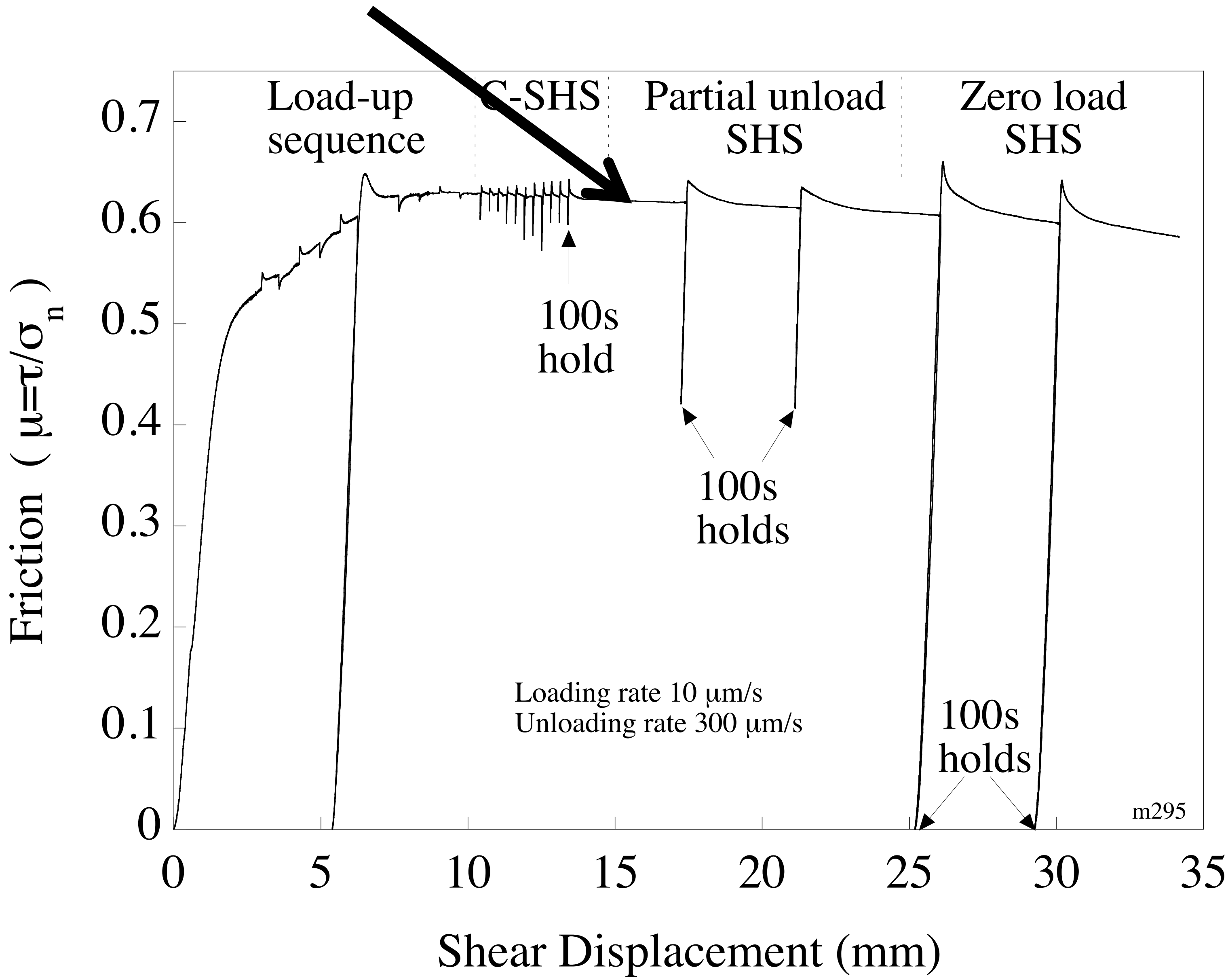
$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$

Friction is the ratio of shear strength to hardness

This is base level friction

Modified from Beeler, 2003

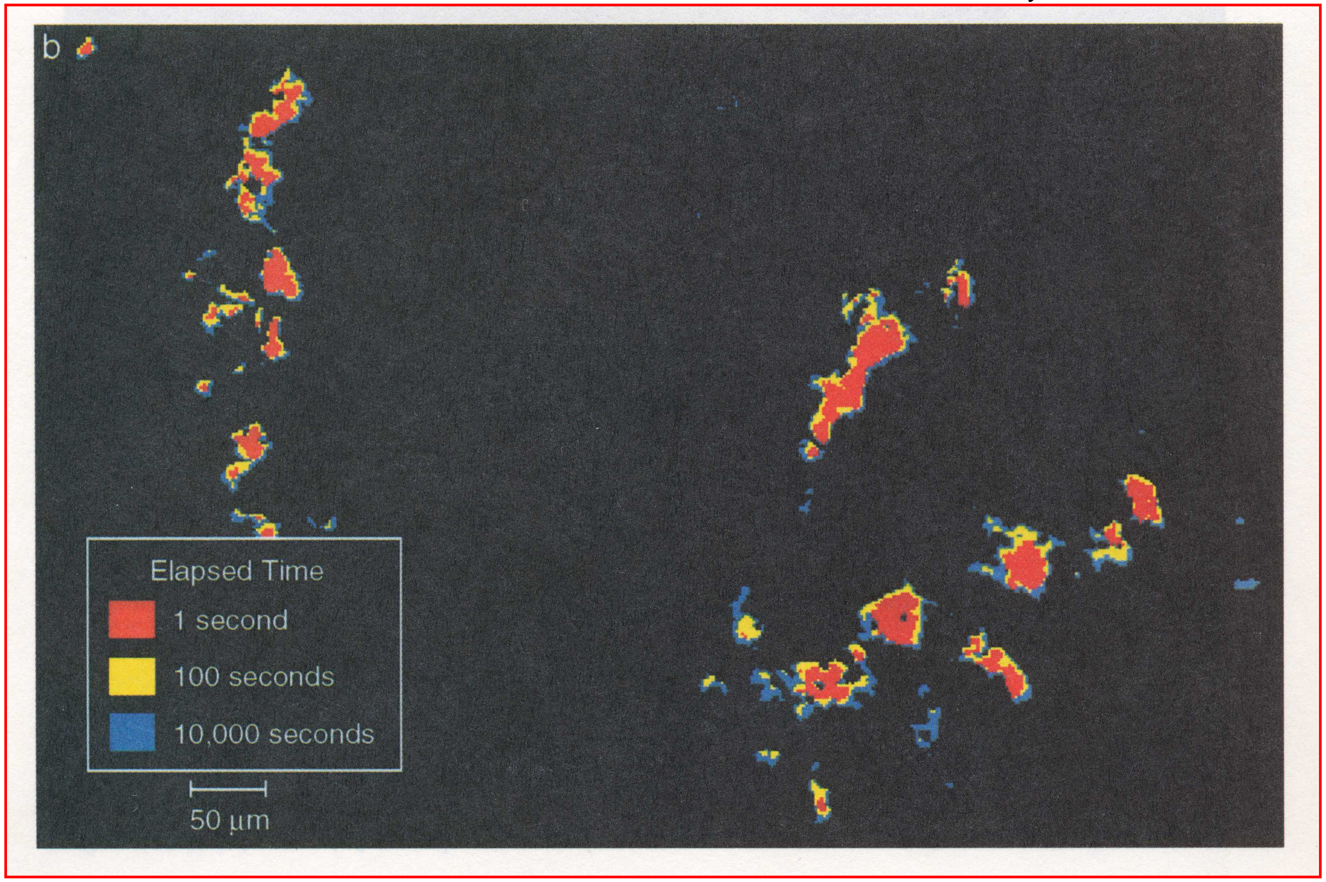
base level friction (~ 0.6 for rocks)



Karner & Marone (GRL 1998, JGR 2001)

Time dependent yield strength:

$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$



Dieterich and Kilgore [1994]

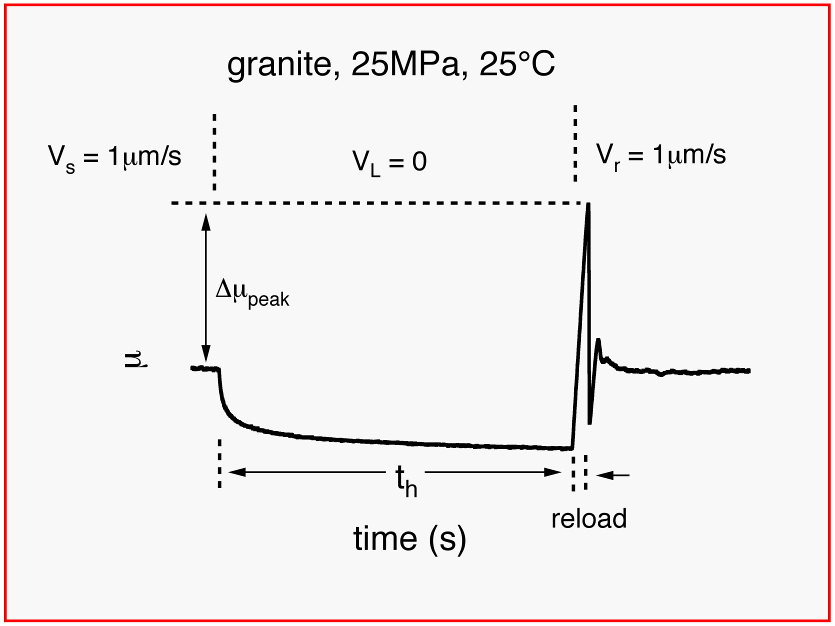
$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$

Time dependent growth of contact (acyrlic plastic)- true static contact

$$\sigma_y = \sigma_o + f(t)$$

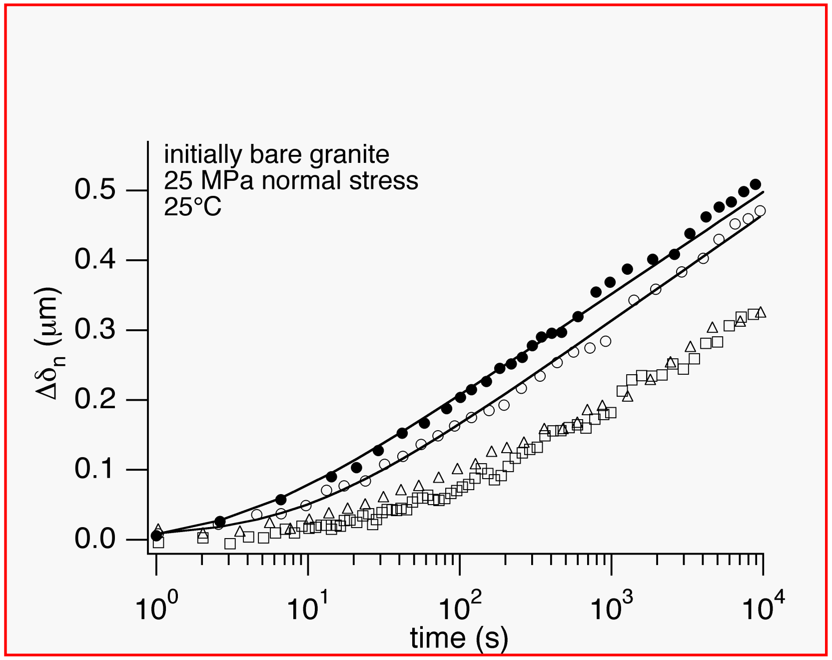
Modified from Beeler, 2003

Other measures of changes in 'static' friction, contact area, or strength



'hold' test

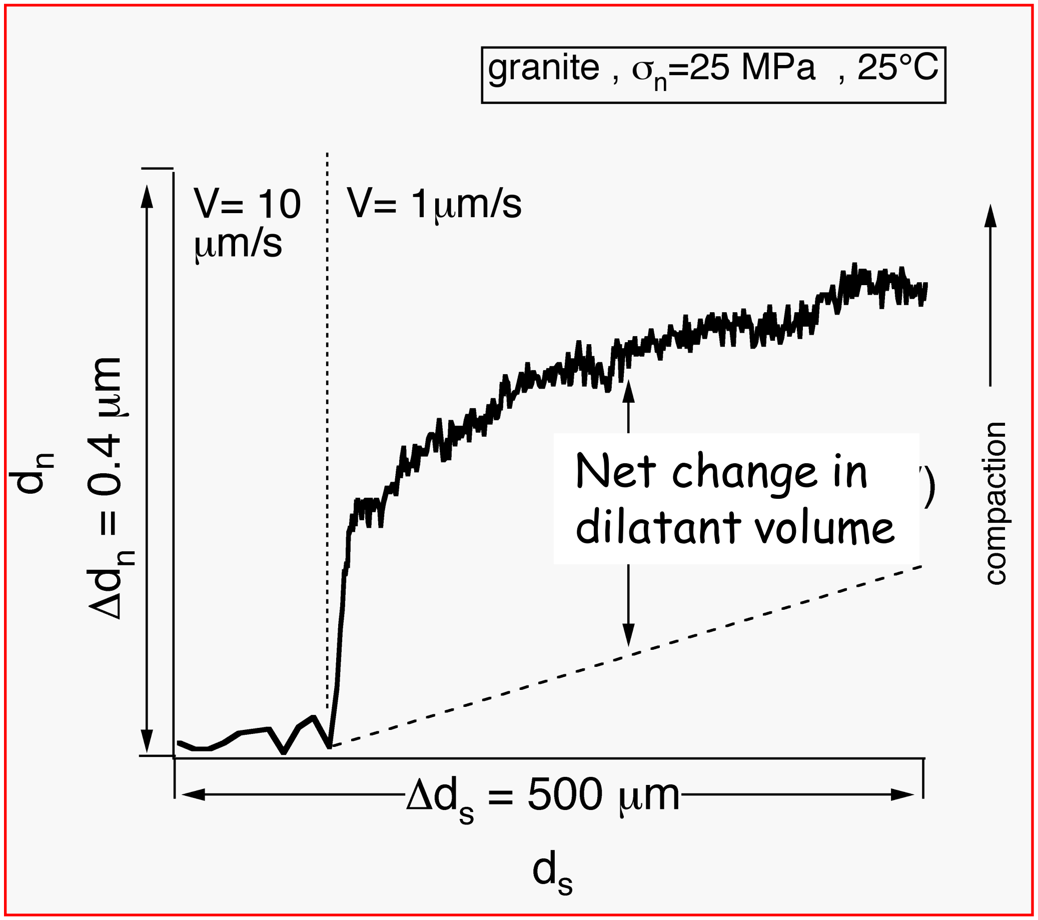
after Dieterich [1972]



time dependent closure (westerly granite)
- approximately static contact

Modified from Beeler, 2003

compaction/dilatancy associated with changes in sliding velocity

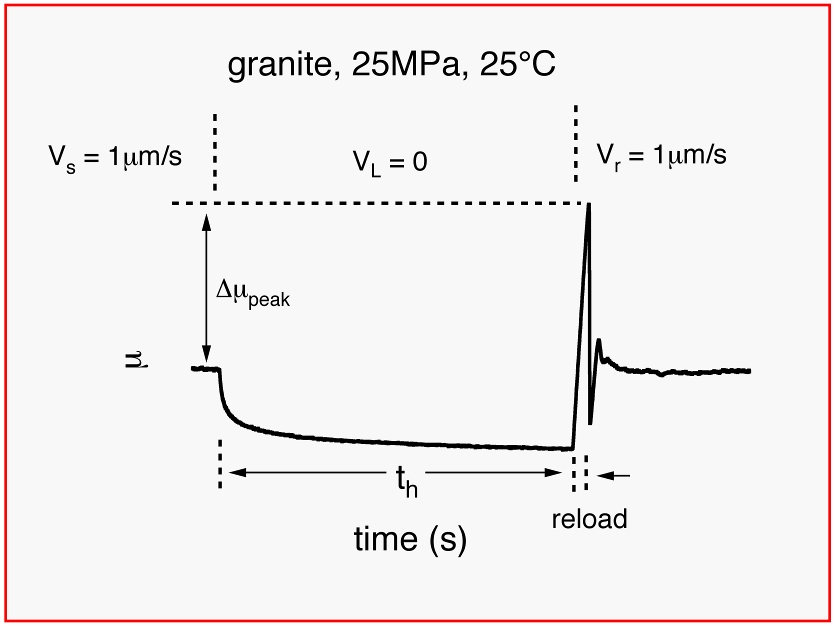


after Marone and Kilgore [1993]

$$\sigma_y = \sigma_o + f(age)$$

Modified from Beeler, 2003

Rate dependence of contact shear strength



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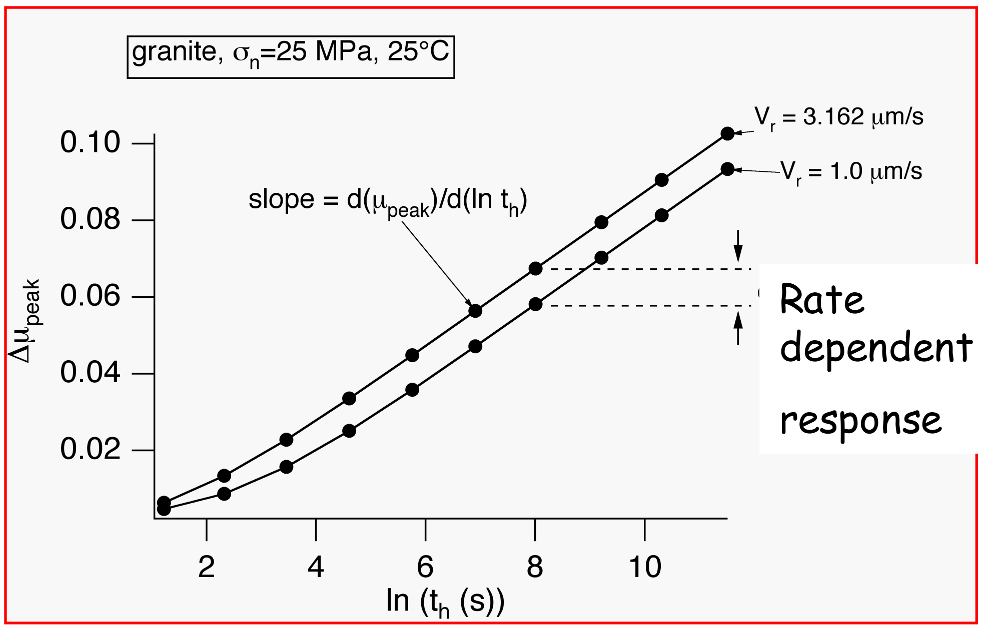

$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$

$$S = S_o + g(V)$$

Modified from Beeler, 2003

Summary of friction observations:

<u>0. Friction is to first order a constant</u>

1. Time dependent increase in contact area (strengthening)
2. Slip dependent decrease in contact area (weakening); equivalently increase in dilatancy
3. Slip rate dependent increase in shear resistance (non-linear viscous)

Modified classic theory of friction:

$$\mu = \frac{S}{\sigma_y} = \frac{S_o + g(V)}{\sigma_o + f(age)}$$

$$\mu = \frac{S_o + g(V)}{\sigma_o + f(age)}\left[\frac{\sigma_o - f(age)}{\sigma_o - f(age)}\right]$$

Discard products of second order terms:

$$\mu = \frac{S_o}{\sigma_o} + \frac{g(V)}{\sigma_o} - \frac{S_o f(age)}{{\sigma_o}^2}$$

[e.g., Dieterich, 1978, 1979]

Modified from Beeler, 2003

$$\mu = \frac{S_o}{\sigma_o} + \frac{g(V)}{\sigma_o} - \frac{S_o f(age)}{\sigma_o^2}$$

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Rate and state equations:

1st order term (0.): μ_0; second order terms: $a \ln \frac{V}{V_0}$ (3.) and $b \ln \frac{V_0 \theta}{D_c}$ (1. & 2.)

