Mechanics of Earthquakes and Faulting

Lecture 7, 17 Sep. 2015

www.geosc.psu.edu/Courses/Geosc508

- Effective stress and energy budget for shear, Dilatancy
- Friction, contact mechanics, hardness, base-level friction coefficient
- Instability, Stick-slip dynamics
- Basic friction theory
  - Amonton’s laws
  - Chemical effects
  - Hydrolytic weakening
- Basic observations of: time-dependent static friction
- velocity-dependent sliding friction
- Adhesive theory of friction, Hertian contact, ploughing

- Read Rabinowicz, 1951
- Read Chapter 2 of Scholz
Consider the implications of dilatancy and volume change for the total work (per unit volume) of shearing, $W$

$$W = \tau_p \, d\gamma + \sigma \, d\theta$$

$W$ is total work of shearing

$$W = \tau \, d\gamma = \sigma \mu \, d\gamma$$
Friction mechanics of 2-D particles

Data from Knuth and Marone, 2007

\[ W = \tau_p \, d\gamma + \sigma \, d\theta \]

\( W \) is total work of shearing

\[ W = \tau \, d\gamma = \sigma \mu \, d\gamma \]
Friction mechanics of 2-D particles

\[ \tau = \sigma \left( \mu_p + \frac{d\theta}{d\gamma} \right) \]

\[ W = \tau_p \, d\gamma + \sigma \, d\theta \]

\[ d\theta = \frac{dV}{V} ; \quad d\gamma = \frac{dx}{h} \]

Data from Knuth and Marone, 2007
Friction mechanics of 2-D particles

\[ \tau = \sigma \left( \mu_p + \frac{d\theta}{d\gamma} \right) \]

\[ \tau = \sigma \left( \mu_p + \frac{dh}{dx} \right) \]

\[ W = \tau_p \, d\gamma + \sigma \, d\theta \]

Dilatancy rate plays an important role in setting the frictional strength.
Macroscopic variations in friction are due to variations in dilatancy rate.

Smaller amplitude fluctuations in dilatancy rate produce smaller amplitude friction fluctuations.
Shear Localization

Strain homogeneity depends on whether dilatancy is restricted

- Homogeneous strain if dilatancy is not opposed
- Strain localization if deformed under finite confining pressure

Shear Bands Form if:

\[
\frac{\partial \theta_{SB}}{\partial \gamma} < \frac{\partial \theta_{homo.}}{\partial \gamma}
\]
Shear Localization

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- Homogeneous strain if dilatancy is not opposed
- Strain localization if deformed under finite confining pressure

Shear Bands Form if:

\[
\frac{\partial \theta^{SB}}{\partial \gamma} < \frac{\partial \theta^{homo.}}{\partial \gamma}
\]

Deformation mode (degree of strain localization) minimizes dilatancy rate

\[
W = \tau_p \, d\gamma + \sigma \, d\theta
\]

\[
\tau = \sigma \left(\mu_p + \frac{d\theta}{d\gamma}\right)
\]

Shear strength depends on friction and dilatancy rate
Mead, 1925
(Geologic Role of Dilatancy)

Shear Localization

Strain homogeneity depends on whether dilatancy is restricted

- Homogeneous strain if dilatancy is not opposed
- Strain localization if deformed under finite confining pressure

Shear Bands Form if:

$$\frac{\partial \theta_{SB}}{\partial \gamma} < \frac{\partial \theta_{homo.}}{\partial \gamma}$$

\[ W = \tau_p \, d\gamma + \sigma \, d\theta \]

\[ \tau = \sigma \left( \mu_p + \frac{d\theta}{d\gamma} \right) \]

Shear strength depends on friction and dilatancy rate

Deformation mode (degree of strain localization) minimizes dilatancy rate
Frank, 1965. Volumetric work, shear localization and stability. Applies to Friction and Fracture

\[ W = W_f + \sigma d\theta \]

\[ W = \tau d\gamma \]

\[ \tau = \tau_f + \sigma \frac{d\theta}{d\gamma} \]

Strain hardening when: \( \frac{d^2\theta}{d\gamma^2} > 0 \)

Strain softening when: \( \frac{d^2\theta}{d\gamma^2} < 0 \)
\[ W = W_f + \sigma d\theta \]
\[ W = \tau d\gamma \]
\[ \tau = \tau_f + \sigma \frac{d\theta}{d\gamma} \]

\[
\frac{d^2 \theta}{d\gamma^2} > 0 \quad \text{or} \quad \frac{d^2 \theta}{d\gamma^2} < 0
\]

Marone, Raleigh & Scholz, 1990
\[ W = W_f + \sigma d\theta \]
\[ W = \tau d\gamma \]
\[ \tau = \tau_f + \sigma \frac{d\theta}{d\gamma} \]

\[ \frac{d^2\theta}{d\gamma^2} > 0 \]
\[ \frac{d^2\theta}{d\gamma^2} < 0 \]

Marone, Raleigh & Scholz, 1990
Pop Quiz:

Derive the relations for shear and normal stress on a plane of arbitrary orientation in terms of principal stresses $\sigma_1$ and $\sigma_2$.

