Mechanics of Earthquakes and Faulting

Lecture 6, 11 Feb 2021

www.geosc.psu.edu/Courses/Geosc508

- Importance of volume change and dilatancy rate (rate of volume strain with shear strain)
- some basic elasticity
- Friction theory
- Amonton's laws
- Chemical effects
- Hydrolytic weakening
- Basic observations of: time-dependent static friction
- velocity-dependent sliding friction
- Adhesive theory of friction
- Hertian contact
- ploughing

- Read *Rabinowicz*, 1951 & 1956 (we will discuss these next week on Feb 11)
- Read Chapter 2 of Scholz (and look ahead at other chapters)

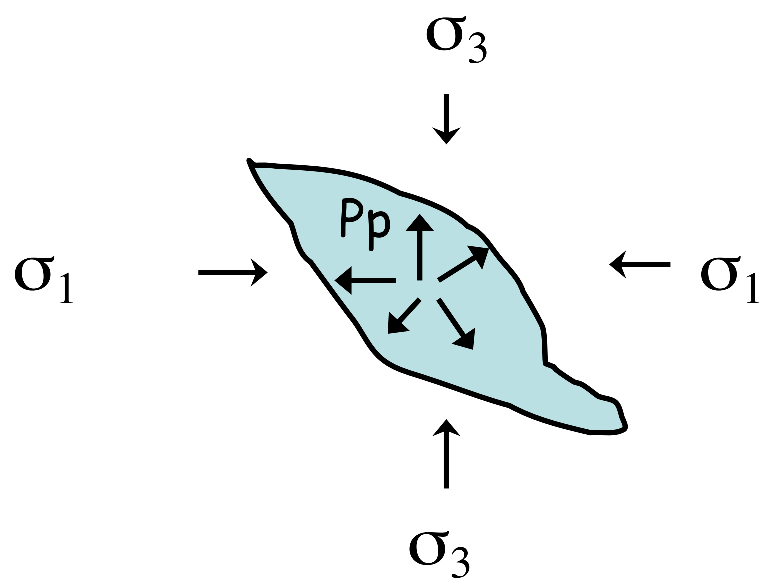
Fluids: Consider the affects on shear strength

- Mechanical Effects
- Chemical Effects

Mechanical Effects: Effective Stress Law

$\sigma_{effective} = \sigma_n - Pp$

$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$



$$\delta_{ij} = 1; i = j$$
$$\delta_{ij} = 0; i \neq j$$

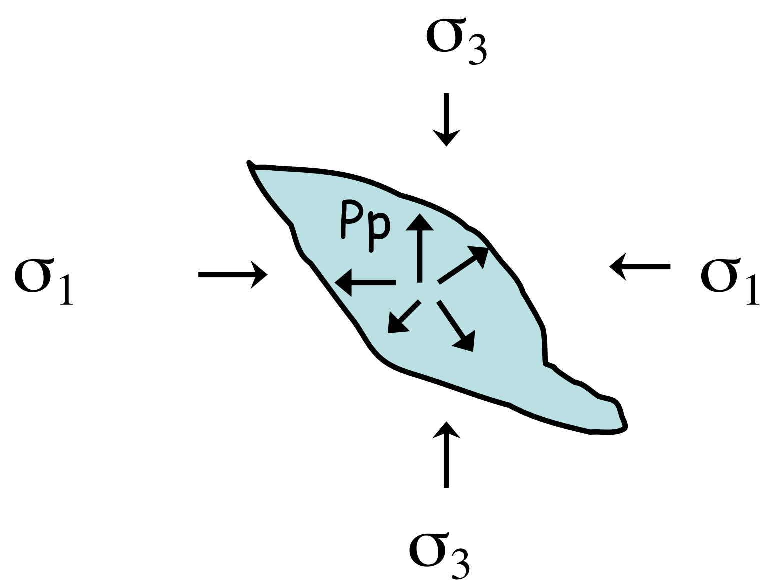
Leopold Kronecker (1823–1891)

Rock properties depend on effective stress: Strength, porosity, permeability, Vp, Vs, etc.