$$\mu = \mu_0 + a \ln \frac{V}{V_0} + b \ln \frac{V_0 \theta}{D_c}$$

θ is contact age

Dieterich [1979]
Rice [1983]
Ruina [1983]

$$\frac{d\theta}{dt} = 1 - \frac{\theta V}{D_c}$$

$$\frac{d\theta}{dt} = \left(\frac{\partial \theta}{\partial t}\right)_d + \left(\frac{\partial \theta}{\partial d}\right)_t V$$

time dependence

$$\left(\frac{\partial \theta}{\partial t}\right)_d = 1$$

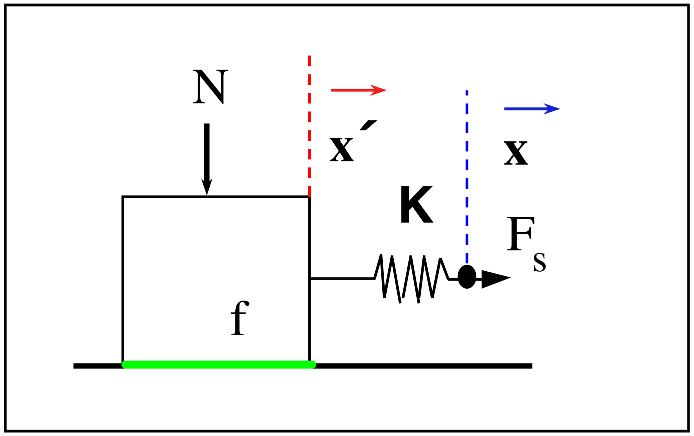
slip dependence

$$\left(\frac{\partial \theta}{\partial d}\right)_t = -\frac{\theta}{D_c}$$

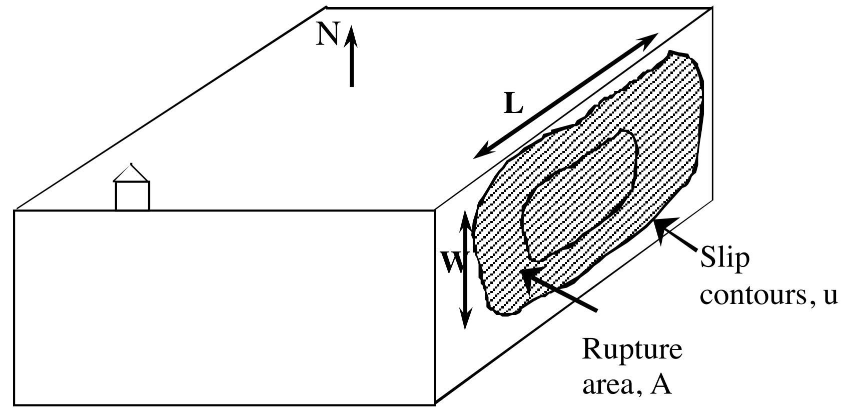
Modified from Beeler, 2003

Brittle Friction Mechanics, Stick-slip

- **Stick-slip (unstable) versus stable shear**



1-D fault zone analog, Stiffness K



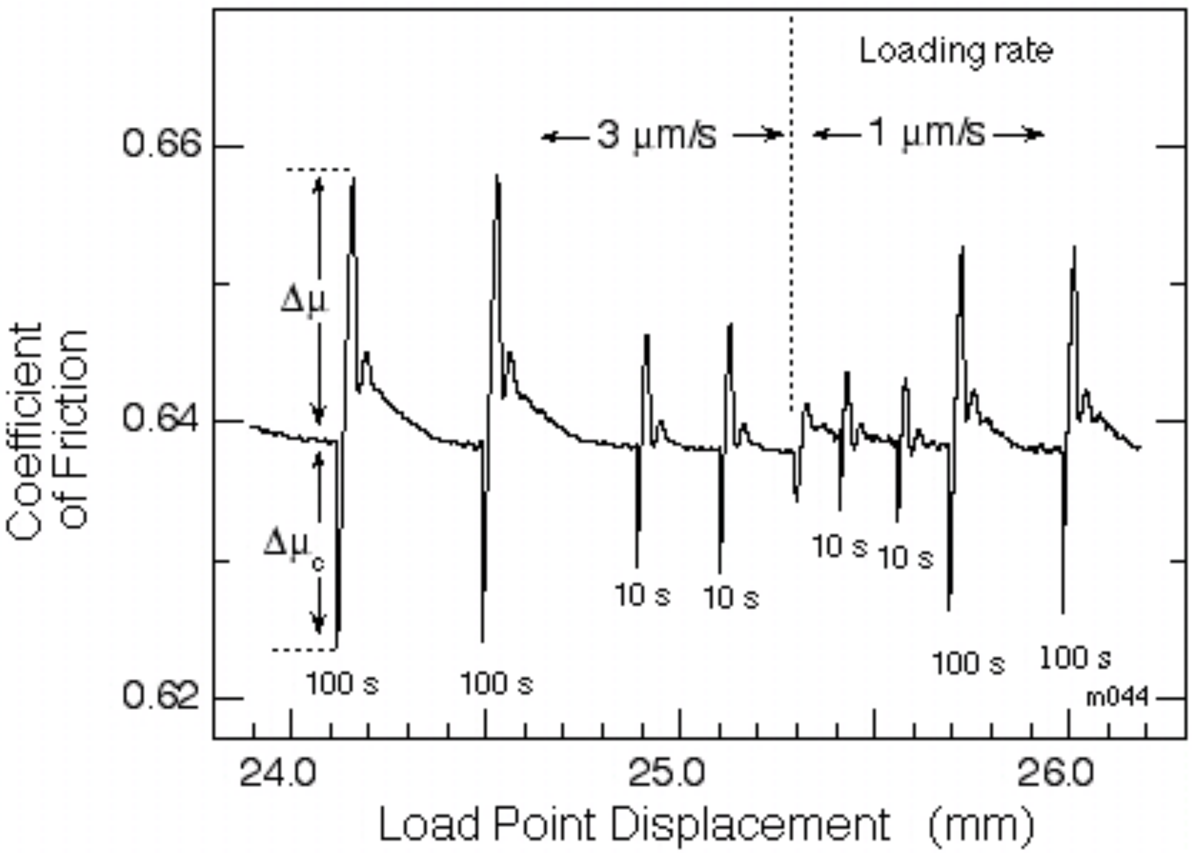
Why is this a reasonable approach?

How do we get at stiffness?

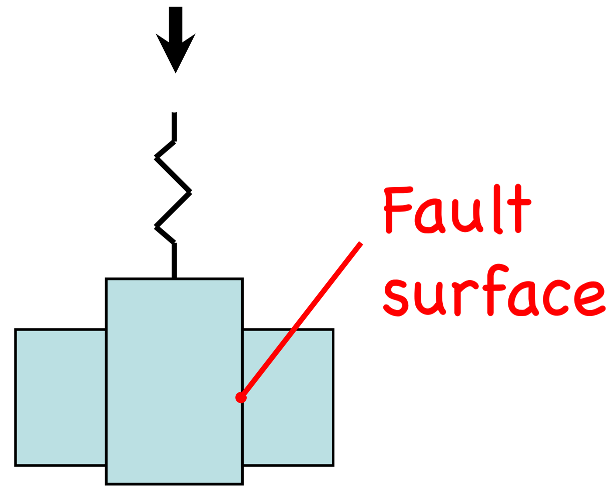
$$\Delta\sigma = \frac{7\pi}{16} G \frac{\Delta\bar{u}}{r}$$

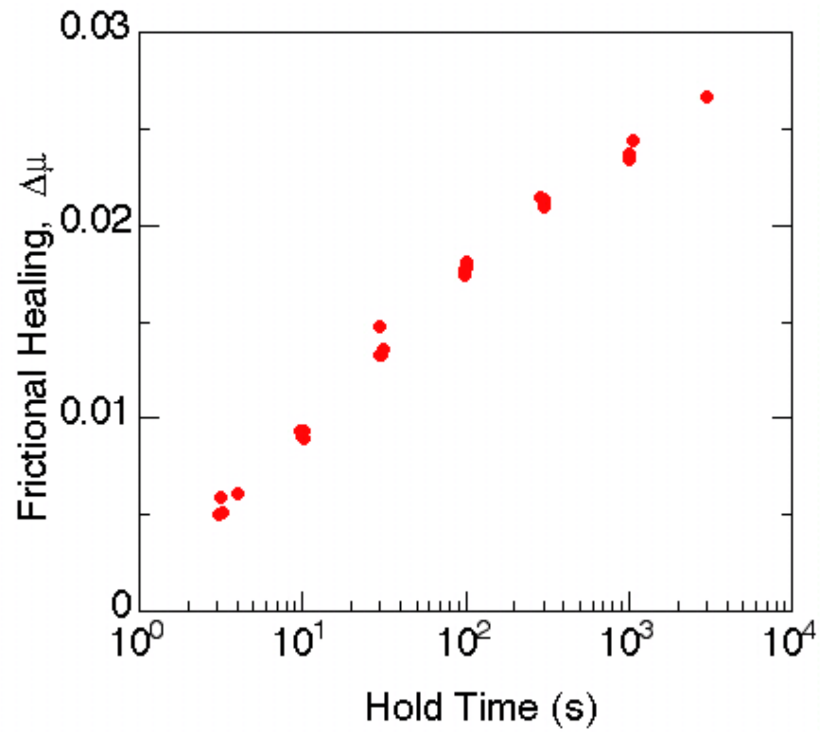
$$K = \frac{\Delta\sigma}{\Delta\bar{u}} = \frac{7\pi}{16} \frac{G}{r}$$

Time dependence of friction in rocks; Macroscopic frictional aging



Marone, Nature, 1998



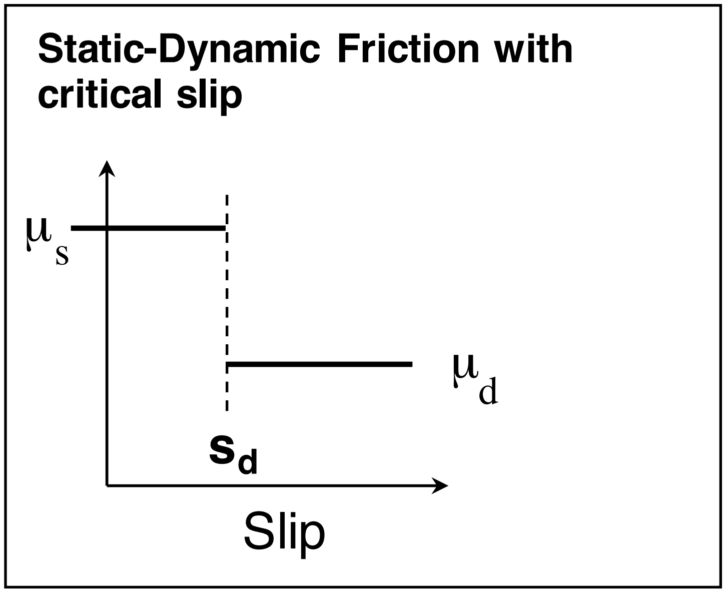


Friction: 2nd order variations, slick-slip and stability of sliding

Rabinowicz 1951, 1956,. 1958
Static vs. dynamic friction & state dependence

$$\left.\begin{array}{l}\mu = \mu_s \quad (s = 0) \\ \mu = \mu_d \quad (s > 0)\end{array}\right\} \text{ Classical view}$$

Rabinowicz recognized that finite slip was necessary to achieve fully dynamic slip



$$\mu = \mu_s \quad (s < s_d)$$

$$\mu = \mu_d \quad (s > s_d)$$

s_d is the critical slip distance

Rabinowicz experiments showed state, memory effects and that μ_d varied with slip velocity.

Friction: 2nd order variations, slick-slip and stability of sliding

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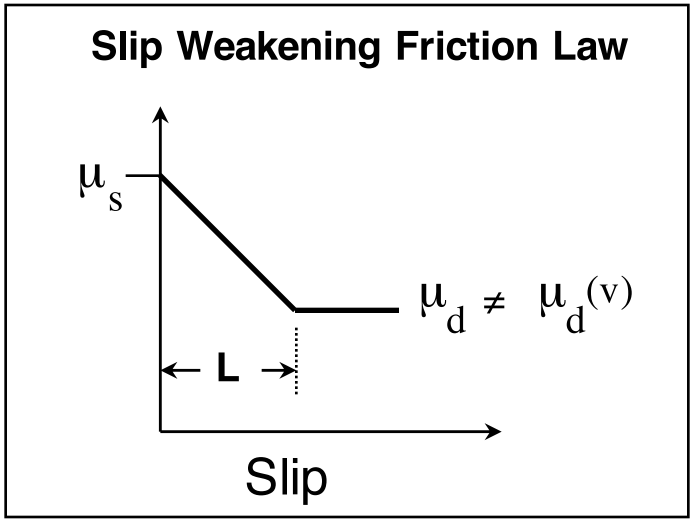

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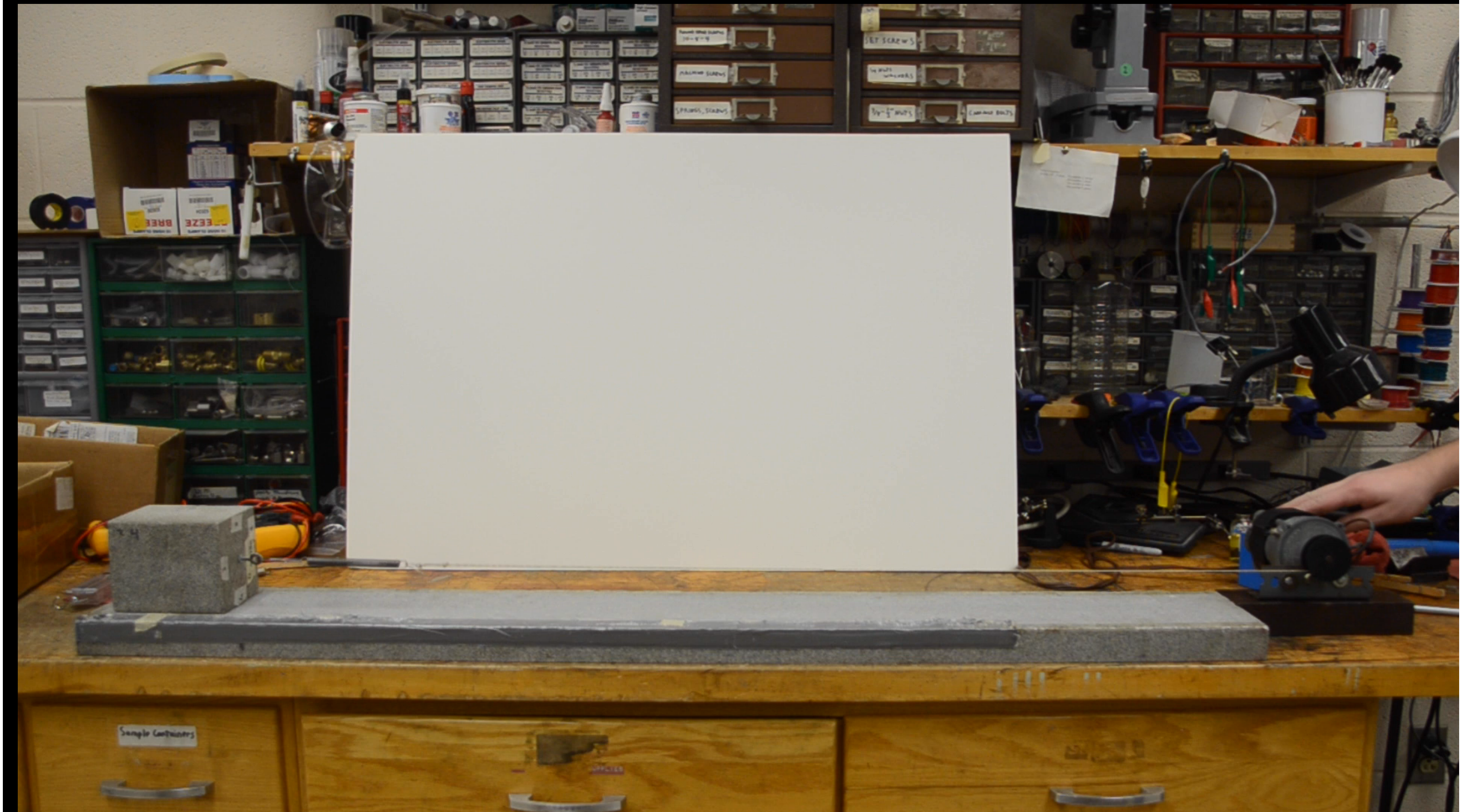
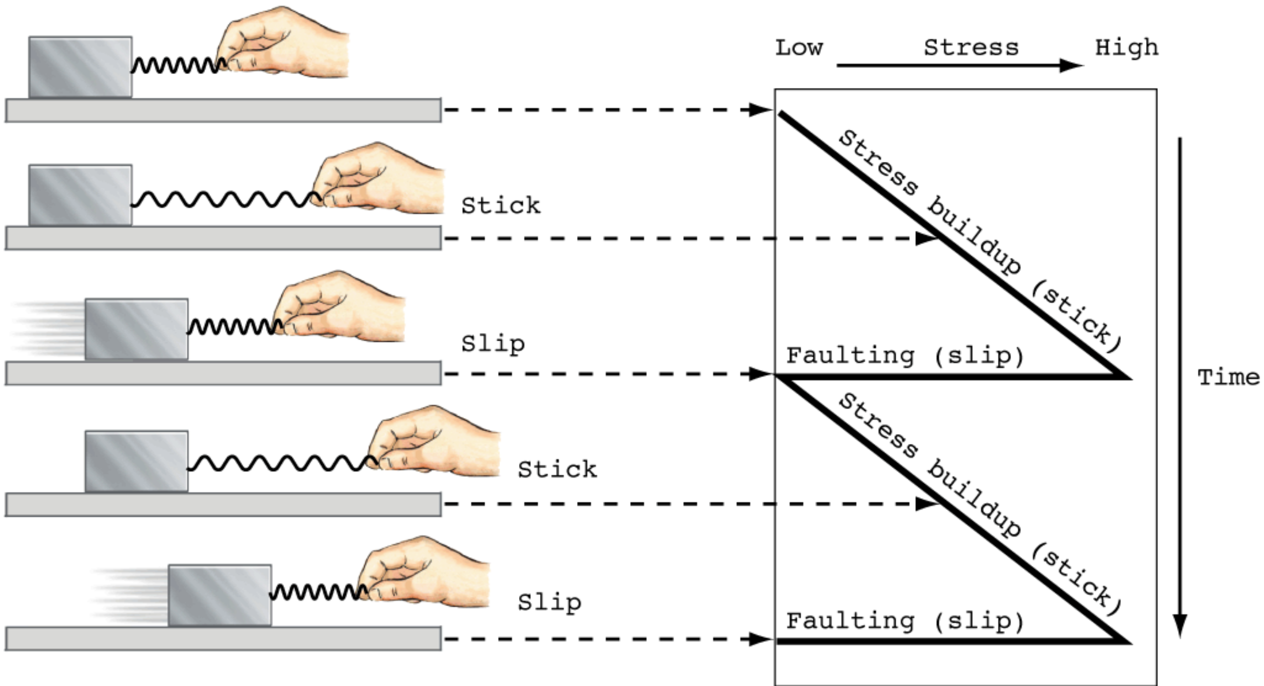
Stick-slip