Hint, use the Mohr Circle.

$$\sigma(\alpha) =$$

$$\tau(\alpha) =$$
Shear and Normal Stress on a Plane of Arbitrary Orientation -- written in terms of Principal Stresses:

\[
\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha
\]

\[
\tau = \frac{\sigma_1 - \sigma_2}{2} \sin(2\alpha)
\]

Mohr Circle.

Length = \(\frac{(\sigma_1 - \sigma_2)}{2}\sin 2\alpha\)

Length = \(\frac{(\sigma_1 - \sigma_2)}{2}\cos 2\alpha\)
• Basic friction theory
• Amonton’s laws
• Chemical effects
• Hydrolytic weakening
• Basic observations of: time-dependent static friction
• velocity-dependent sliding friction
• Adhesive theory of friction
• Hertian contact
• ploughing
Base Friction vs. 2nd order variations

For metals: $\mu_0 \sim 1/3$

For rocks: $\mu_0 \sim 2/3$

(Frye and Marone, GRL 2002)
Amonton’s Laws (1699)  

(Both apply to base friction, $\mu_o$)

1st Friction force independent of the size of surface contact dimension $A$

$$\tau_o \neq \tau_o(A) \quad \mu_o \neq \mu_o(A)$$

2nd Friction force is proportional to normal load

$$\tau_o = \mu_o \sigma$$

Contact area $A$
Amonton’s Laws (1699)

Friction force is the same for objects small and large as long as is $\sigma \sim$ equal

$\mu_0 \sim 1/3$ regardless of surface or material for a wide range of metals and technological materials, excluding lubricated surfaces and modern polymers such as teflon

Why does it hold?

Friction is a contact problem. Therefore base friction is primarily a surface property and not a material property (we’ll have to relax this a bit when we talk about 2nd order variations in friction

Friction $\sim$ independent of surface roughness for low normal loads and unmated surfaces
Adhesive Theory of Friction

1st Friction force independent of the size of surface contact dimension $A$

Why does it hold?

Solution to Amonton’s Problem: Asperities and contact junctions

$A_r = \sum a_i$

$A_r \neq A$

Real area of contact $\sim 10\% \, A$ for unmated rough surfaces -- doesn’t apply for very light loads, mirror-smooth surfaces or lubricated surfaces

But we still have the problem of $A_r \propto \sigma$

Why is this a problem?
Adhesive Theory of Friction

But we still have the problem of $A_r \propto \sigma$
and $\mu_o \sim$ independent of material

Why is this a problem?

consider a hemispherical contact against a flat, under a shear load

(Bowden & Tabor, 1950)

Two assumptions:
1) Yielding at asperities is just sufficient to support normal load

$$\sigma = p A_r$$
where, $p$ is penetration hardness

2) Slip involves shearing of adhesive contacts and/or asperities

$$\tau = s A_r$$
where, $s$ is shear strength

combing these equations shows why $\mu_o \sim$ independent of material

$$\mu_o = \frac{\tau}{\sigma} = \frac{s A_r}{p A_r} = \frac{s}{p}$$
friction is the ratio of two material properties
Adhesive Theory of Friction

(Bowden & Tabor, 1950)

\[
\mu_0 = \frac{\tau}{\sigma} = \frac{s A_r}{p A_r} = \frac{s}{p}
\]

friction is the ratio of two material properties

Generally see that \( p \sim 3 \sigma_y \) compressive yield strength and \( s \sim \sigma_y / 2 \)
This gives \( \mu_0 = 1/6 \) --but recall that observation is that \( \mu_0 \sim 1/3 \).

--difference due to unaccounted effects, such as ploughing, wear and surface production, interlocking, dilational work, etc.