$$\sigma'_{ij} = \sigma_{ij} - P_p\delta_{ij}$$

$\sigma_{effective} = \sigma_n - Pp$

$$\delta_{ij} = 1; i = j$$
$$\delta_{ij} = 0; i \neq j$$



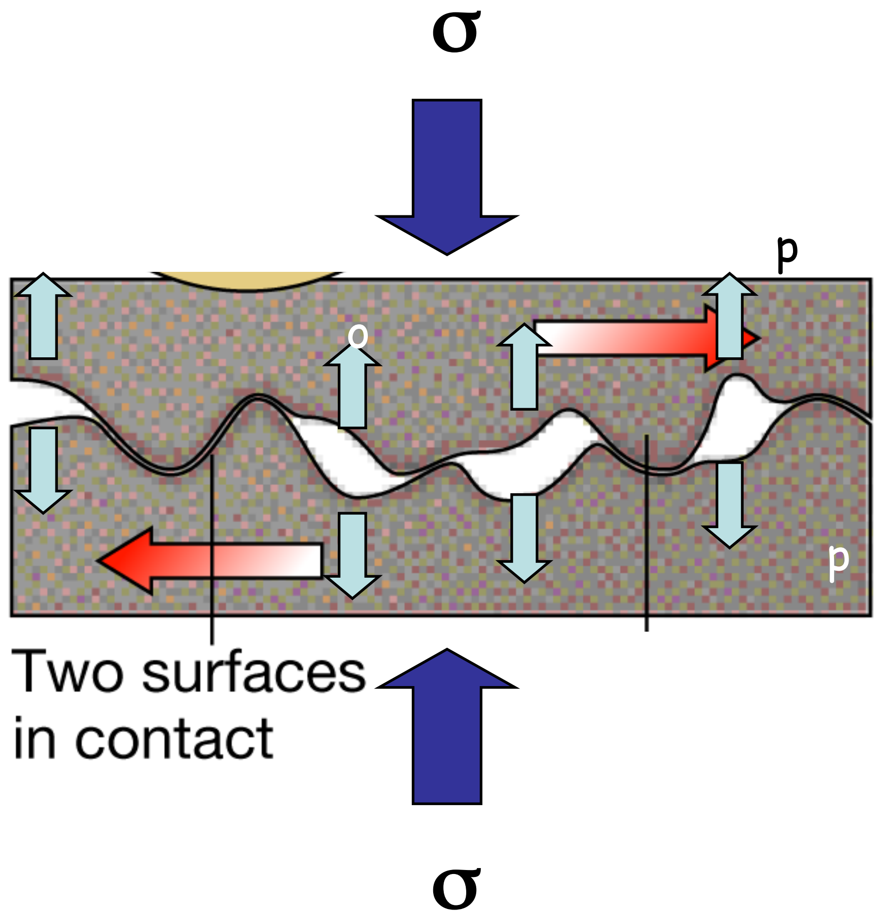
Exercise: Follow through the implications of Kronecker's delta to see that pore pressure only influences normal stresses and not shear stresses. Hint: see the equations for stress transformation that led to Mohr's circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

Void space filled with a fluid at pressure Pp

But what if $Ar \neq A$?



Fluids play a role by opposing the normal stress

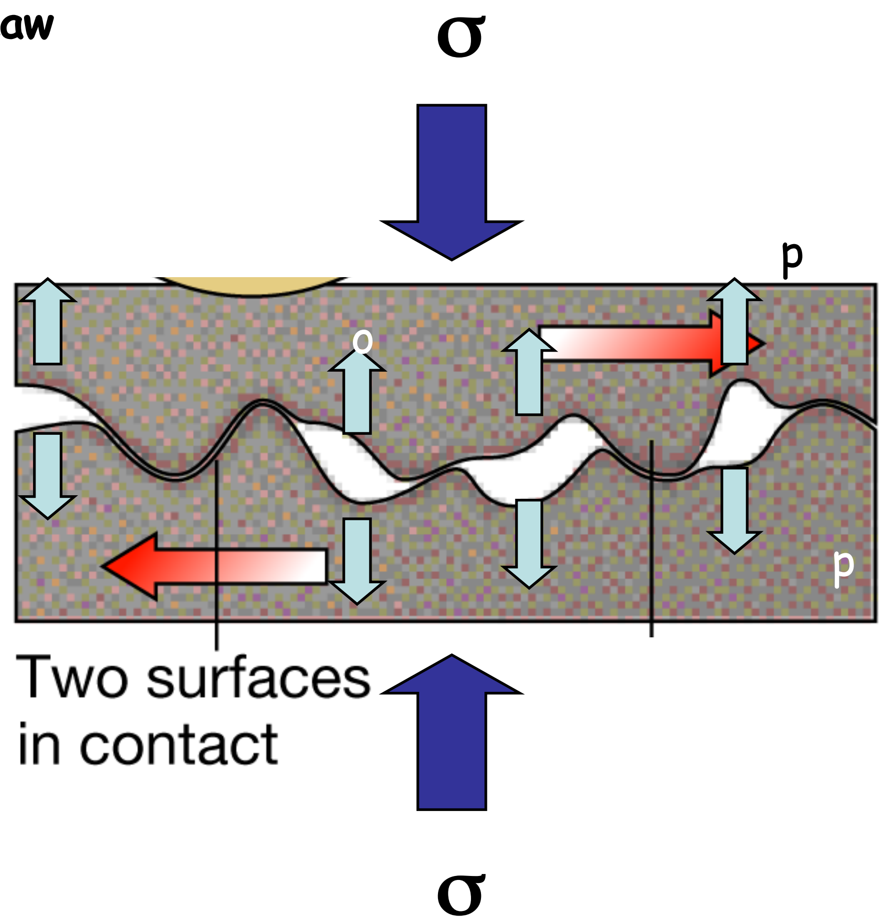
Mechanical Effects: Effective Stress Law