Earth, S. Marshak, W.W. Norton

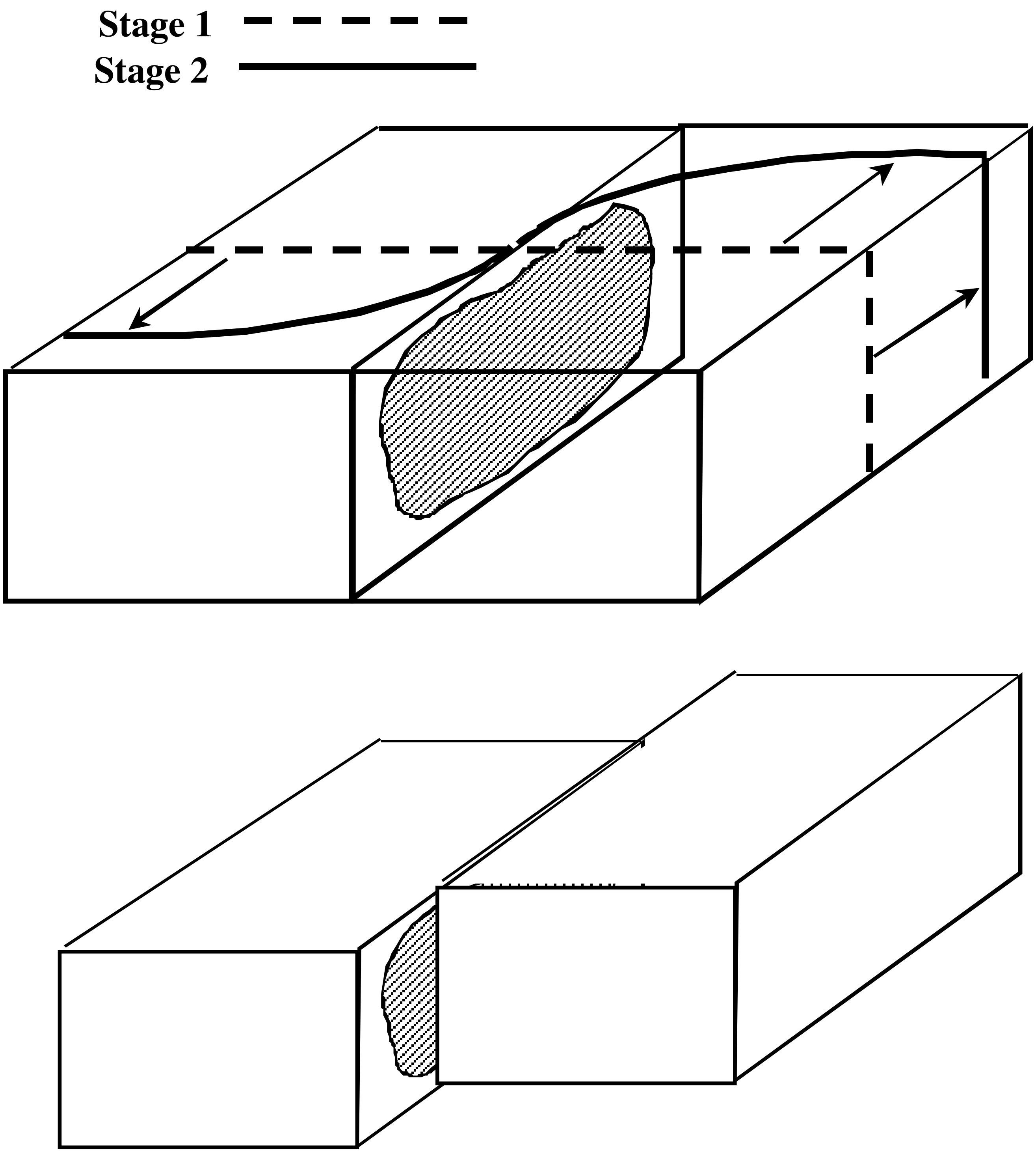
Reid's Hypothesis of Elastic Rebound

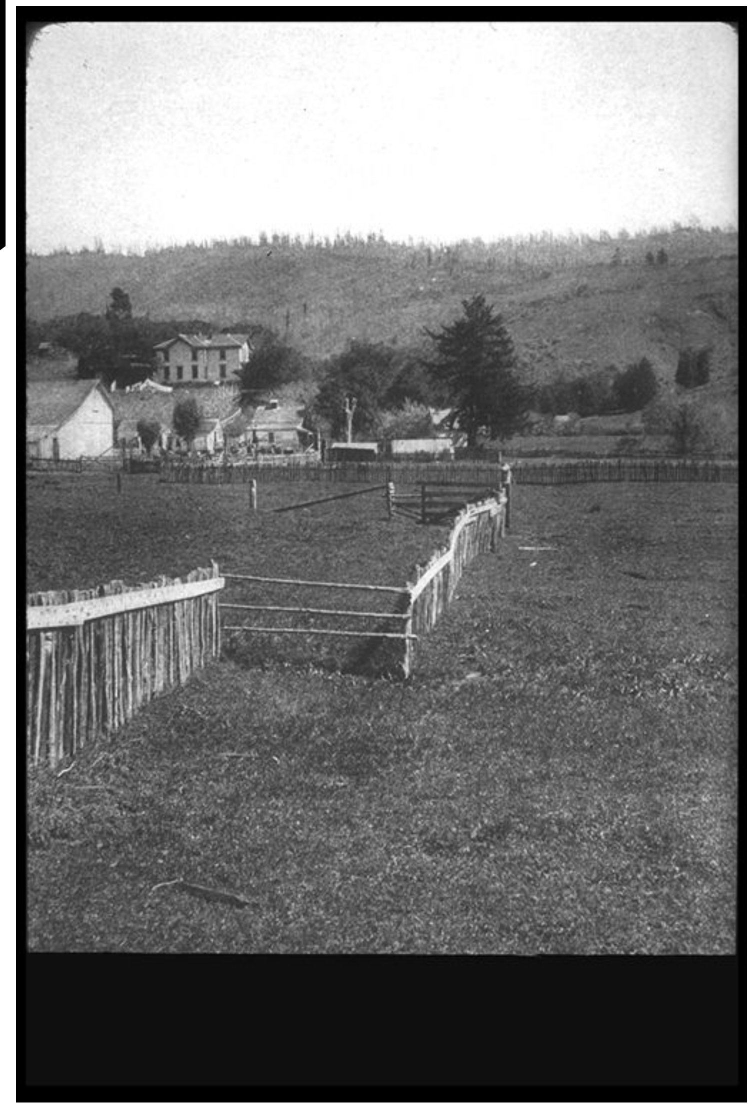
Reid, H.F., The mechanics of the earthquake, v. 2 of The California earthquake of April 18, 1906. Report of the State Earthquake Investigation Commission, Carnegie Institution of Washington Publication 87, 1910.

Elastic strain accumulates during the interseismic period and is released during an earthquake. The elastic strain *causes* the earthquake –in the sense that the elastic energy stored around the fault drives earthquake rupture.

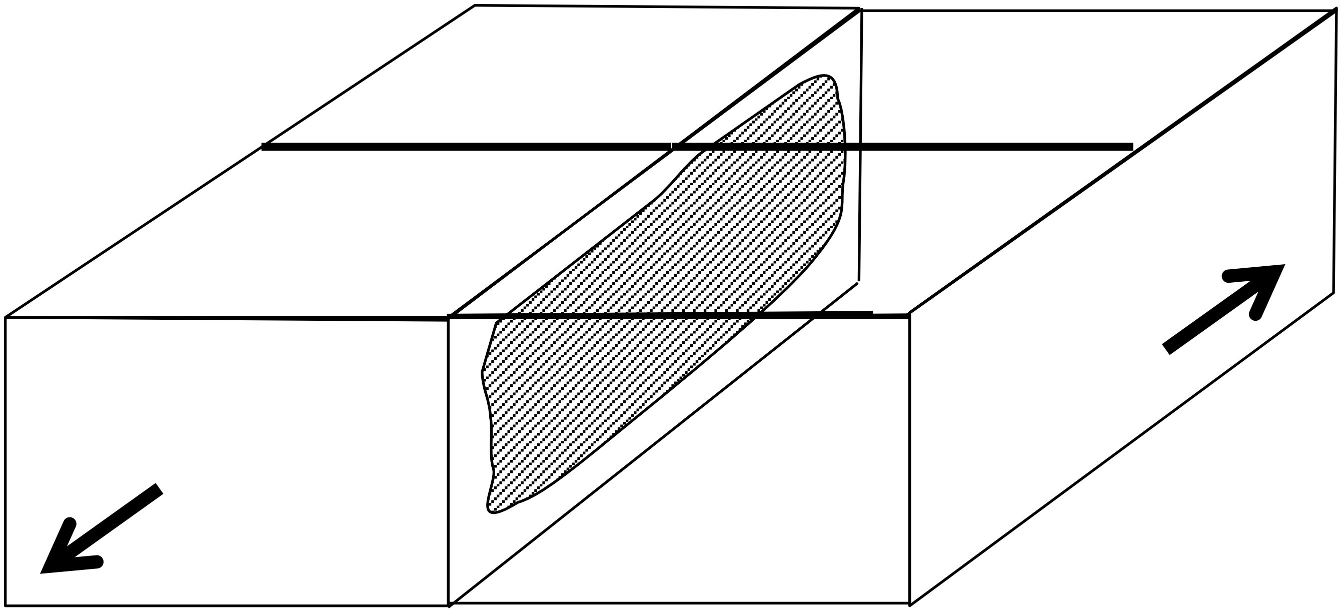
There are three basic stages in Reid's hypothesis.

1) Stress accumulation (e.g., due to plate tectonic motion --but what about intra-plate earthquakes?)
2) Stress reaches or exceeds the (frictional) failure strength
3) Failure, seismic energy release (elastic waves), and fault rupture propagation

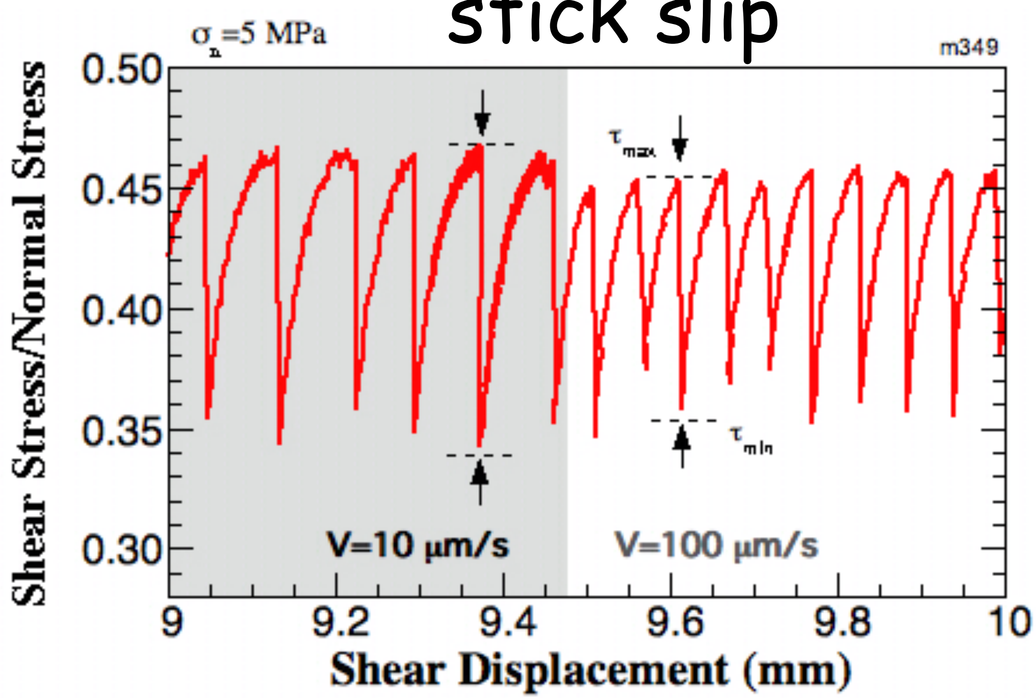

Stage 1
Stage 2

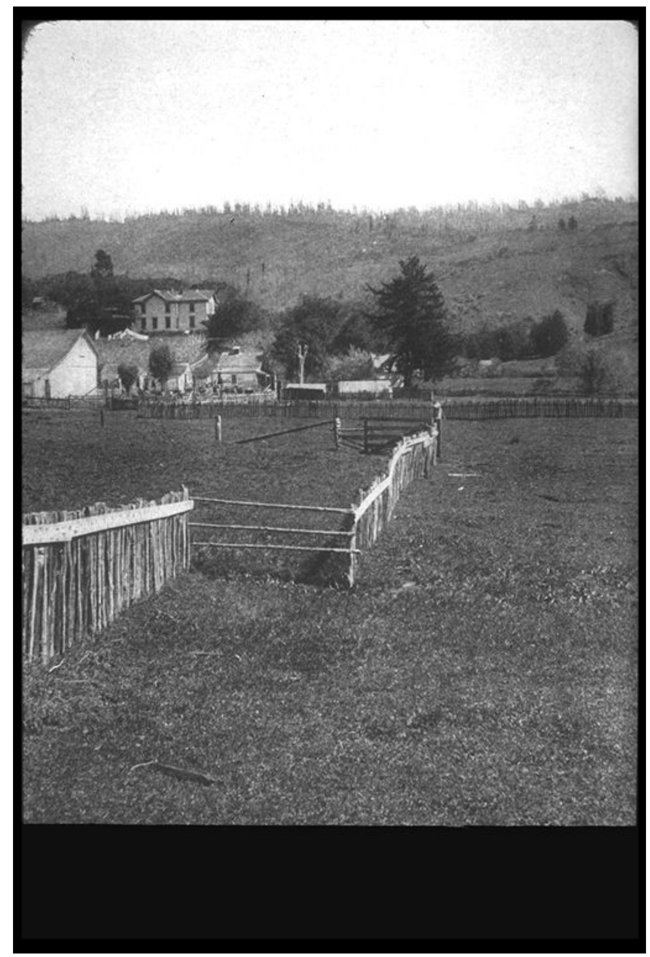


Seismic cycle as repetitive stick slip instability



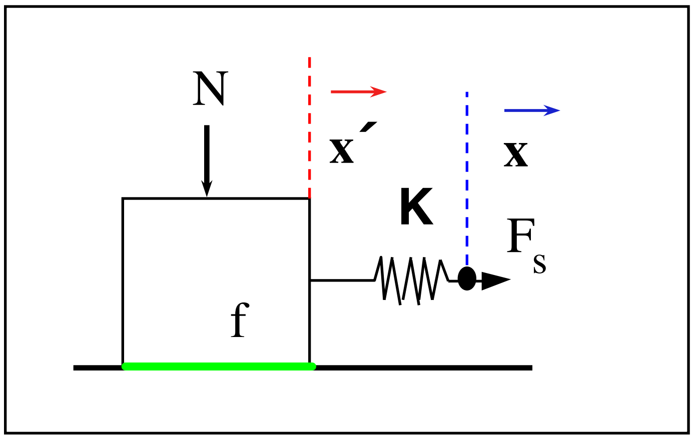
stick slip



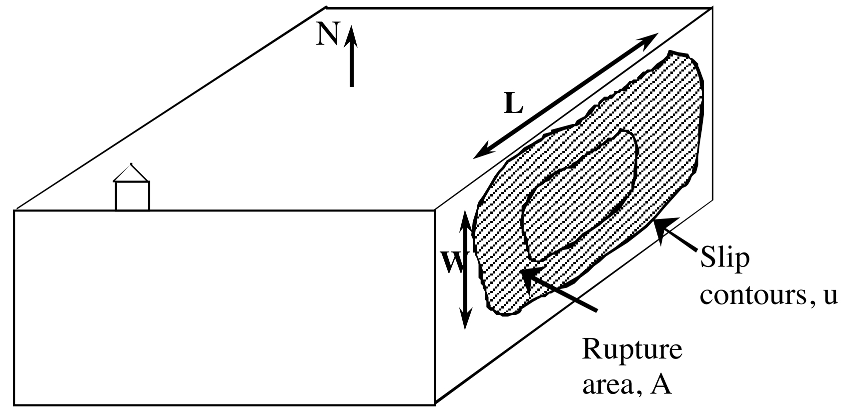


Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear



1-D fault zone analog, Stiffness K



Why is this a reasonable approach?

How do we get at stiffness?

$$\Delta\sigma = \frac{7\pi}{16} G \frac{\Delta\bar{u}}{r}$$

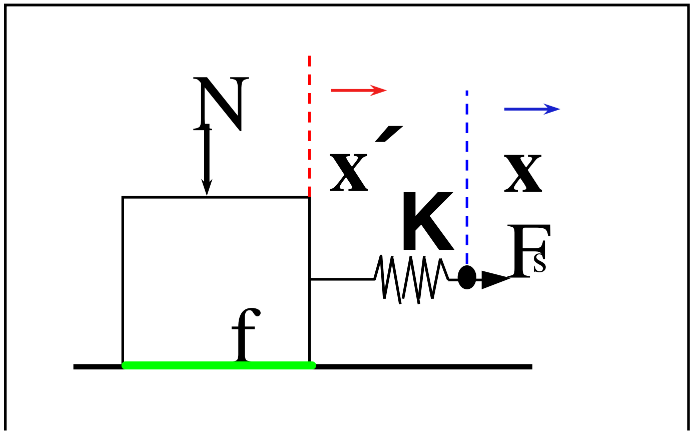
Relation between stress and slip on a dislocation of radius r. Therefore, the local stiffness around the slip patch is:

$$K = \frac{\Delta\sigma}{\Delta\bar{u}} = \frac{7\pi}{16} \frac{G}{r}$$

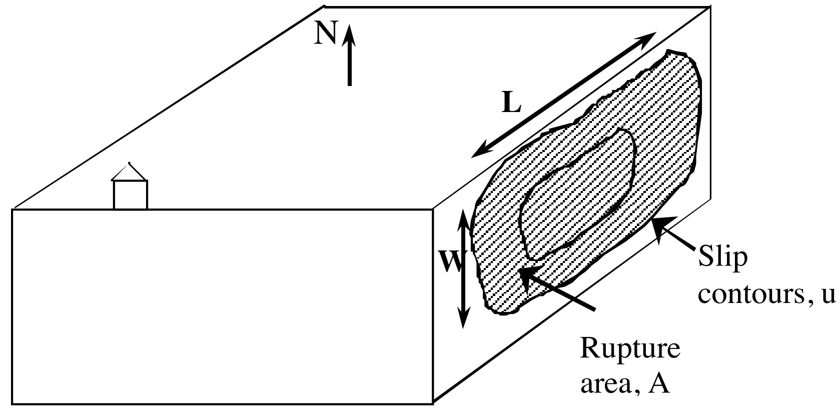
That is, stiffness decreases as the patch enlarges.

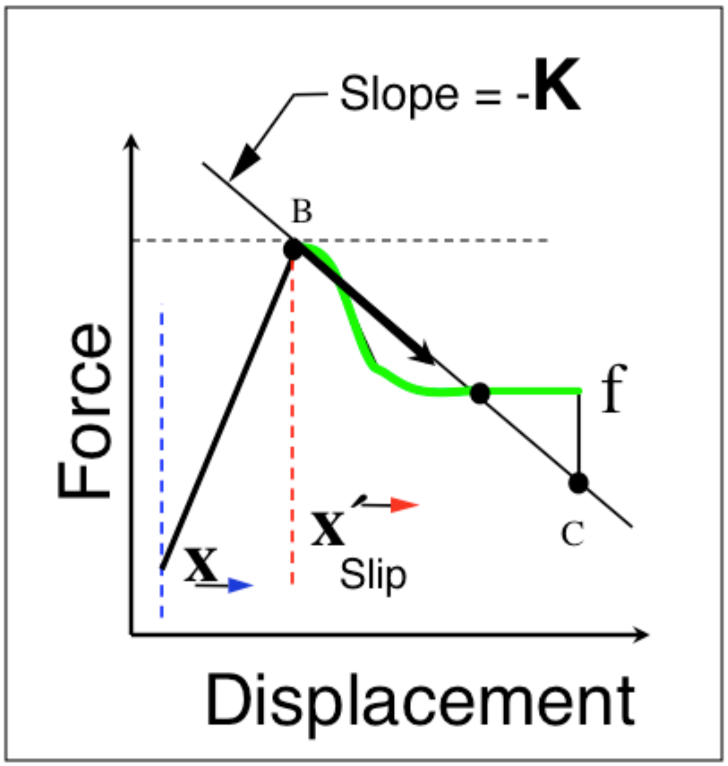
Brittle Friction Mechanics, Stick-slip

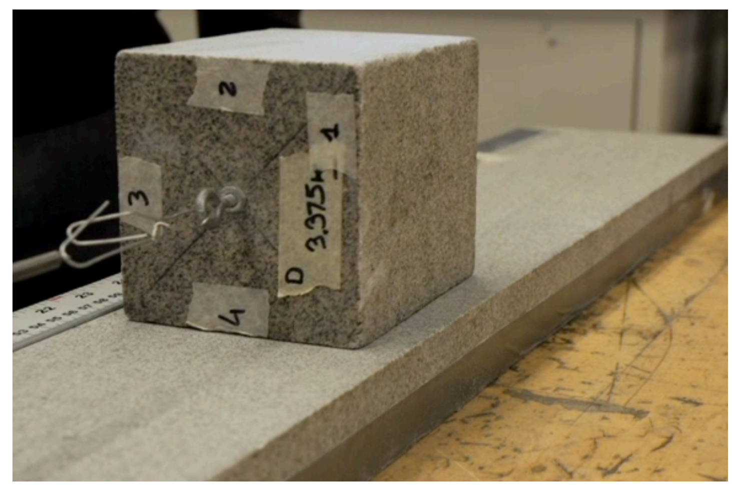
Stick-slip (unstable) versus stable shear



1-D fault
zone analog,
Stiffness K







Brittle Friction Mechanics, Stick-slip

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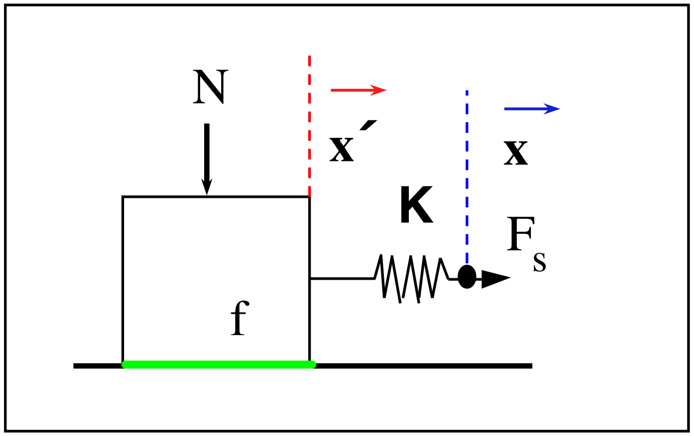
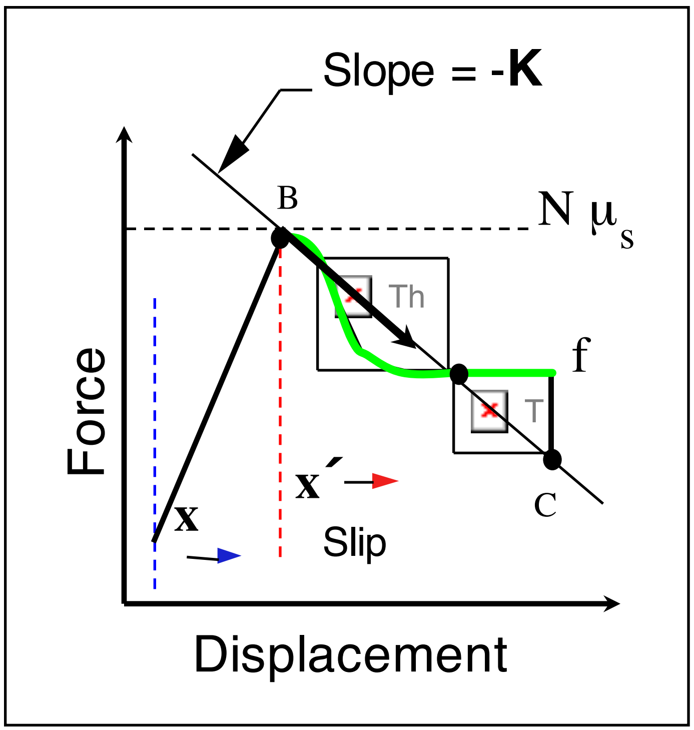