But we still have the problem of linearity between \( \tau_0 \) and \( \sigma \)

Hertzian contact predicts \( A_r \propto \sigma^{2/3} \)

but, this is dealt with by realistic descriptions of surface roughness: asperities have asperities on them.... Archard (1957), Greenwood and Williamson (1966)
Friction: Observations & Geophysical Experimental Studies

See Scholz Fig 2.5 for common experimental configurations

Rock Mechanics Lab Studies
- Experiments designed to investigate mechanisms and processes, not scale model experiments
- Application of friction/fracture studies to earthquakes/fault behavior
- Scaling problem.
  - Lab: cm-sized samples, Field: earthquake source dimensions 10’ s to 100’ s km
- Friction is scale invariant to 1st order (Amonton) --i.e. $\mu$ is a dimensionless constant. But will this extend to 2nd order characteristics of friction that control slip stability

Byerlee’s Law (Byerlee, 1967, 1978)

Base Friction is:
~ independent of rock type and normal stress
~ the same for bare, ground surfaces and gouge

\[
\tau = 0.85 \sigma_n \text{ for } \sigma_n < 200 \text{ MPa} \\
\tau = 50 + 0.6 \sigma_n \text{ for } \sigma_n > 200 \text{ MPa}
\]

This applies (only) to ground surfaces, primarily Westerly granite

For granular materials, powders, and fault gouge: \( \tau = 0.6 \sigma_n \)

Note that Byerlee’s law is just Coulomb Failure. It’s simply a statement about brittle (pressure sensitive) deformation and failure.
Byerlee’s Law (Byerlee, 1967, 1978)

\[ \tau = 0.85 \sigma_n \text{ for } \sigma_n < 200 \text{ MPa} \]
\[ \tau = 50 + 0.6 \sigma_n \text{ for } \sigma_n > 200 \text{ MPa} \]

For granular materials, powders, and fault gouge: \( \tau = 0.6 \sigma_n \)
Byerlee's Law for Rock Friction (Coulomb's Criterion)

MAXIMUM FRICTION

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>REFERENCE</th>
<th>ROCK TYPE</th>
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<tbody>
<tr>
<td>2F</td>
<td></td>
<td>Granite, fractured</td>
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<tr>
<td>2G</td>
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<td>Granite, ground surface</td>
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<tr>
<td>3</td>
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<td>5</td>
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<td>Granite, ground surface</td>
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<td></td>
<td>Weber Sandstone, faulted</td>
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<tr>
<td>6S</td>
<td></td>
<td>Weber Sandstone, saw cut</td>
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<tr>
<td>9</td>
<td></td>
<td>Granodiorite</td>
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<tr>
<td>13</td>
<td></td>
<td>Gneiss and Mylonite</td>
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<tr>
<td>16</td>
<td></td>
<td>Plaster in joint of Quartz Monzonite</td>
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<tr>
<td>20</td>
<td></td>
<td>Quartz Monzonite joints</td>
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<tr>
<td>25</td>
<td></td>
<td>Westerly Granite, Chlorite, Serpentinite, Illite, Kaolinite, Halloysite, Montmorillonite, Vermiculite</td>
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<tr>
<td>26</td>
<td></td>
<td>Granite</td>
</tr>
<tr>
<td>27</td>
<td></td>
<td>Kaolinite, Halloysite, Illite, Montmorillonite, Vermiculite</td>
</tr>
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</table>

\[ T = 0.5 + 0.6 \sigma_n \]

\[ \mu = 0.6 \]

Byerlee, 1978
Friction of Fault Zones

Penn State Lab, ~ 2000 samples

\[ \tau = 0.6 \sigma_n \]

Sliding Friction

Maximum Friction

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Friction: 2nd order variations, slick-slip and stability of sliding

Rabinowicz 1951, 1956, 1958

Static vs. dynamic friction & state dependence

\[ \mu = \mu_s \quad (s = 0) \]
\[ \mu = \mu_d \quad (s > 0) \]

Classical view

Rabinowicz recognized that finite slip was necessary to achieve fully dynamic slip

\[ \mu = \mu_s \quad (s < s_d) \]
\[ \mu = \mu_d \quad (s > s_d) \]

\( s_d \) is the critical slip distance

Rabinowicz experiments showed state, memory effects and that \( \mu_d \) varied with slip velocity.
Friction: 2nd order variations, slick-slip and stability of sliding

Rabinowicz’ s work solved a major problem with friction theory: he introduced a way to deal with the singularity in going from $\mu_s$ to $\mu_d$

$$\mu(x) = \mu_s - \frac{x}{L} \Delta \mu \quad \text{(for } L > x > 0)$$

$$\mu(x) = \mu_s - \Delta \mu \quad \text{(for } x > L)$$

Palmer and Rice, 1973; Ide, 1972; Rice, 1980

For solid surfaces in contact (without wear materials), the slip distance $L$ represents the slip necessary to break down adhesive contact junctions formed during ‘static’ contact.