$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$

For brittle conditions,
Ar / A ~ 0.1

$$\sigma'_{ij} = \sigma_{ij} - \alpha \, P_p \delta_{ij}$$

$$\alpha = (1 - \frac{A_r}{A})$$

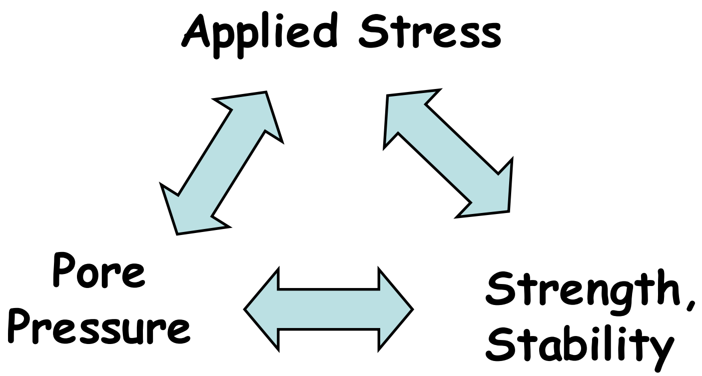


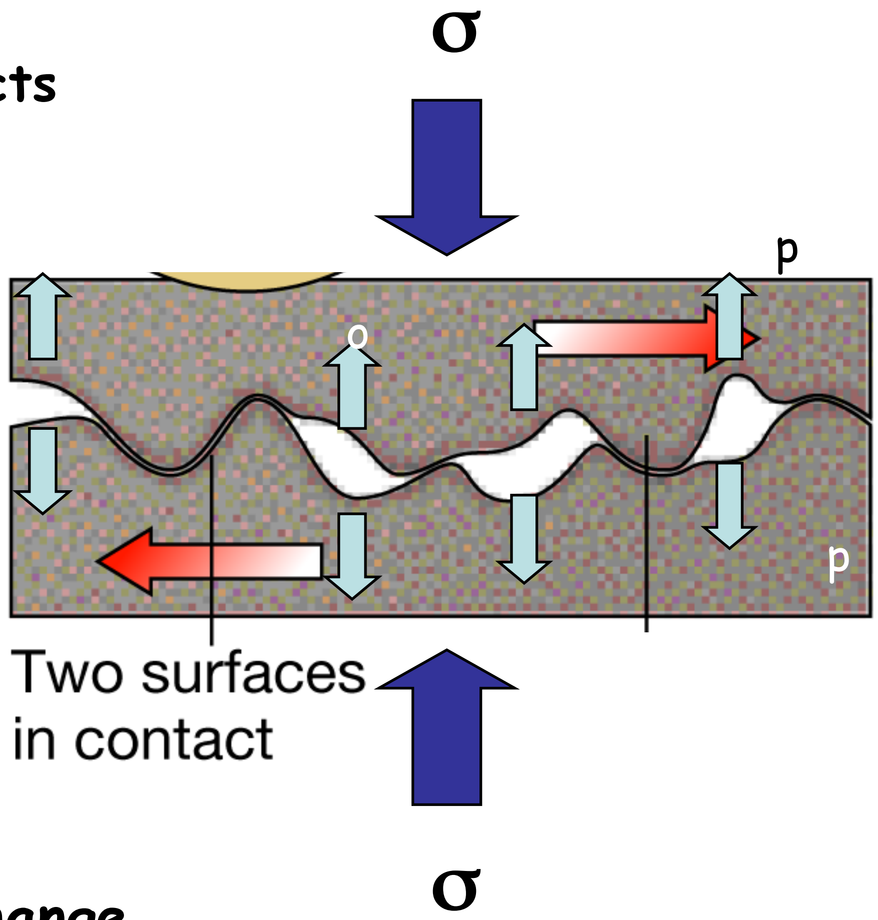
$$\tau = C + \mu_i \sigma'$$

Exercise: Consider how a change in applied stress would differ from a change in Pp in terms of its effect on Coulomb shear strength. Take α = 0.9