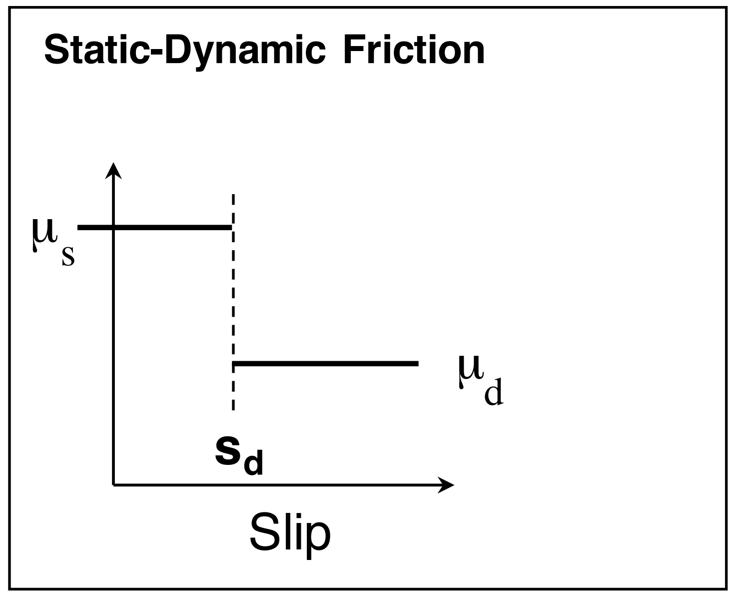


Stick-slip dynamics

$$m\ddot{x}' + \Gamma\dot{x}' + f(\dot{x}', x', t, \theta) = F_s$$

$$m\ddot{x}' + \Gamma\dot{x}' + f(\dot{x}', x't, \theta) = K(v_{lp} - v)t$$

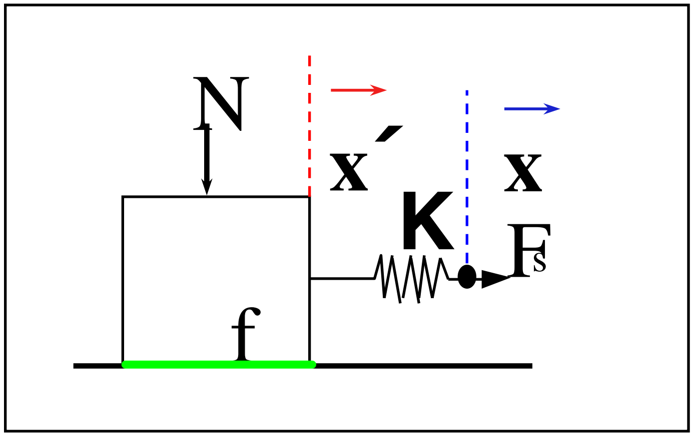
$$m\ddot{x}' + Fx' = K(v_{lp} - v)t$$

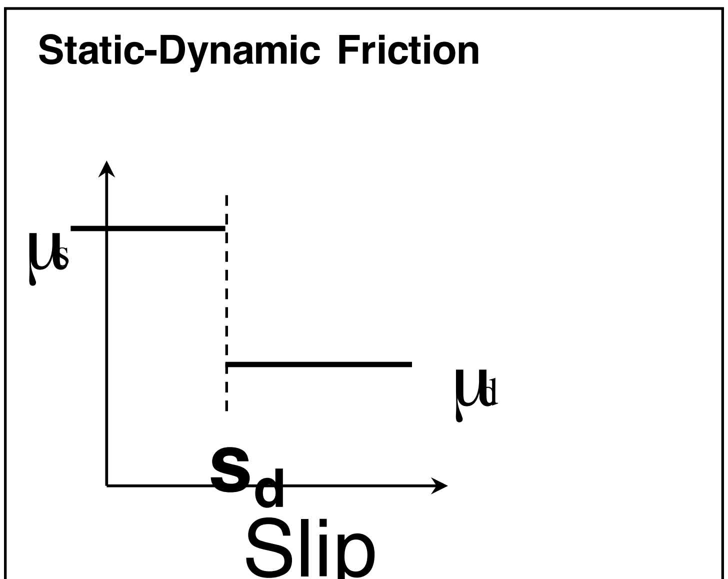


$$f = \Delta\mu N$$

Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear





Stick-slip dynamics

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$$m\ddot{x}' + \Gamma\dot{x}' + f(\dot{x}', x't, \theta) = K(v_{lp} - v)t$$

$$m\ddot{x}' + f(x') = K(v_{lp} - v)t$$

$$m\ddot{x}' + Kx' = \Delta\mu N$$

$$x'(t) = \frac{\Delta\mu N}{K}(1 - cos\kappa t)$$

$$v(t) = \frac{\Delta\mu N}{\sqrt{Km}} sin\kappa t$$

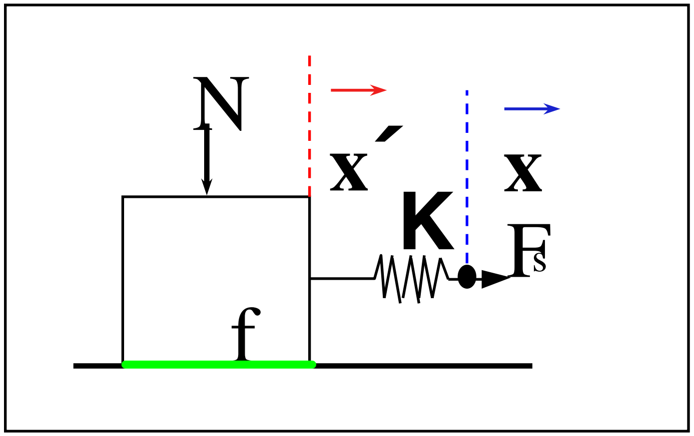
$$\kappa = \sqrt{\frac{K}{m}}$$

$$t_r = \pi\sqrt{\frac{m}{K}}$$

slip duration = rise time

Brittle Friction Mechanics, Stick-slip

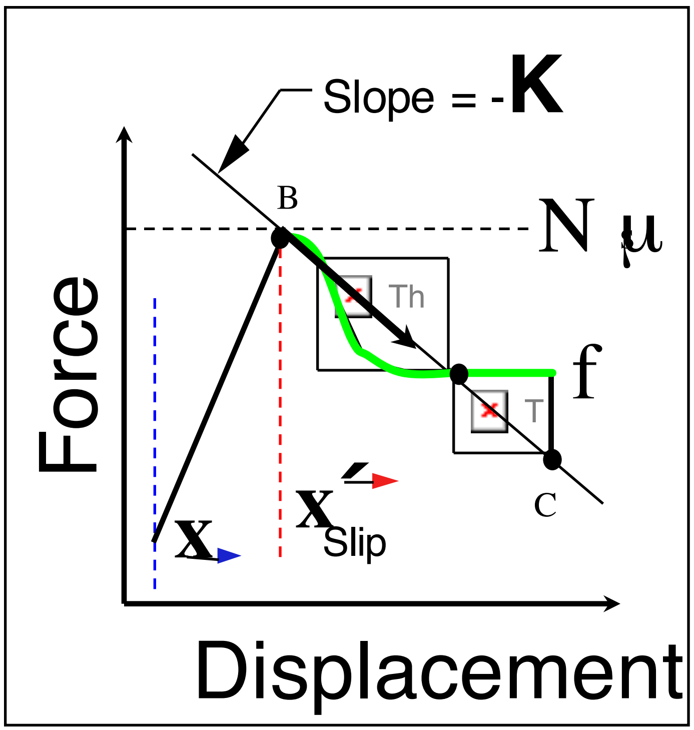
- Stick-slip (unstable) versus stable shear



$$m\ddot{x}' + Kx' = \Delta\mu N$$

$$x'(t) = \frac{\Delta\mu N}{K}(1 - cos\kappa t)$$

$$v(t) = \frac{\Delta\mu N}{\sqrt{Km}} sin\kappa t \qquad \kappa = \sqrt{\frac{K}{m}}$$



$$t_r = \pi\sqrt{\frac{m}{K}}$$

slip duration = rise time

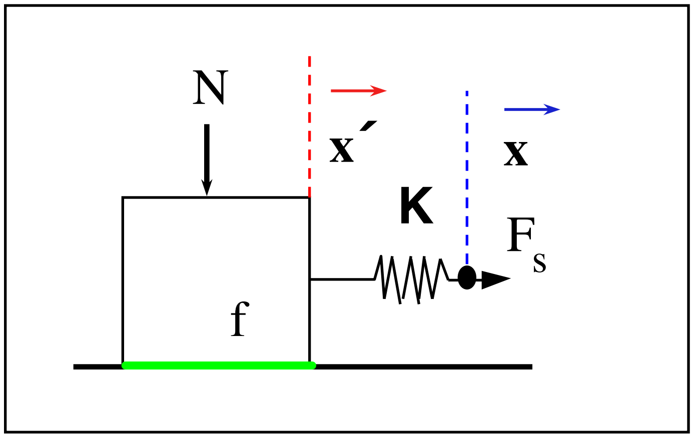
$$\Delta x' = \frac{2\Delta\mu N}{K}$$

total slip, particle velocity, and accel. all depend on friction drop (stress drop)

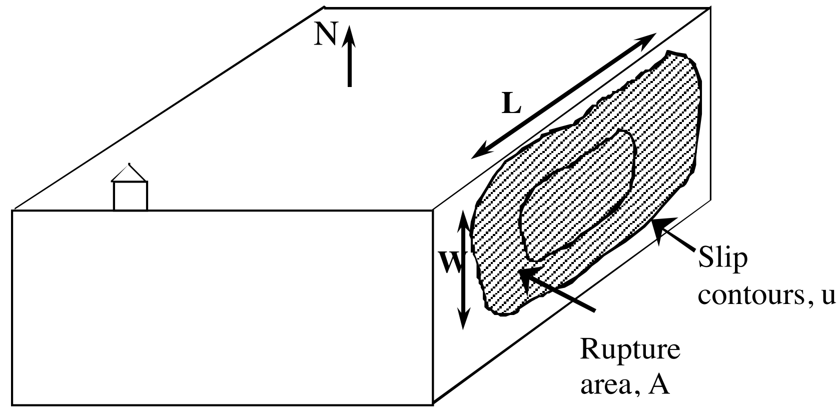
$$\Delta\sigma = 2(\mu_s - \mu_d)\sigma_n$$

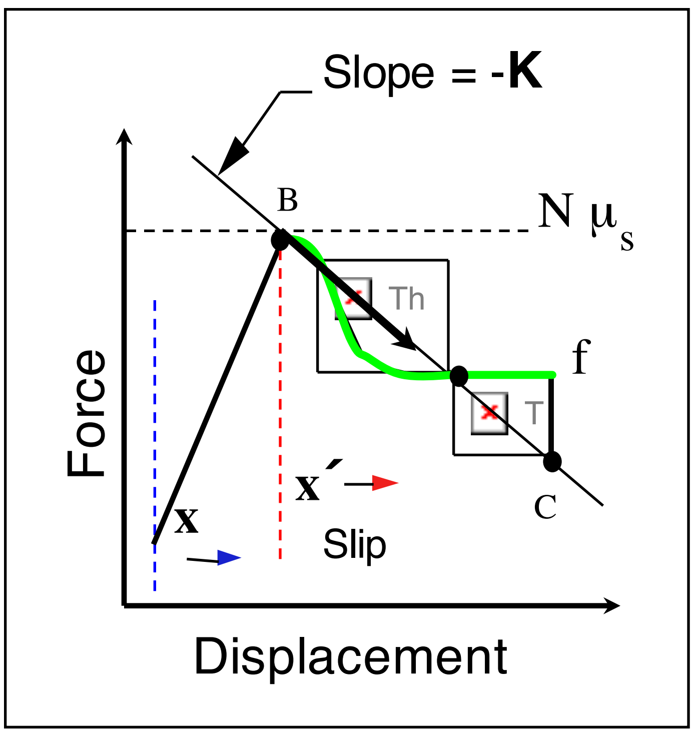
Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear



1-D fault zone analog, Stiffness K





Frictional stability is determined by the combination of

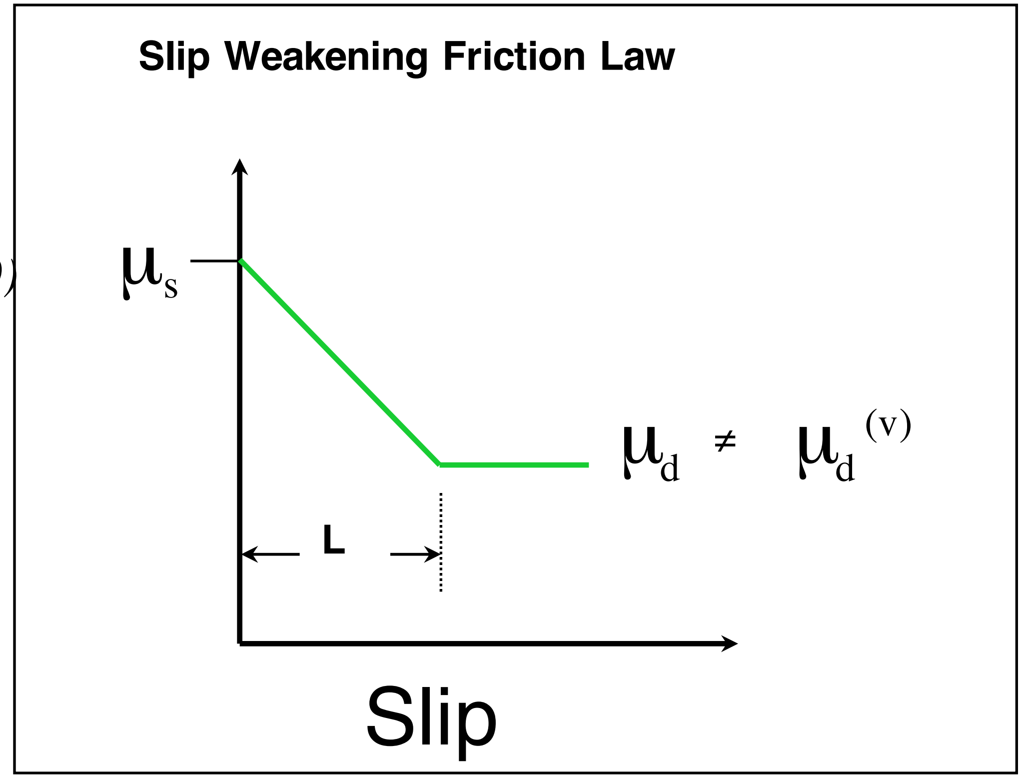
1) fault zone frictional properties and

2) elastic properties of the surrounding material

$$\mu(x) = \mu_s - \frac{x}{L}\Delta\mu \qquad (for\ L > x > 0)$$

$$\mu(x) = \mu_s - \Delta\mu \qquad (for\ x > L)$$

Palmer and Rice, 1973; Ide, 1972; Rice, 1980



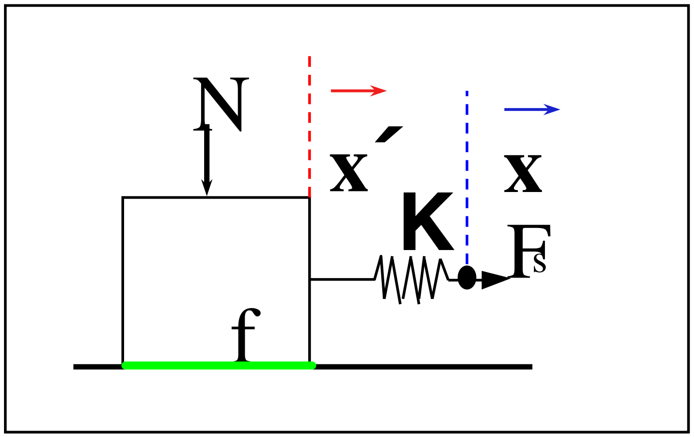
Quasistatic Stability Criterion

$$K_c = \frac{\sigma_n(\mu_s - \mu_d)}{L}$$

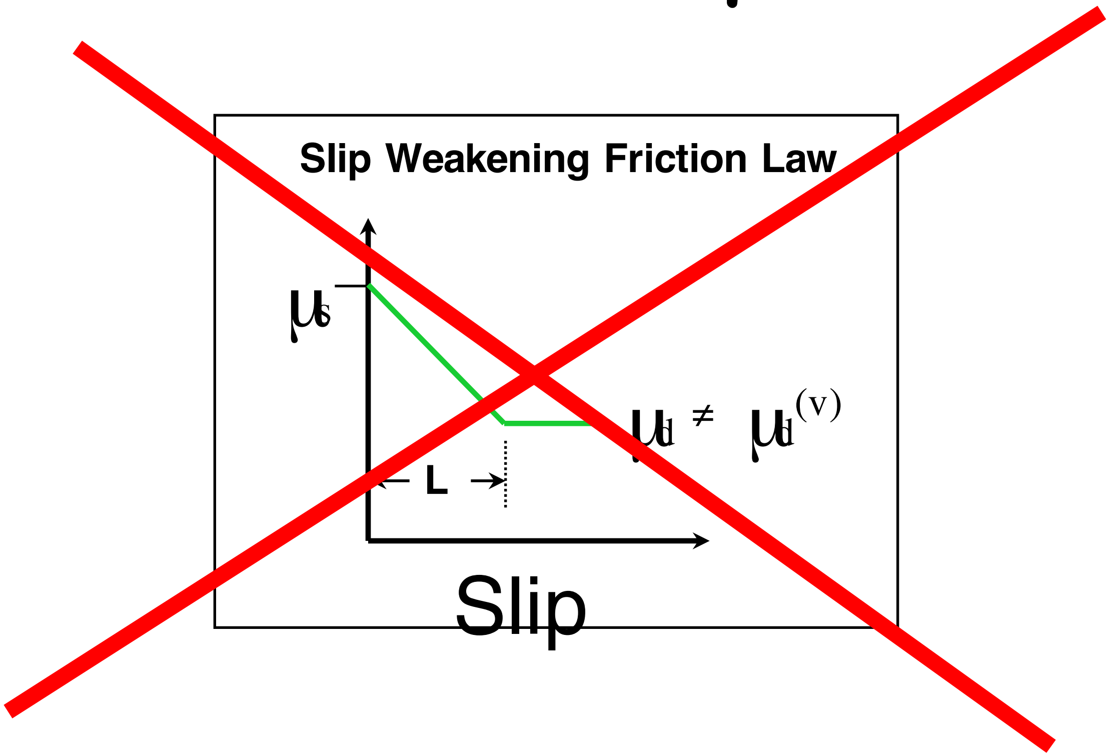
K< K_c; Unstable, stick-slip

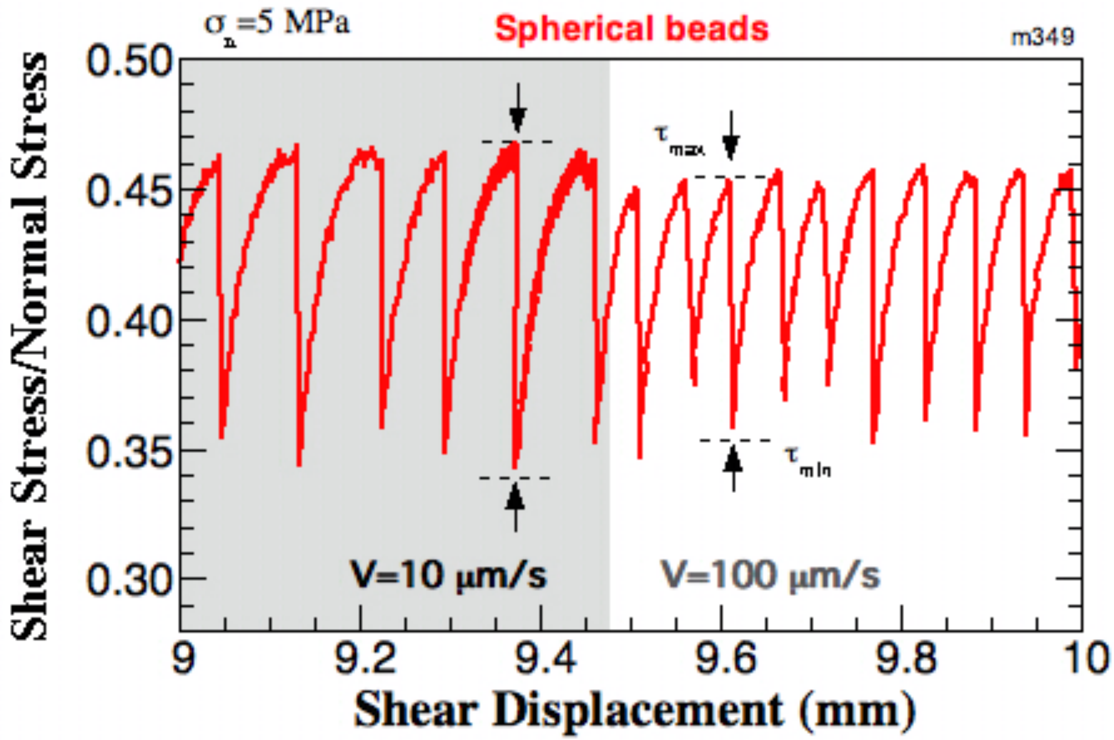
K > K_c; Stable sliding

Laboratory Studies



But, there's a problem…….