The slip weakening distance is also known as the critical slip or the breakdown slip.

This slip distance helps with the stress singularity at propagating crack tips, because the stress concentration is smeared out over the region with slip $< L$. 
Friction: 2nd order variations, slick-slip and stability of sliding

**Slip Weakening Friction Law**

\[ \mu(x) = \mu_s - \frac{x}{L} \Delta \mu \quad (\text{for } L > x > 0) \]

\[ \mu(x) = \mu_s - \Delta \mu \quad (\text{for } x > L) \]

Critical friction distance represents slip necessary to erase existing contact.

Adhesive Theory of Friction

For a surface with a distribution of contact junction sizes, \( L \), will be proportional to the average contact dimension.

Critical friction distance scales with surface roughness.
Friction

Base-level friction coefficient in terms of contact mechanics and hardness
Time dependent yield strength:  

$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$

Time dependent growth of contact (acrylic plastic)- true static contact
Friction

Base-level friction coefficient in terms of contact mechanics and hardness

Adhesive Theory of Friction (Bowden and Tabor)
- Real contact area << nominal area
- Contact junctions at inelastic (plastic) yield strength
- Contacts grow with “age”
- Add: Rabinowicz’s observations of static/dynamic friction
- “Static” friction is higher than “Dynamic” friction because contacts are older (larger)
- \( \rightarrow \) implies that contact size decreases as velocity increases
Friction

Base-level friction coefficient in terms of contact mechanics and hardness

Adhesive Theory of Friction (Bowden and Tabor)

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- -> implies that contact size decreases as velocity increases
Classic theory of friction

Bowden and Tabor [1960]

\[ \tau = \frac{SA_c}{A_T} \]

\[ \mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y} \]

Friction is the ratio of shear strength to hardness

This is base level friction

Modified from Beeler, 2003
base level friction (~ 0.6 for rocks)

Friction ($\mu = \tau / \sigma_n$)

Shear Displacement (mm)

Karner & Marone (GRL 1998, JGR 2001)
Time dependent yield strength:

\[ \mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y} \]

Dieterich and Kilgore [1994]

Time dependent growth of contact (acrylic plastic)- true static contact

\[ \sigma_y = \sigma_o + f(t) \]

Modified from Beeler, 2003
Other measures of changes in ‘static’ friction, contact area, or strength

\[ \Delta \mu_{\text{peak}} \]

‘hold’ test

\[ \mu_s = 1 \mu m/s \]
\[ V_L = 0 \]
\[ V_r = 1 \mu m/s \]

Modified from Beeler, 2003

after Dieterich [1972]

Time dependent closure (westerly granite)
- approximately static contact
compaction/dilatancy associated with changes in sliding velocity

\[ \sigma_y = \sigma_o + f(\text{age}) \]

after Marone and Kilgore [1993]

Modified from Beeler, 2003
Rate dependence of contact shear strength

\[ \mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y} \]

\[ S = S_o + g(V) \]

Modified from Beeler, 2003
Summary of friction observations:
0. Friction is to first order a constant
1. Time dependent increase in contact area (strengthening)
2. Slip dependent decrease in contact area (weakening); equivalently increase in dilatancy
3. Slip rate dependent increase in shear resistance (non-linear viscous)

Modified classic theory of friction:

\[ \mu = \frac{S}{\sigma_y} = \frac{S_o + g(V)}{\sigma_o + f(age)} \]

\[ \mu = \frac{S_o + g(V)}{\sigma_o + f(age)} \left[ \frac{\sigma_o - f(age)}{\sigma_o - f(age)} \right] \]

Discard products of second order terms:

\[ \mu = \frac{S_o + g(V)}{\sigma_o} - \frac{S_o f(age)}{\sigma_o^2} \]

[e.g., Dieterich, 1978, 1979]

Modified from Beeler, 2003
Summary of friction observations:

0. Friction is to first order a constant
1. Time dependent increase in contact area (strengthening)
2. Slip dependent decrease in contact area (weakening); equivalently increase in dilatancy
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\[
\mu = \frac{S_o}{\sigma_o} + \frac{g(V)}{\sigma_o} - \frac{S_o f(\text{age})}{\sigma_o^2}
\]

Rate and state equations:

\[
\mu = \mu_0 + a \ln \frac{V}{V_0} + b \ln \frac{V_0 \theta}{D_c}
\]

\[
\frac{d\theta}{dt} = 1 - \frac{\theta V}{D_c}
\]

1st order term  second order terms

rate dependence    slip dependence

0. 1. & 2. 3.

Dieterich [1979]
Rice [1983]
Ruina [1983]

Modified from Beeler, 2003
Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear

\[ \Delta \sigma = \frac{16}{7\pi} \mu \frac{\Delta u}{r} \]

\[ \Delta \sigma = \frac{24}{7\pi} \mu \frac{\Delta u_{\text{max}}}{r} \]

Why is this a reasonable approach?

How do we get at stiffness?
Elastic strain accumulates during the interseismic period and is released during an earthquake. The elastic strain causes the earthquake—in the sense that the elastic energy stored around the fault drives earthquake rupture.

There are three basic stages in Reid’s hypothesis.

1) Stress accumulation (e.g., due to plate tectonic motion—but what about intra-plate earthquakes?)
2) Stress reaches or exceeds the (frictional) failure strength
3) Failure, seismic energy release (elastic waves), and fault rupture propagation

Reid’s Hypothesis of Elastic Rebound

Reid’s Hypothesis of Elastic Rebound
Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear

Why is this a reasonable approach?

\[ \Delta \sigma = C \mu \frac{\Delta \bar{u}}{r} \]

Relation between stress and slip on a dislocation of radius \( r \). Therefore, the local stiffness around the slip patch is:

\[ K = \frac{\Delta \sigma}{\Delta \bar{u}} = C \frac{\mu}{r} \]

That is, stiffness decreases as the patch enlarges.

How do we get at stiffness?
Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear

Frictional stability is determined by the combination of

1) fault zone frictional properties and
2) elastic properties of the surrounding material
Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear

Stick-slip dynamics

\[ m \ddot{x} + \Gamma \dot{x} + f(x', x', t, \theta) = F_s \]

\[ m \ddot{x} + \Gamma \dot{x} + f(x', x't, \theta) = K(v_{lp} - v)t \]

\[ m \ddot{x} + F \dot{x}' = K(v_{lp} - v)t \]

Static-Dynamic Friction

\[ f = \Delta \mu N \]
Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear

Stick-slip dynamics

\[
mx'' + \Gamma \dot{x}' + f(x', x', t, \theta) = F_s
\]

\[
m\ddot{x}' + \Gamma \dot{x}' + f(x', x't, \theta) = K(v_{lp} - v)t
\]

\[
m\ddot{x}' + f(x') = K(v_{lp} - v)t
\]

\[
m\ddot{x}' + Kx' = \Delta \mu N
\]

\[
x'(t) = \frac{\Delta \mu N}{K} (1 - \cos \kappa t)
\]

\[
v(t) = \frac{\Delta \mu N}{\sqrt{Km}} \sin \kappa t
\]

\[
\kappa = \sqrt{\frac{K}{m}}
\]

\[
t_r = \pi \sqrt{\frac{m}{K}}
\]

slip duration = rise time
Brittle Friction Mechanics, Stick-slip

- Stick-slip (unstable) versus stable shear

\[ m\ddot{x}' + Kx' = \Delta\mu N \]

\[ x'(t) = \frac{\Delta\mu N}{K} (1 - \cos\kappa t) \]

\[ v(t) = \frac{\Delta\mu N}{\sqrt{Km}} \sin\kappa t \]

\[ \kappa = \sqrt{\frac{K}{m}} \]

\[ t_r = \pi \sqrt{\frac{m}{K}} \]

- Slip duration = rise time

- Total slip, particle velocity, and accel. all depend on friction drop (stress drop)

\[ \Delta x' = \frac{2\Delta\mu N}{K} \]

\[ \Delta\sigma = 2(\mu_s - \mu_d)\sigma_n \]
Laboratory Studies

Plausible Mechanisms for Instability

Slip Weakening Friction Law

\[ \mu_i \neq \mu_i^{(v)} \]

Quasistatic Stability Criterion

\[ K_c = \frac{\sigma_n (\mu_s - \mu_d)}{L} \]

- \( K < K_c \): Unstable, stick-slip
- \( K > K_c \): Stable sliding
But, there’s a problem......
Duality of time and displacement dependence of friction.

“Static” and “dynamic” friction are just special cases of a more general behavior called “rate and state friction”

Stick-slip stress-drop amplitude varies with loading rate.

Mair, Frye and Marone, JGR 2002