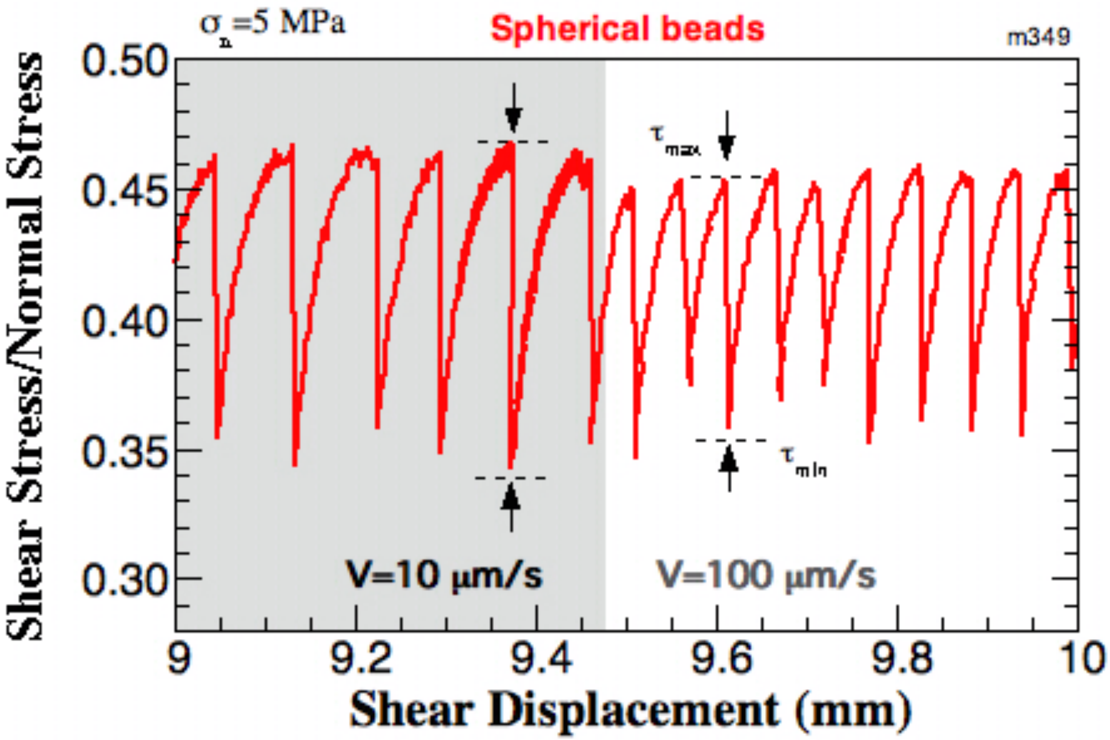




Mair, Frye and Marone, JGR 2002

Repetitive stick-slip

Seismic cycle of interseismic stress accumulation and repeating earthquakes

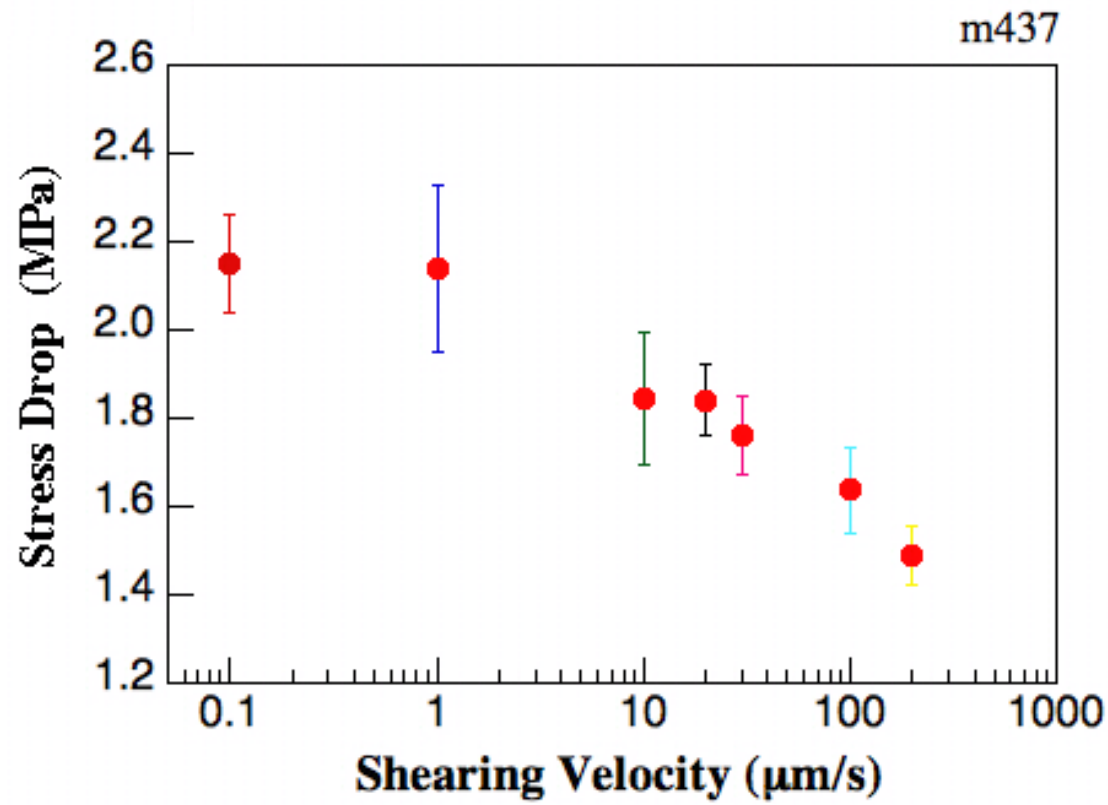


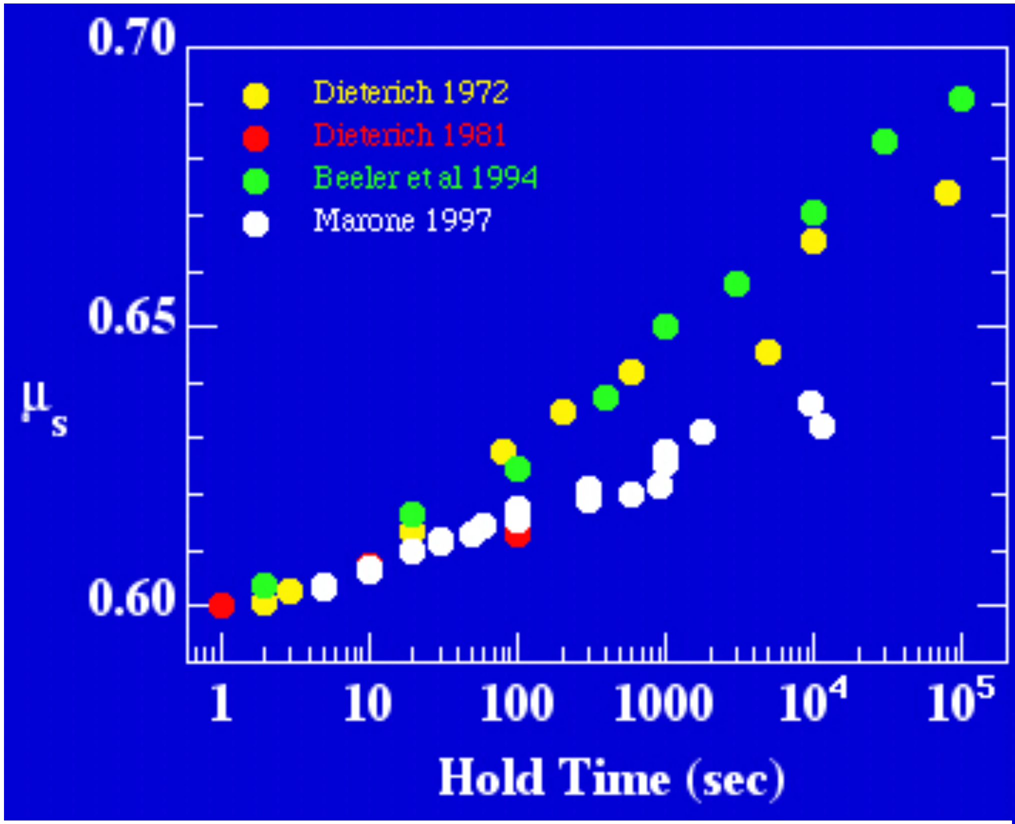
Stick-slip stress-drop amplitude varies with loading rate.

Mair, Frye and Marone, JGR 2002

Duality of time and displacement dependence of friction.

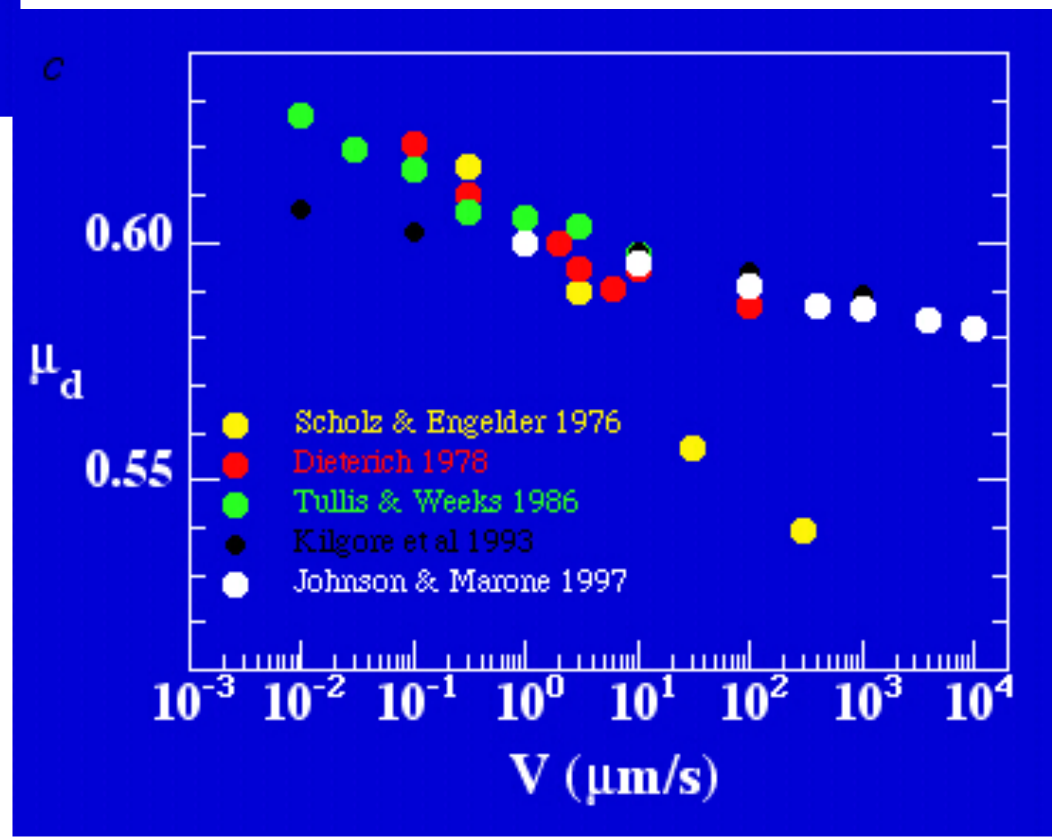
“Static” and “dynamic” friction are just special cases of a more general behavior called “rate and state friction”





Time (state) dependence of friction: Healing

Velocity (rate) dependence of friction.

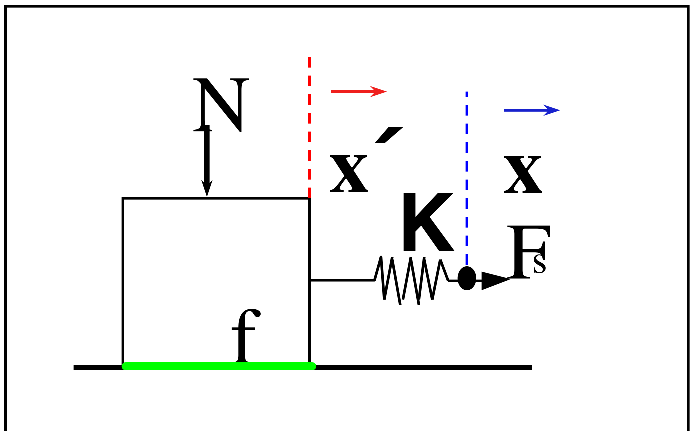


Duality of time and displacement dependence of friction.

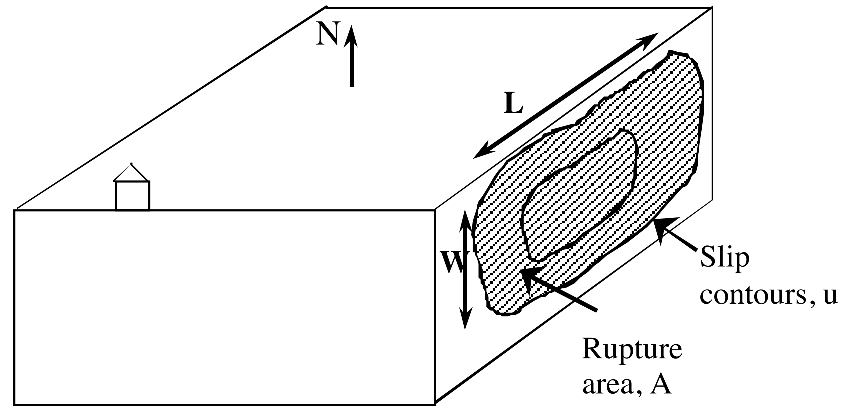
"Static" and "dynamic" friction are just special cases of a more general behavior called "rate and state friction"

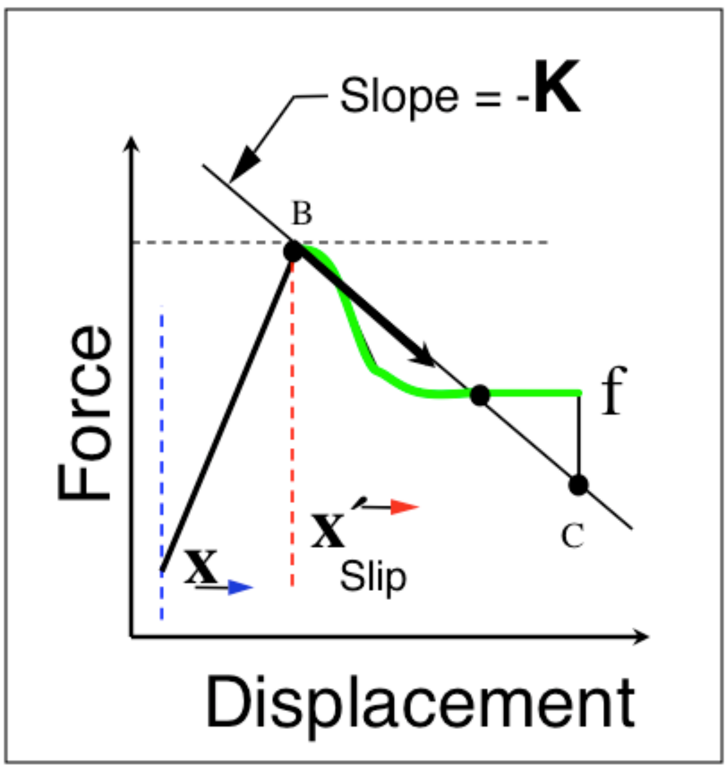
Brittle Friction Mechanics, Stick-slip

Stick-slip (unstable) versus stable shear



1-D fault zone analog, Stiffness K





What causes this weakening?

In the context of the seismic cycle, this happens repeatedly.

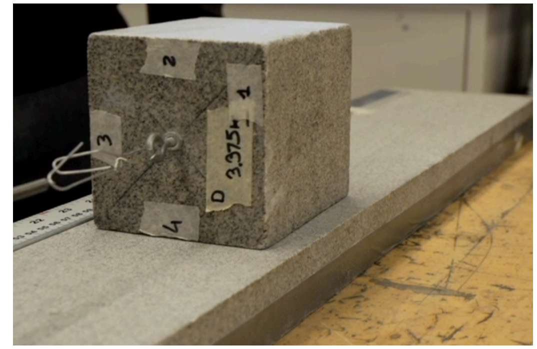
How does that work? What causes repeated weakening?

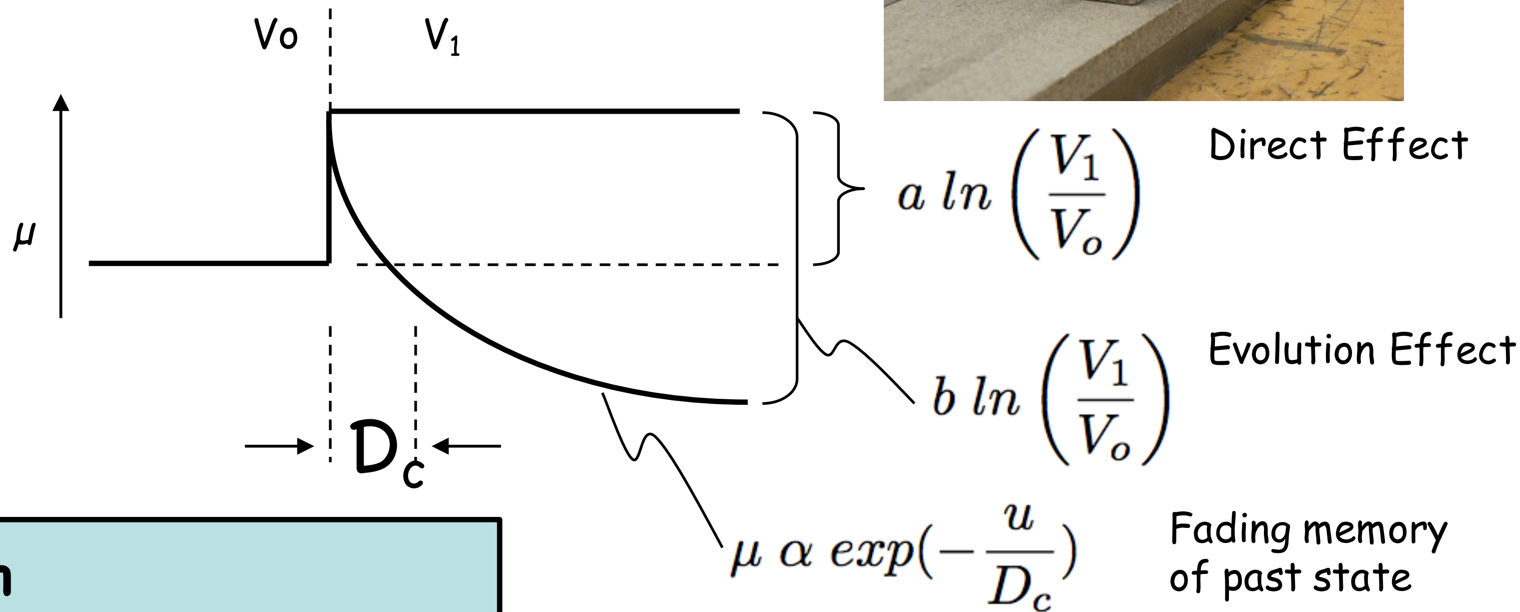
Stick-Slip Instability Requires Some Form of Weakening:

Velocity Weakening, Slip Weakening, Thermal/hydraulic Weakening

1) $\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$

2) $\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$





Stability Criterion

$$K_c = \frac{\sigma_n(b-a)}{D_c}[1 + \frac{mV_o{}^2}{\sigma_n a D_c}]$$

(b > a), K < K_c Unstable, stick-slip

(a > b), K > K_c Stable sliding

$K/K_c < 1$

Rate (v) and State (θ) Friction Constitutive Laws

Recall (as motivation for going beyond other friction laws)

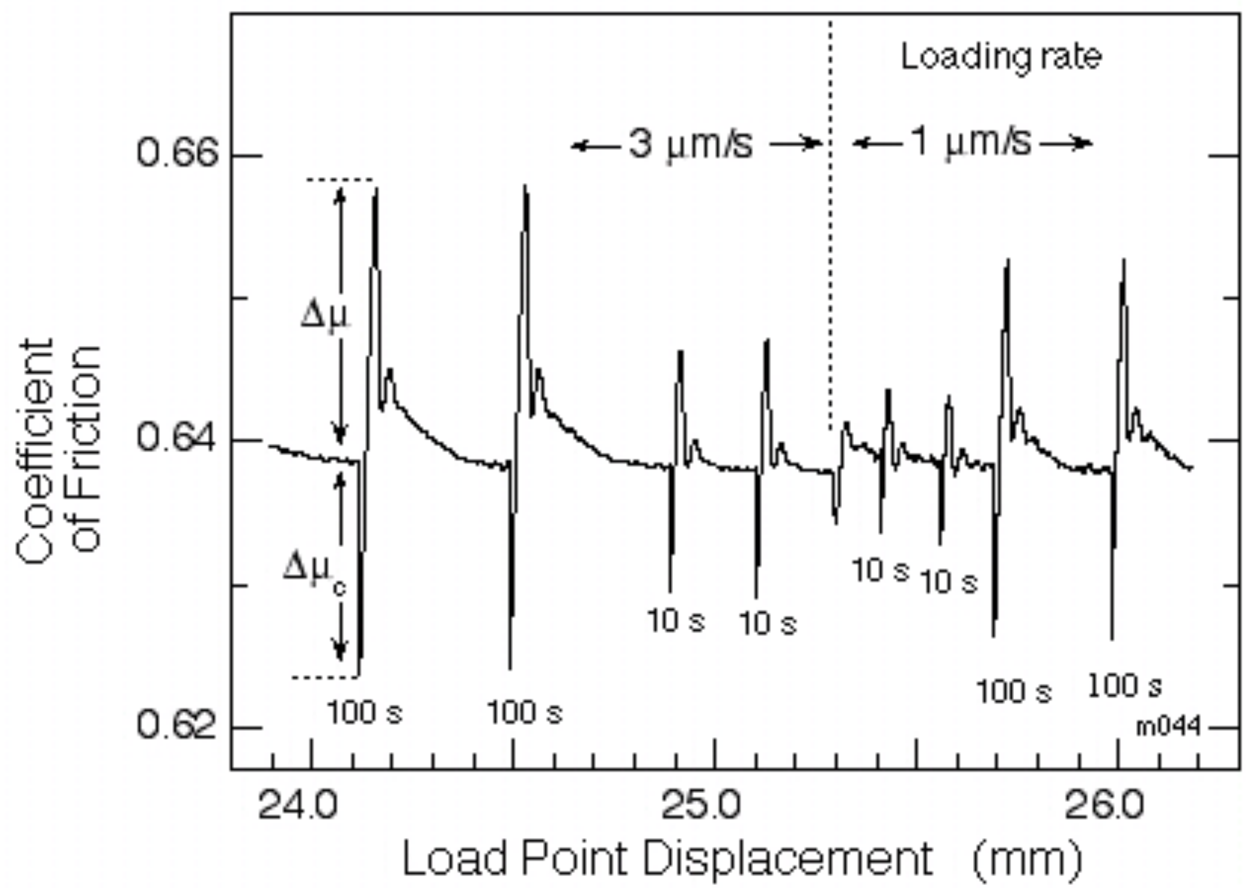
- **Time-dependent static friction**
- **Velocity dependent sliding friction**
- **Memory effects, state dependence**
- **Repetitive stick-slip instability**

Key Observations

- log-time strengthening
- log-velocity dependence

Application to earthquakes

- One set of constitutive relations to describe 'entire' seismic cycle



Rate (v) and State (θ) Friction Constitutive Laws

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

state variable, characterizes physical state of surface or shearing region

critical slip distance

reference velocity

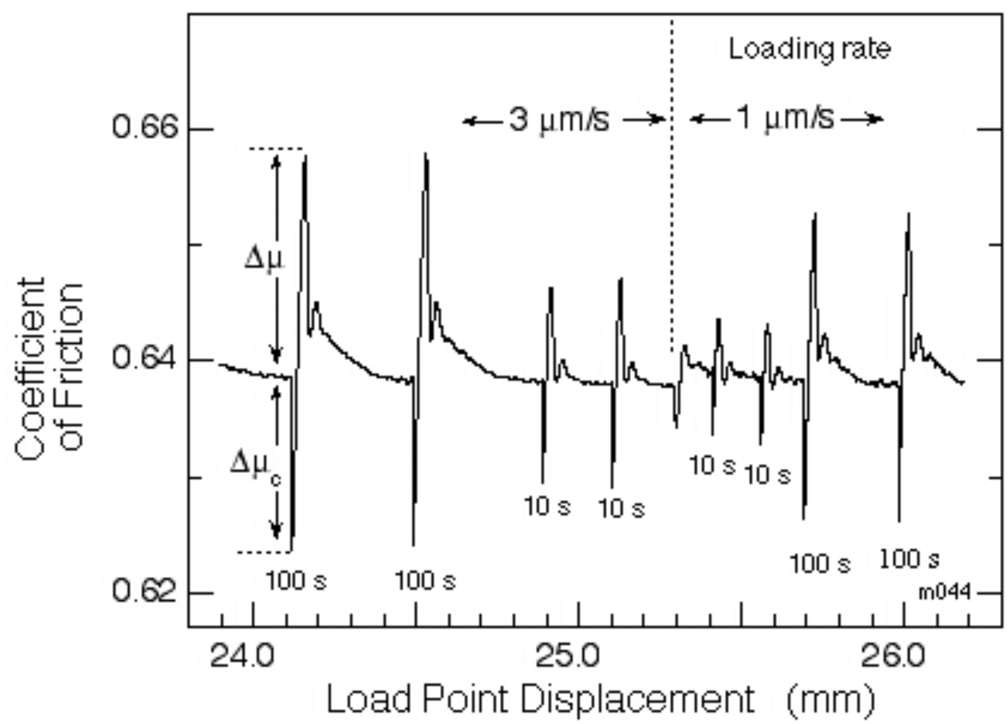
reference value of base friction

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Dieterich, aging law

$$\frac{d\theta}{dt} = -\frac{V\theta}{D_c} ln(\frac{V\theta}{D_c})$$

Ruina, slip law

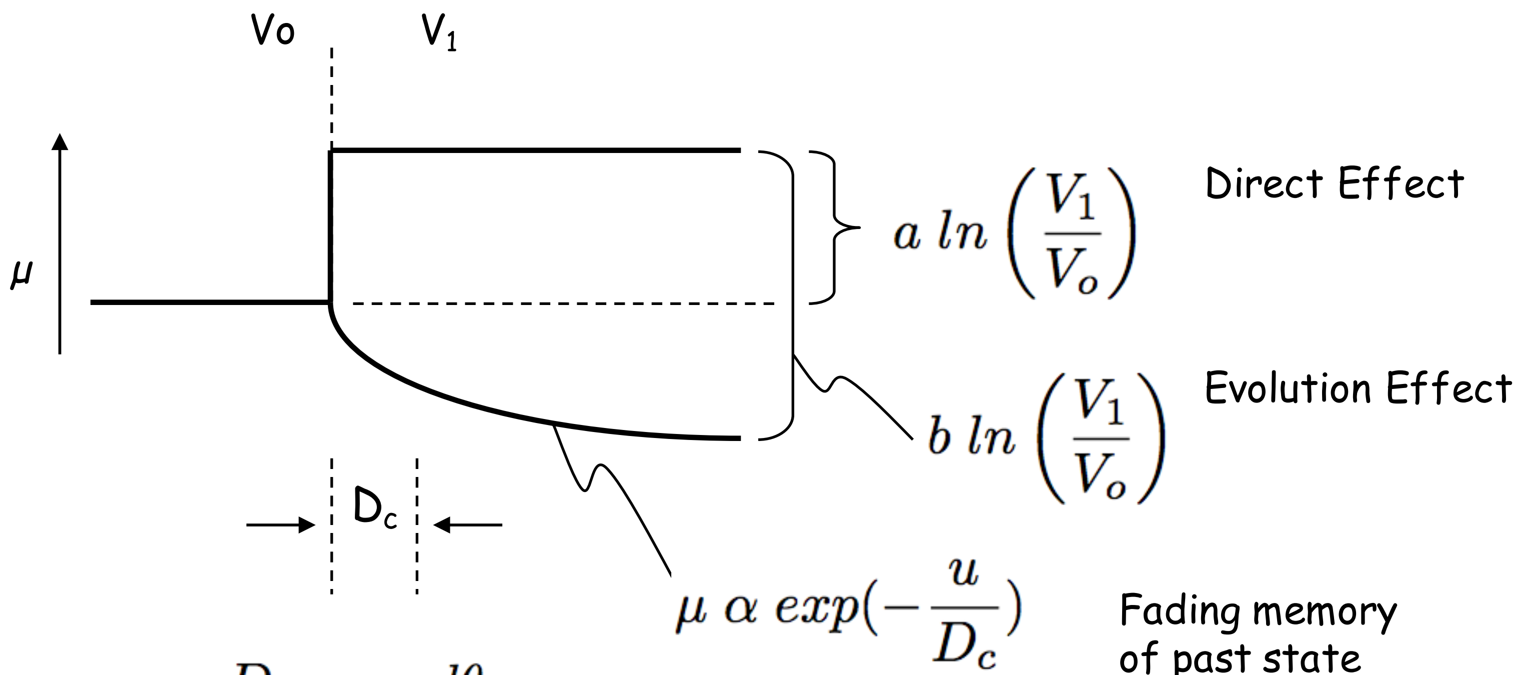


Rate (v) and State (θ) Friction Constitutive Laws

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Implies:



Steady-state sliding: $\theta_{ss} = \frac{D_c}{V_1}$ => $\frac{d\theta}{dt} = 0$

then (1) becomes: $$\mu_1 - \mu_o = (a-b) ln(\frac{V_1}{V_o})$$

$$(a-b) = \frac{\Delta\mu}{\Delta lnV}$$

Rate (v) and State (θ) Friction Constitutive Laws

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

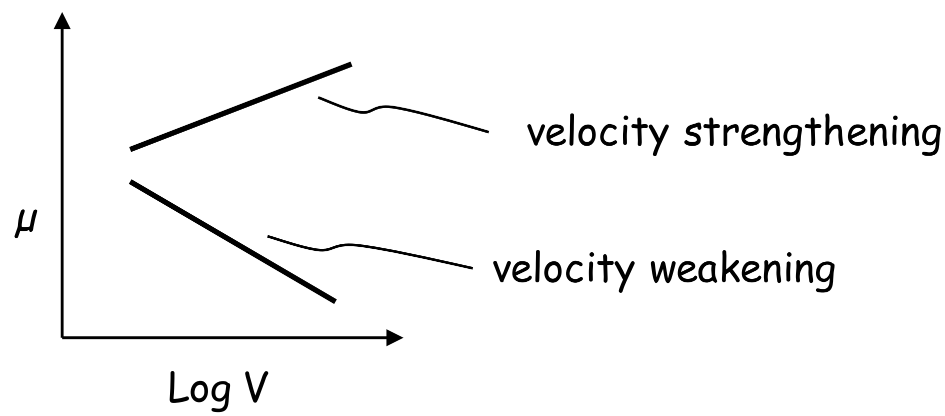
2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Convention is to use a, b for friction and A, B for Stress

$$\tau(\theta, v) = \tau_o + A\, ln\left(\frac{V}{V_o}\right) + B\, ln\left(\frac{V_o\theta}{D_c}\right)$$

$$A - B = \frac{\Delta\tau}{\Delta lnV}$$

Steady-state velocity strengthening if a-b > 0,
velocity weakening if a-b < 0

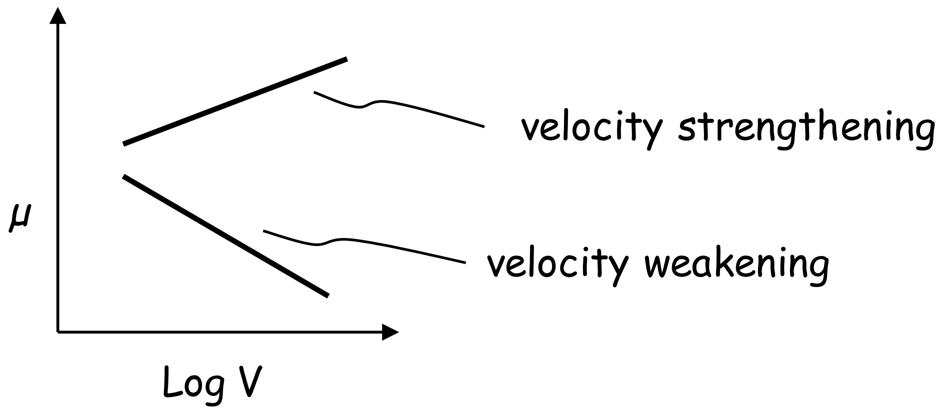


Rate (v) and State (θ) Friction Constitutive Laws

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Steady-state velocity strengthening if a-b > 0,
velocity weakening if a-b < 0



a & b are small, dimensionless constants determined from experiments

Dc has units of length

Modeling experimental data

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling

Rate (v) and State (θ) Friction Constitutive Laws

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Modeling experimental data

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling

$$V = V_o\, exp\left[\frac{\mu - \mu_o - b\, ln(\frac{V_o\theta}{D_c})}{a}\right]$$

Solve:

$$\frac{d\mu}{dt} = k\left(V_{lp} - V_o\, exp\left[\frac{\mu - \mu_o - b\, ln(\frac{V_o\theta}{D_c})}{a}\right]\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

1) $\mu(\theta, V) = \mu_o + a\ ln\left(\frac{V}{V_o}\right) + b\ ln\left(\frac{V_o\theta}{D_c}\right)$

2) $\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$

3) $\frac{d\mu}{dt} = k(V_{lp} - V)$

Typical Values of the RSF parameters (Marone et al., 1990)

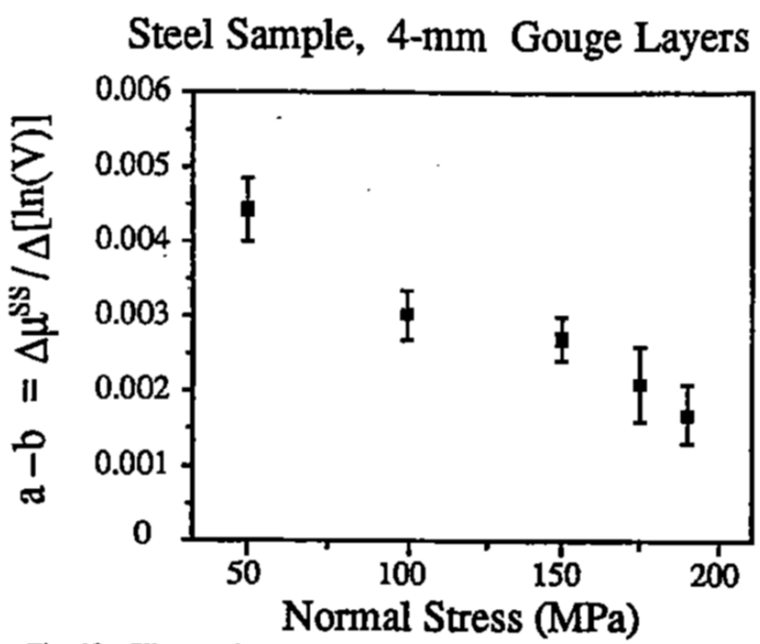


Fig. 13. Slip rate dependence of steady state friction (a–b* Table 2) as a function of normal stress for gouge sheared within rough steel surfaces. The mean value ±1 standard deviation is plotted for 110 measurements, roughly evenly distributed over the five normal stresses. The parameter a–b varies inversely with normal stress as a result of decreasing a and slightly increasing b with increasing normal stress (see Table 2).

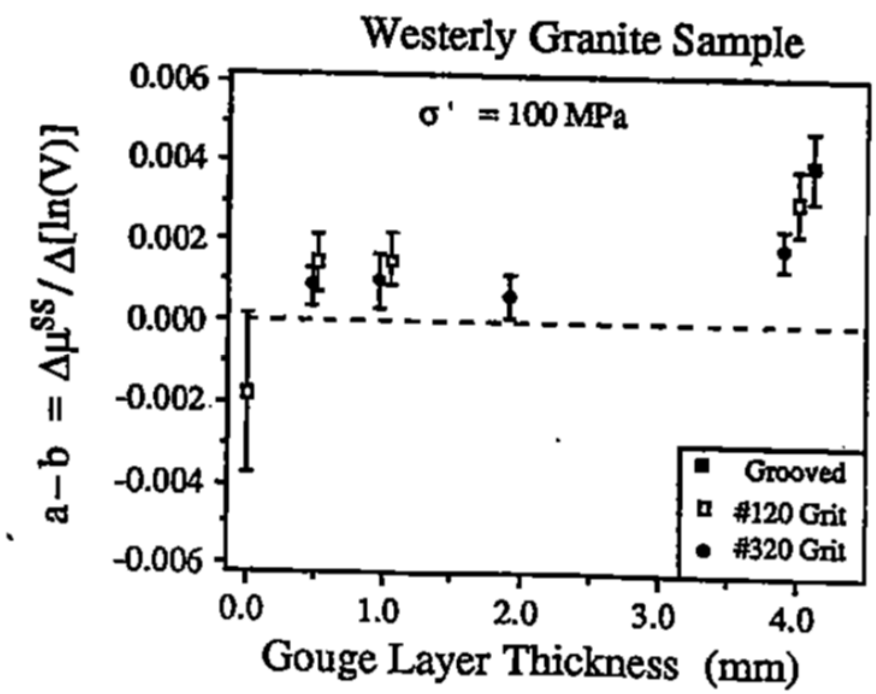


Fig. 17. Mean values of a–b ±1 standard deviation are plotted as a function of gouge layer thickness and surface roughness (data from Table 2). Data for 320 grit and grooved surfaces are offset horizontally for clarity. The data are corrected for jacket and apparatus effects; a–b decreased with decreasing gouge thickness and, at a given gouge thickness, was lower for smoother surfaces. Initially bare 320 grit surfaces exhibited dominantly unstable slip, and therefore we did not measure a–b but would infer velocity weakening from the unstable nature of slip.

1) $\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o \theta}{D_c}\right)$

2) $\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$

3) $\frac{d\mu}{dt} = k(V_{lp} - V)$

Typical Values of the RSF parameters (Marone et al., 1990)

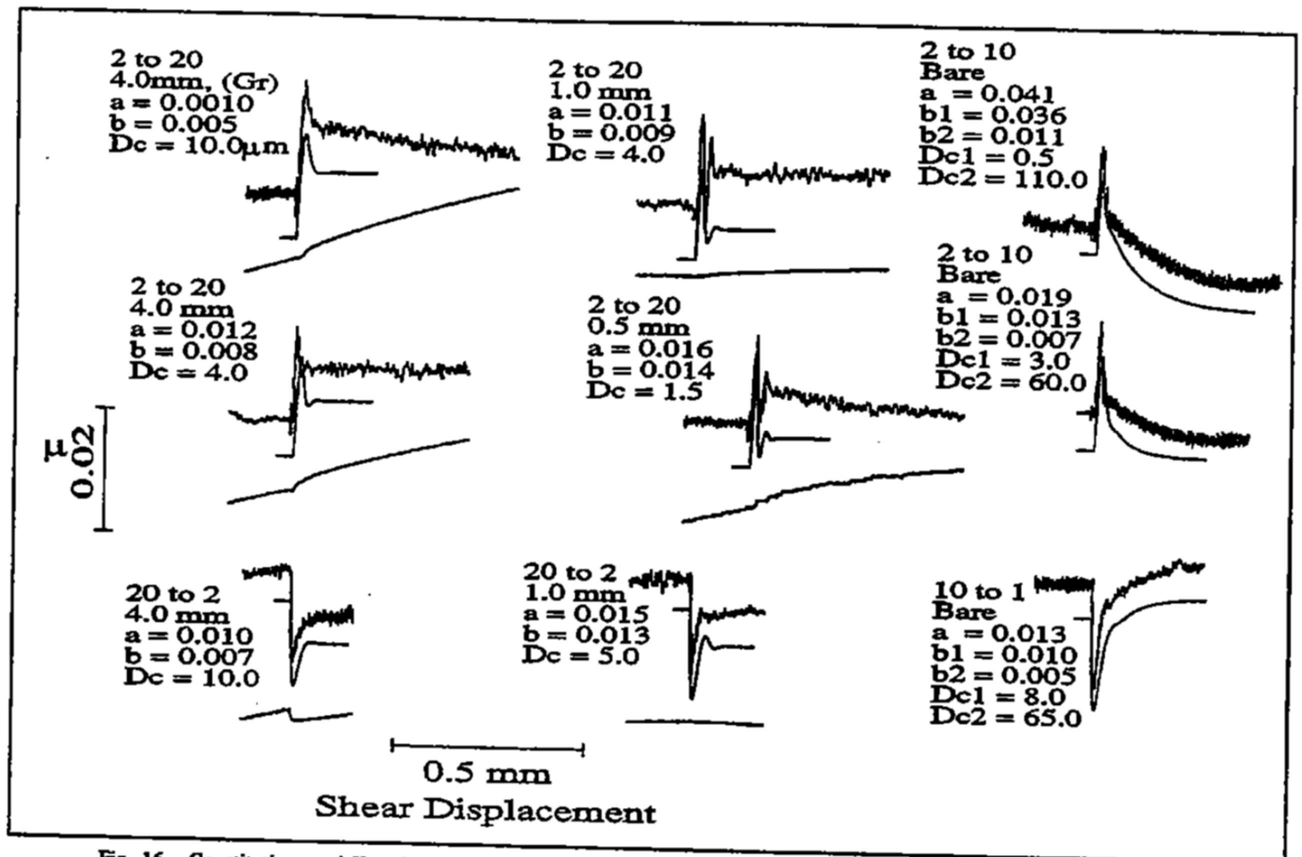


Fig. 16. Constitutive modeling for gouge layers sheared within granite surfaces. In each case, the raw friction data (note electrical noise) are plotted above a rate and state variable numerical simulation, which has been offset downward for clarity, and porosity is plotted below (except for the cases of initially bare surfaces.) The load point velocity (μm/s) before and after the velocity step is given first, followed by the gouge thickness and constitutive parameters. The data are for slip within 120 grit surfaces, except for that at the top left labeled (Gr), which is for shear within a grooved sample. Note that velocity strengthening occurs for shear of an initial gouge layer, whereas initially bare granite surfaces exhibit velocity weakening.

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o \theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$

Typical Values of the RSF parameters (Carpenter, Ikari & Marone 2016)

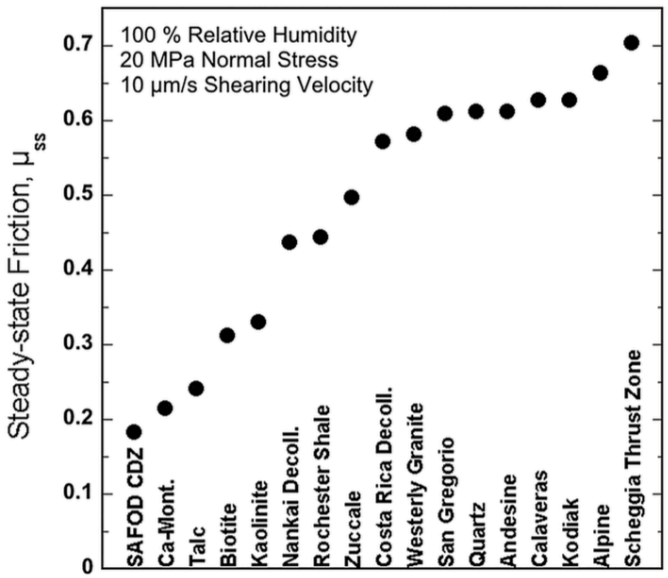


Figure 3. The steady state coefficient of friction for each sample used in this study. The value of μ_{ss} was determined at 20 MPa normal stress, 10 μm/s shearing velocity, 100% relative humidity, and just prior to SHS tests (see Figure 1). Note that our samples exhibit μ_{ss} values ranging from 0.19 to 0.71.

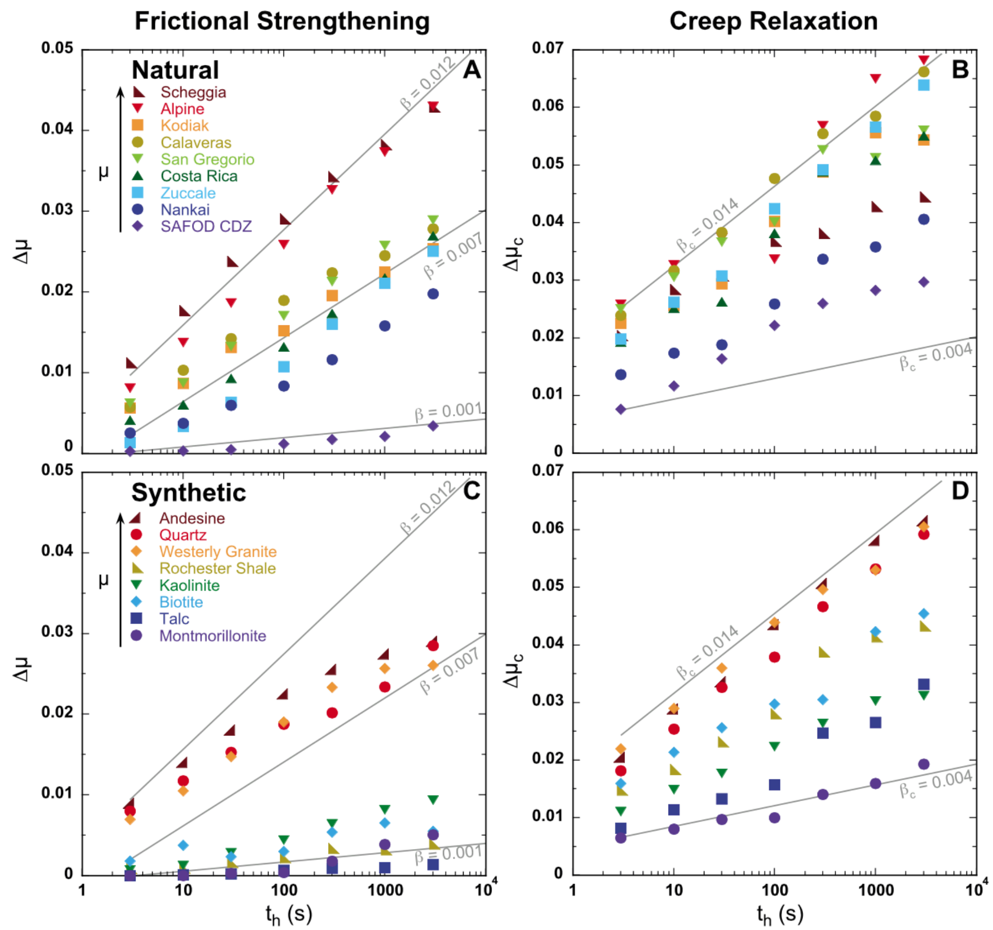
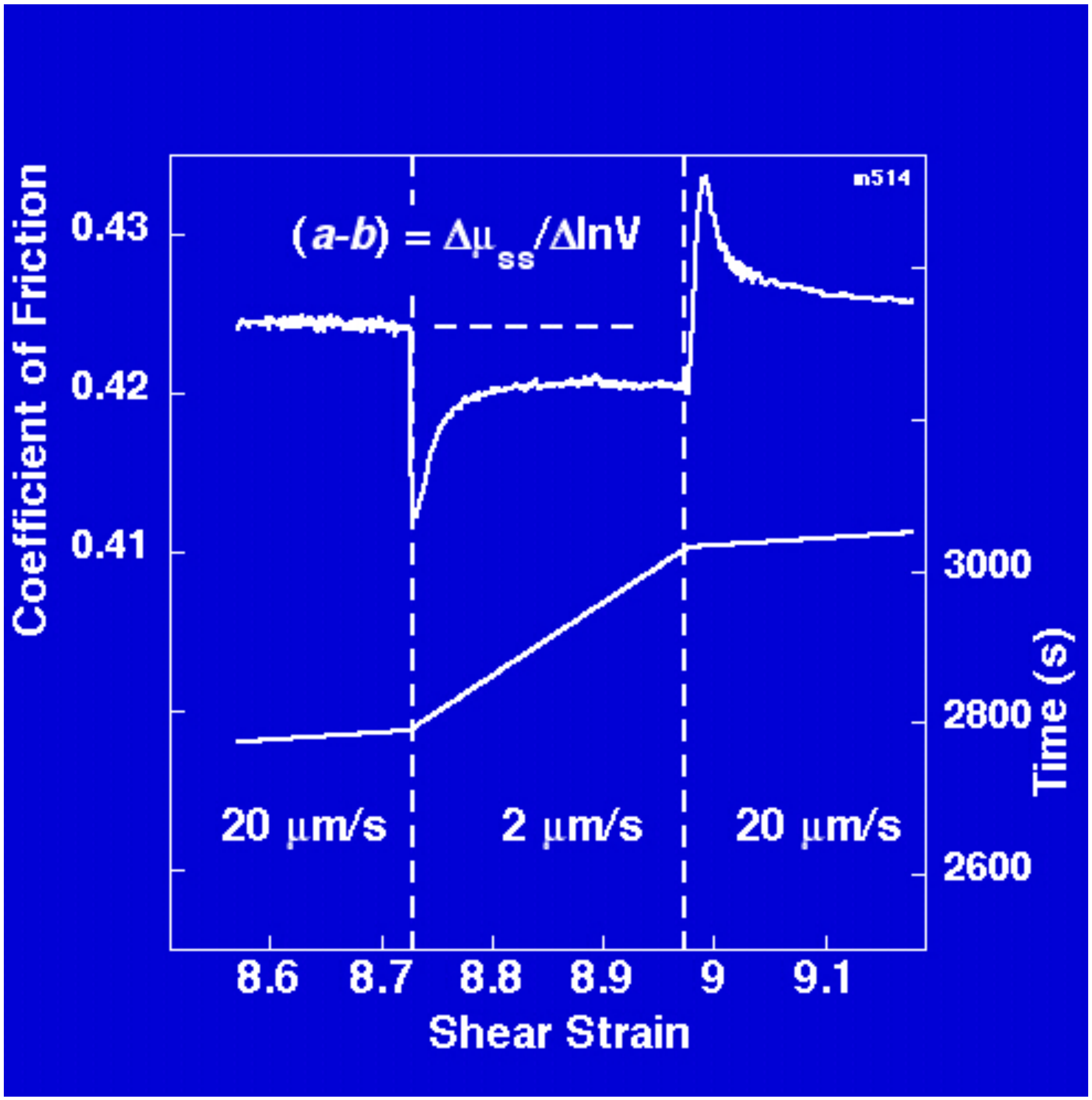


Figure 4. Slide-hold-slide parameters $\Delta\mu$ and $\Delta\mu_c$ are plotted against hold time (t_h) for (a, b) natural and (c, d) synthetic samples. Samples are listed in order from low to high friction (arrow points toward higher friction), and friction increases from cool to warm colors. This color scheme is used on many of the following figures. Note the three populations of strengthening behaviors for natural samples: (1) low-strengthening rate (SAFOD), (2) intermediate rate, and (3) high-strengthening rate (Alpine and Scheggia Fault). Synthetic samples exhibit two populations of strengthening rates. Also note that samples with larger friction exhibit greater creep relaxation compared to weaker samples. Fiducial lines show noted rates.

Measuring the velocity dependence of friction

Frictional Instability Requires (a-b) < 0



Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

$$\mu(\theta,v) = \mu_0 + a\ln\left(\frac{v}{v_o}\right) + b\ln\left(\frac{v_o\theta}{D_c}\right)$$

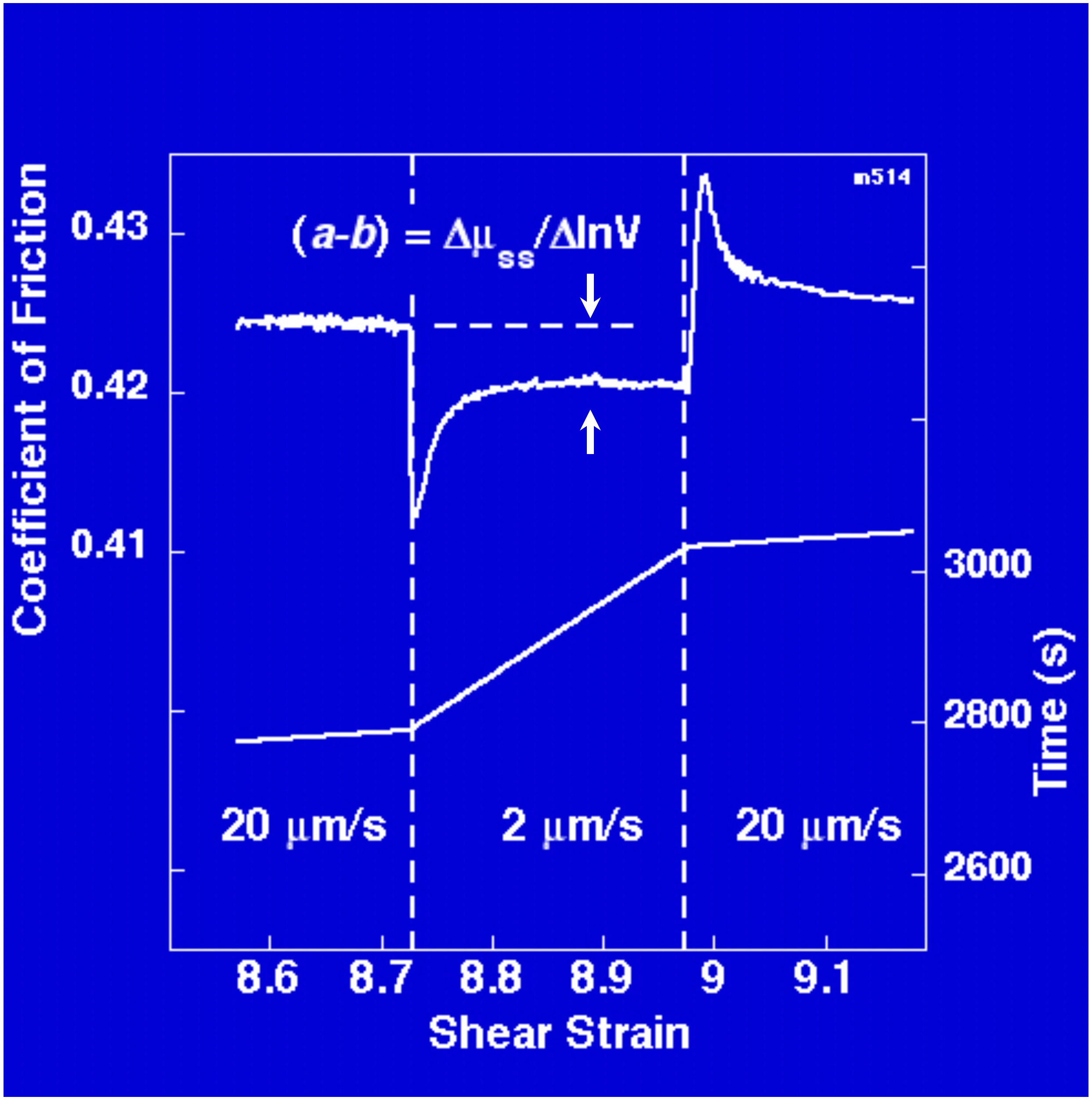
$$\frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c}$$

$$\theta_{ss} = \frac{D_c}{v}$$

$$\Delta\mu_{ss} = (a-b)\ln\left(\frac{v}{v_o}\right)$$

$$\frac{d\mu}{dt} = k'\,(v_{lp} - v)$$

Results: Velocity stepping
Measuring the velocity dependence of friction

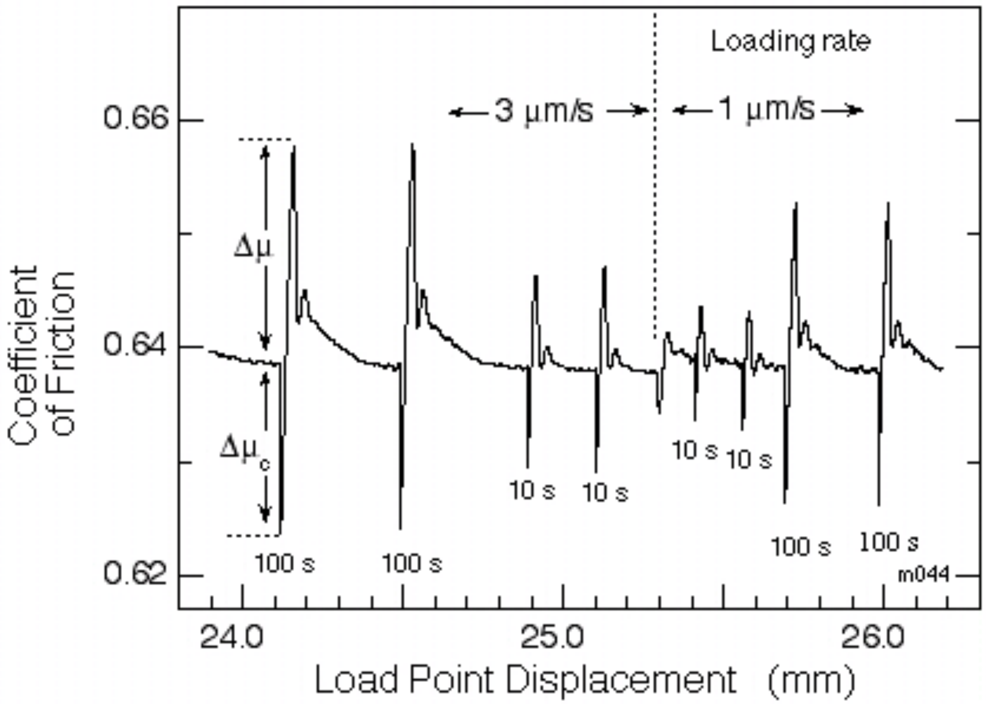


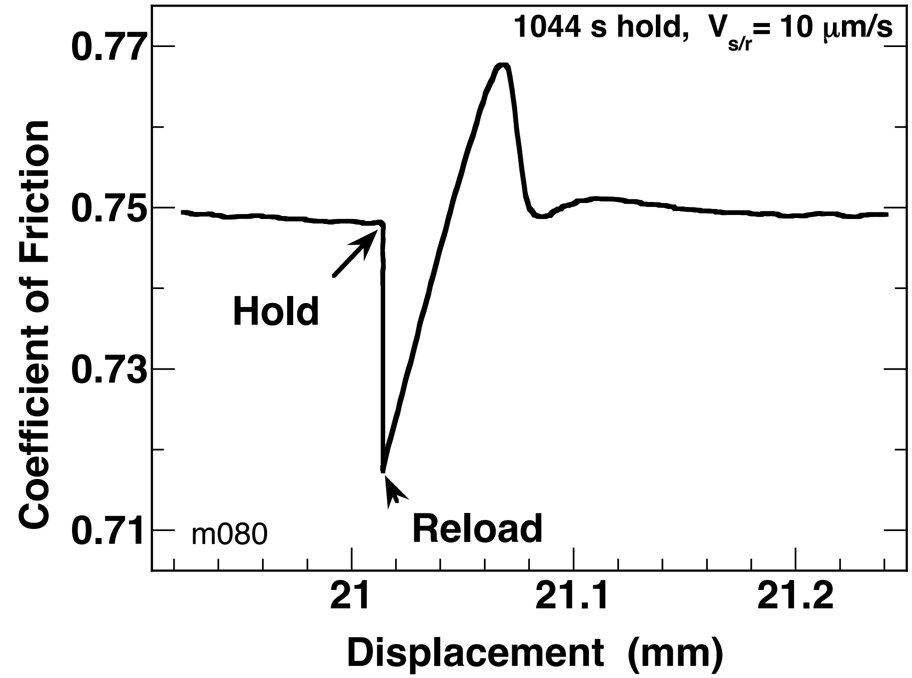
Frictional Instability

Requires $K < K_c$

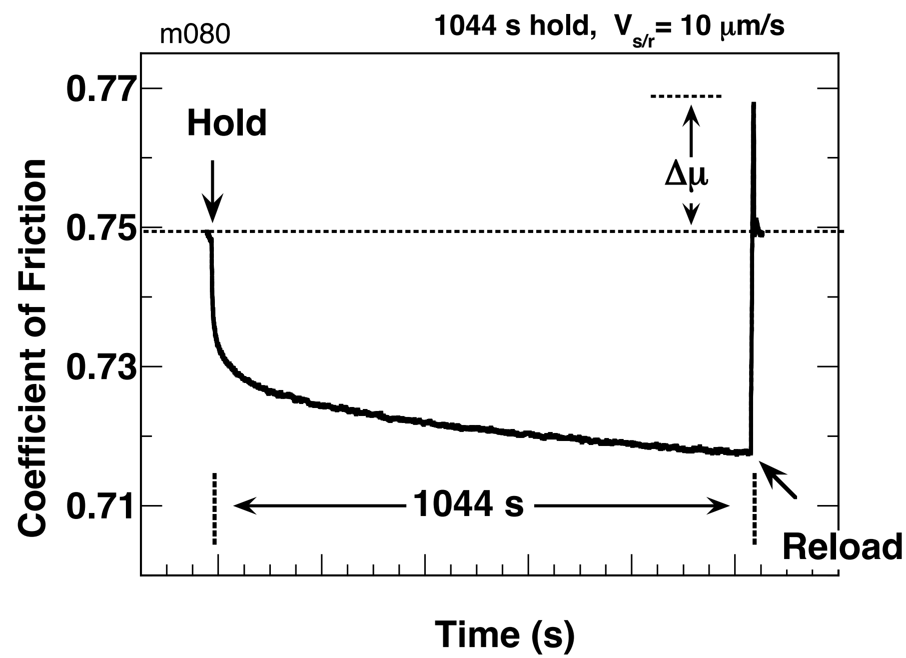
$$K_c = \frac{\sigma_n (b-a)}{D_c}$$

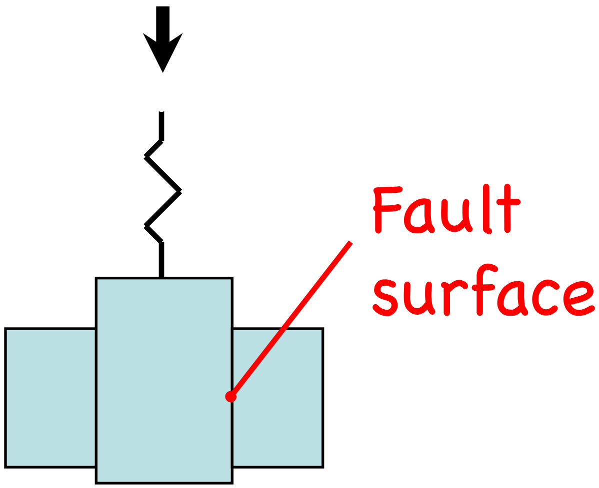
This example shows steady-state velocity strengthening: $(a-b) > 0$

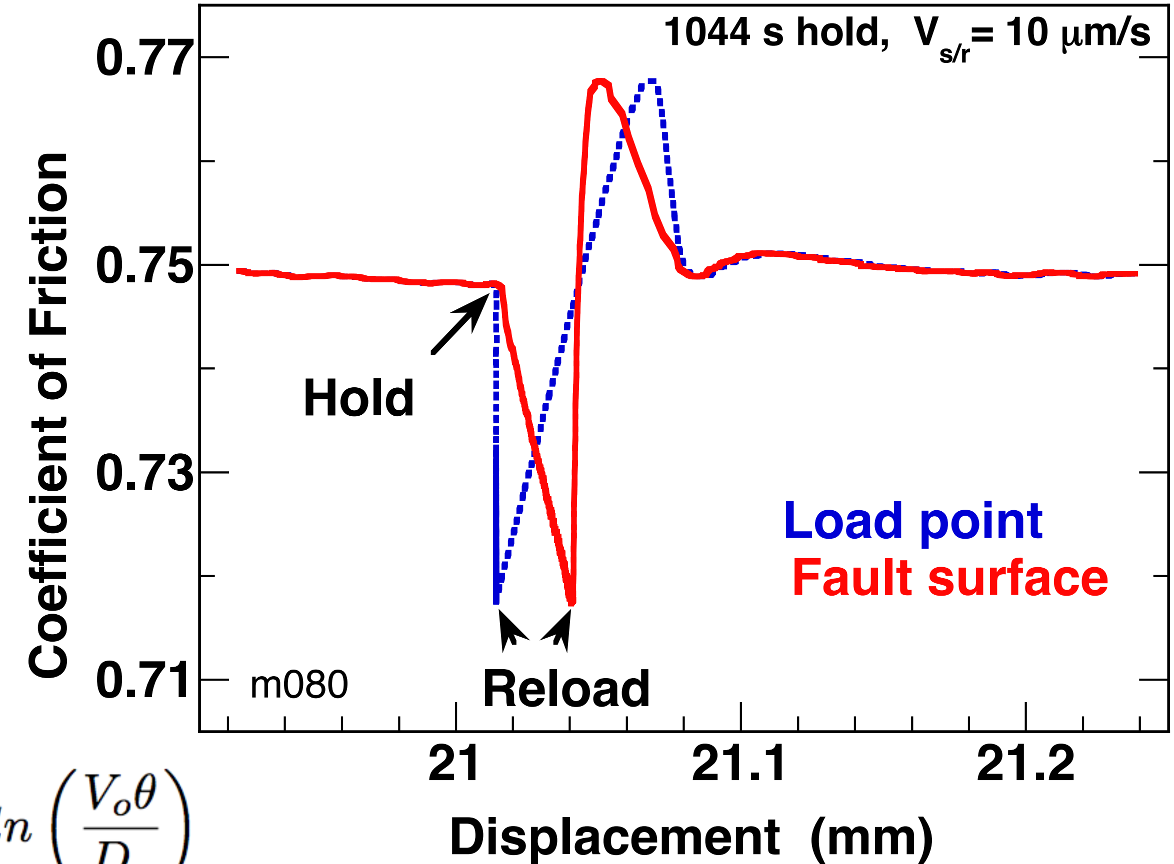




Sheared layer of quartz particles (100-150 μm), 25 MPa normal stress . Marone, 1998







1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

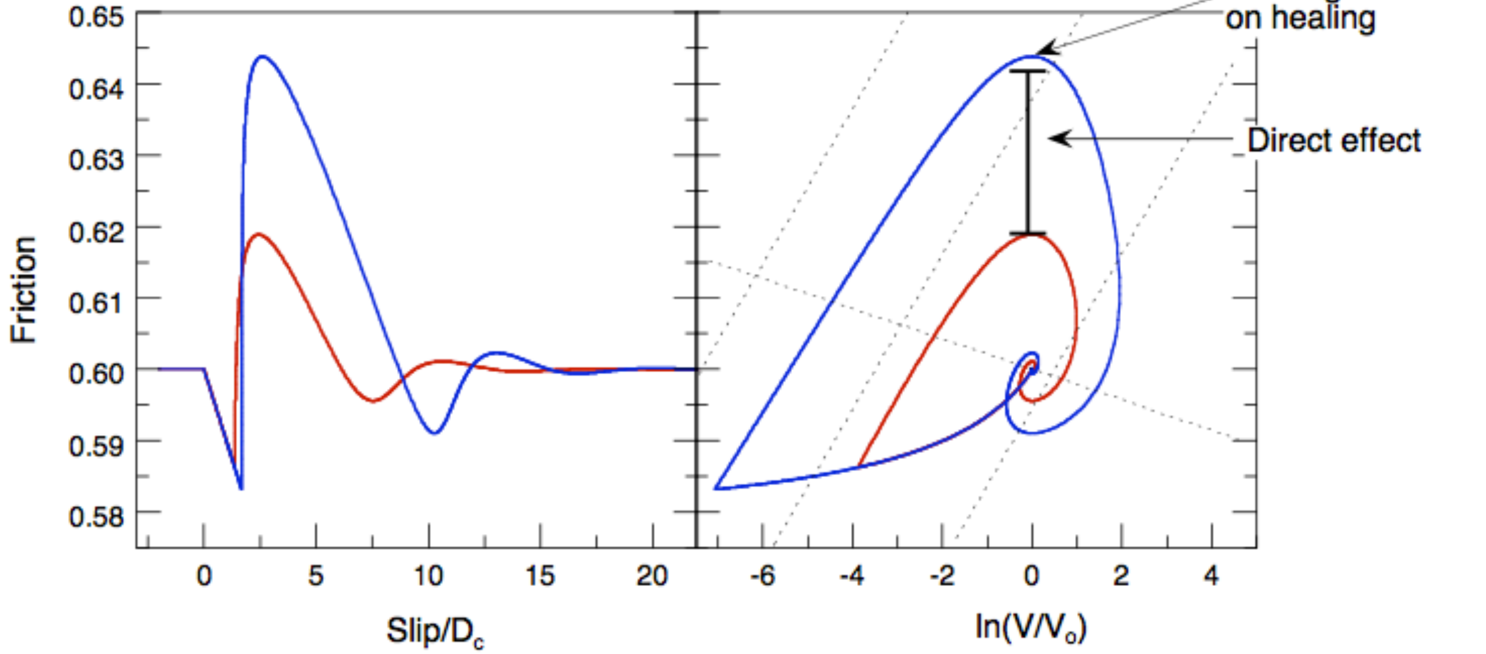
2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling

$$\frac{d\mu}{dt} = k\left(V_{lp} - V_o\, exp\left[\frac{\mu - \mu_o - b\, ln(\frac{V_o\theta}{D_c})}{a}\right]\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Derivation of the healing rate

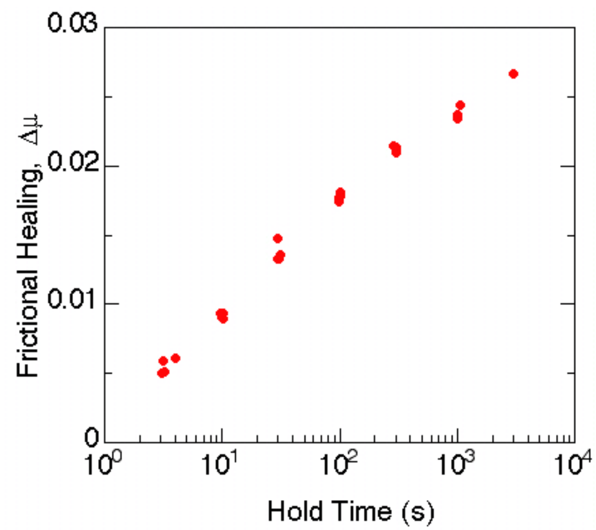


Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o \theta}{D_c}\right)$$

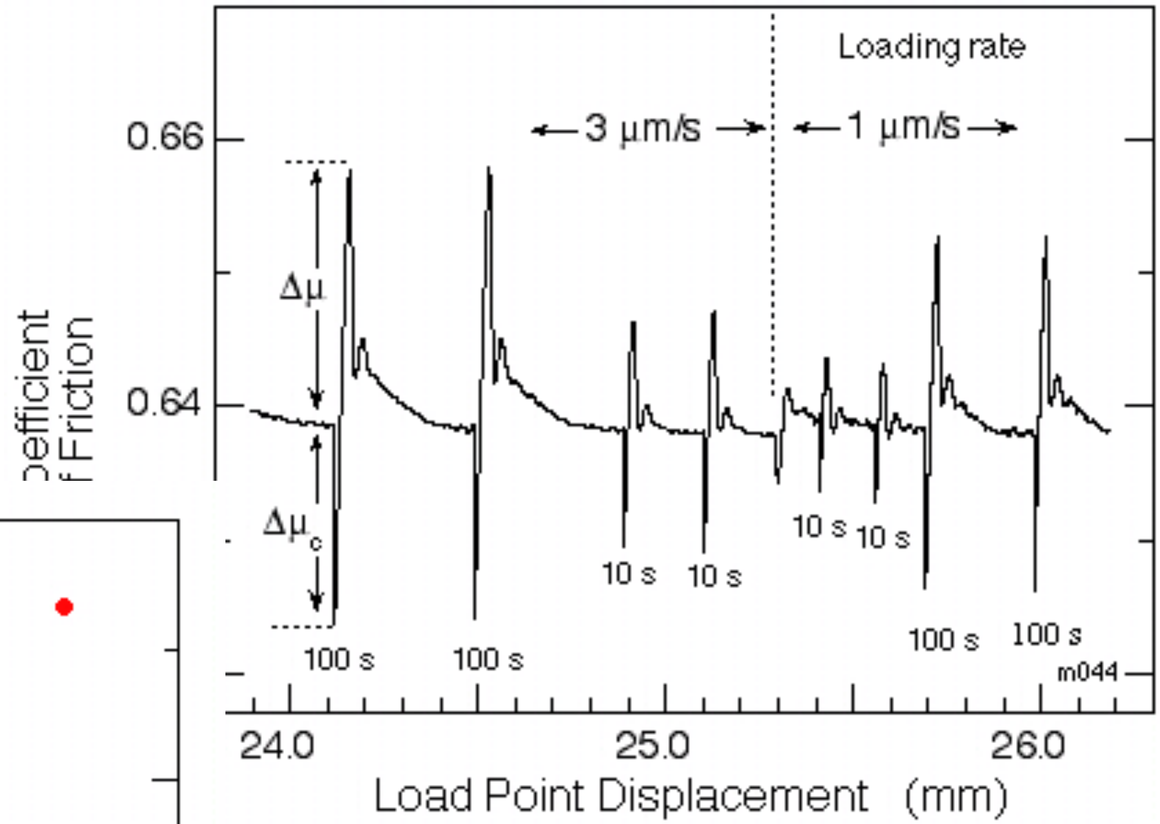
$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

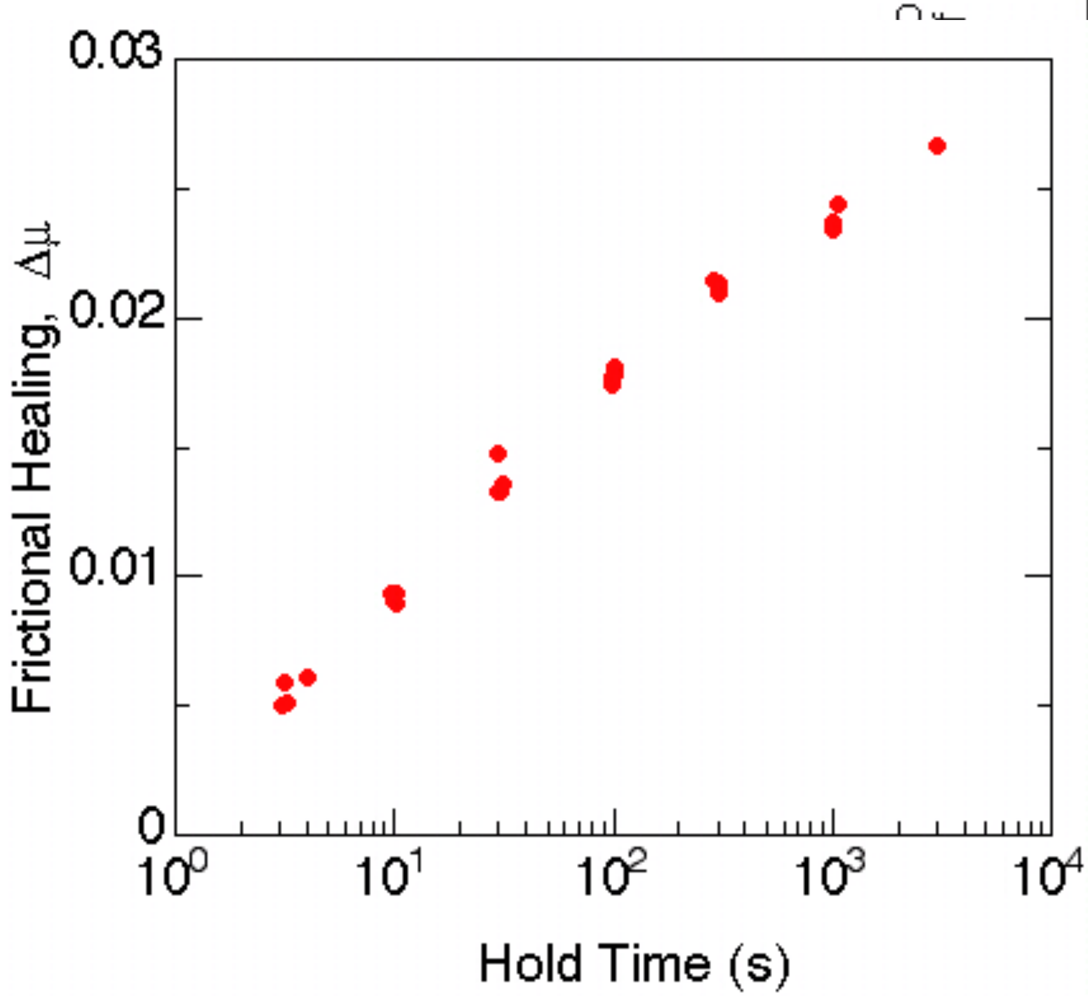
$$\frac{d\mu}{dt} = k(V_{lp} - V)$$



Time Dependence of "static" friction

Stressed Aging





Monodisperse,
angular quartz
particles

$$\mu = \frac{S_o}{\sigma_o} + \frac{g(V)}{\sigma_o} - \frac{S_o f(age)}{\sigma_o^{\ 2}}$$

Summary of friction observations:

0. Friction is to first order a constant
1. Time dependent increase in contact area (strengthening)
2. Slip dependent decrease in contact area (weakening); equivalently increase in dilatancy
3. Slip rate dependent increase in shear resistance (non-linear viscous)

Rate and state equations:

1st order term second order terms

$$\mu = \mu_0 + a \ln \frac{V}{V_0} + b \ln \frac{V_0 \theta}{D_c}$$

0. 3. 1. & 2.

θ is contact age

Dieterich [1979]
Rice [1983]
Ruina [1983]

$$\frac{d\theta}{dt} = 1 - \frac{\theta V}{D_c}$$

$$\frac{d\theta}{dt} = \left(\frac{\partial \theta}{\partial t}\right)_d + \left(\frac{\partial \theta}{\partial d}\right)_t V$$

time dependence

$$\left(\frac{\partial \theta}{\partial t}\right)_d = 1$$

slip dependence

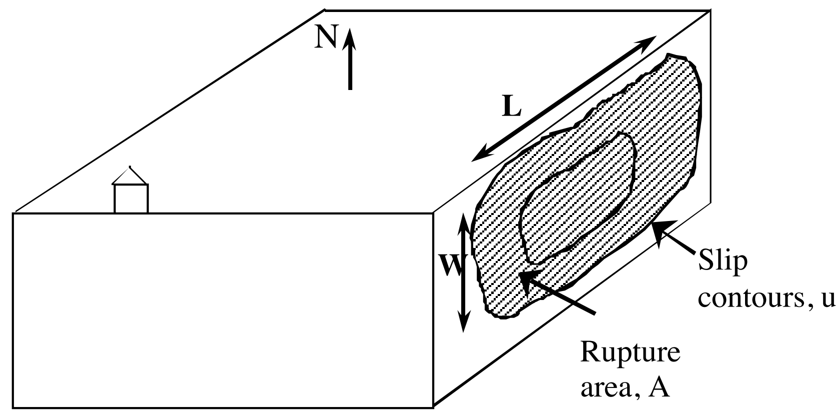
$$\left(\frac{\partial \theta}{\partial d}\right)_t = -\frac{\theta}{D_c}$$

Modified from Beeler, 2003

Stick-Slip Instability Requires Some Form of Weakening:
Velocity Weakening, Slip Weakening, Thermal/hydraulic Weakening

1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o \theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$



Stability Criterion

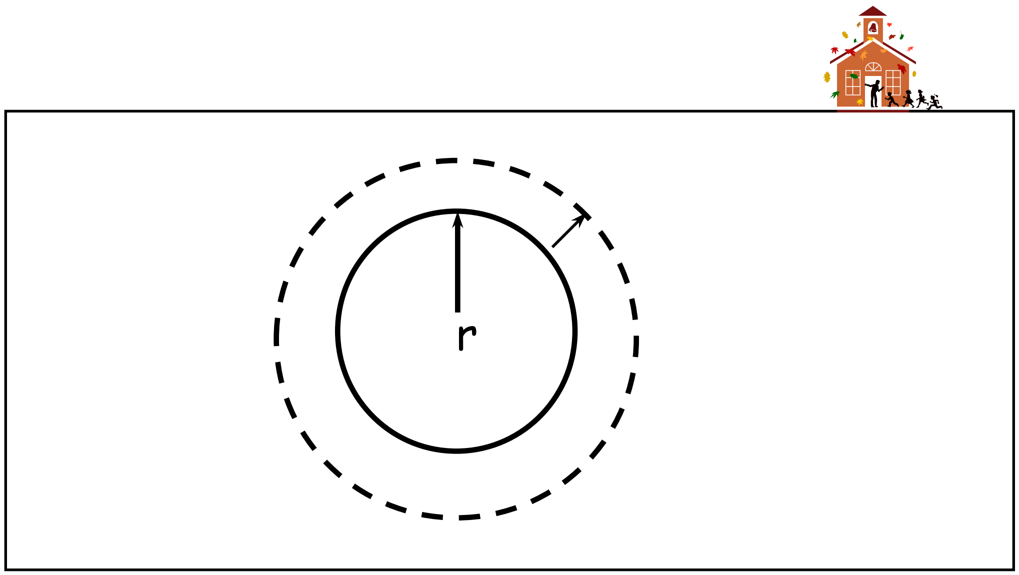
$$K_c = \frac{\sigma_n (b-a)}{D_c}\left[1 + \frac{m{V_o}^2}{\sigma_n a D_c}\right]$$

(b > a), K < K_c Unstable, stick-slip

(a > b), K > K_c Stable sliding

$K/K_c < 1$

Dislocation model for fault slip and earthquake rupture



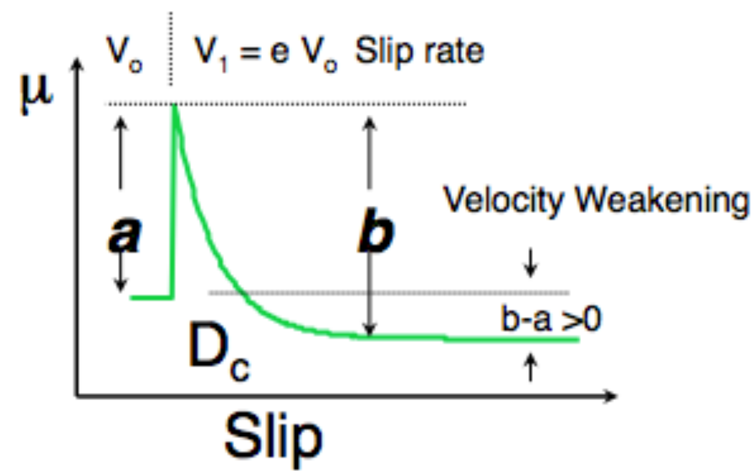
Relation between stress drop and slip:

$$\Delta\sigma = \frac{16}{7\pi} G \frac{\Delta\bar{u}}{r}$$

$$K = \frac{\Delta\sigma}{\Delta\bar{u}} = \frac{16}{7\pi}\frac{G}{r}$$

$K/K_c < 1$ Unstable, stick-slip

$K/Kc > 1$ Stable, aseismic slip



$$K_c = \frac{\sigma_n (b-a)}{D_c}$$

$$r_c = \frac{24}{7\pi}\frac{D_c G}{\sigma(b-a)}$$