Effective Stress Law

Coupled Effects



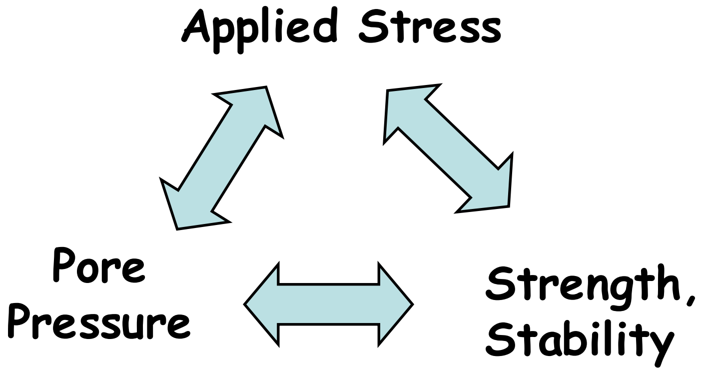


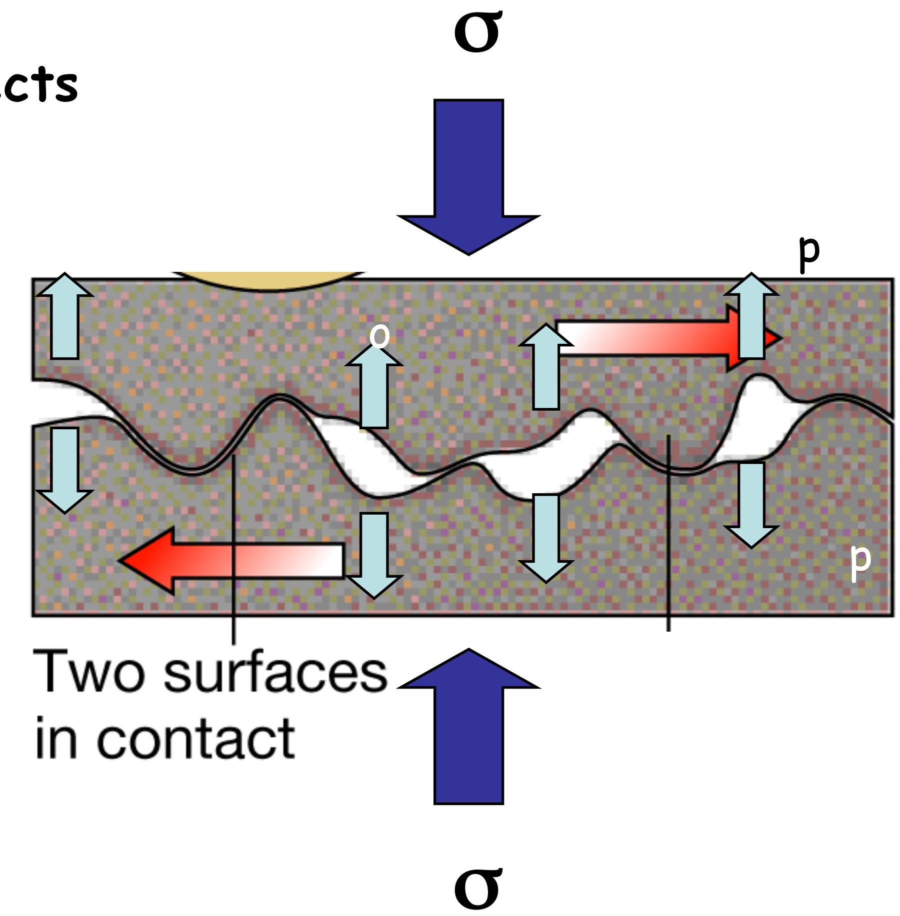
Dilatancy: Shear driven volume change

Exercise: Make the dilatancy demo described by Mead (1925) on pages 687-688. You can use a balllon, but a plastic bottle with a tube works better. Bring to class to show us. Feel free to work in groups of two.

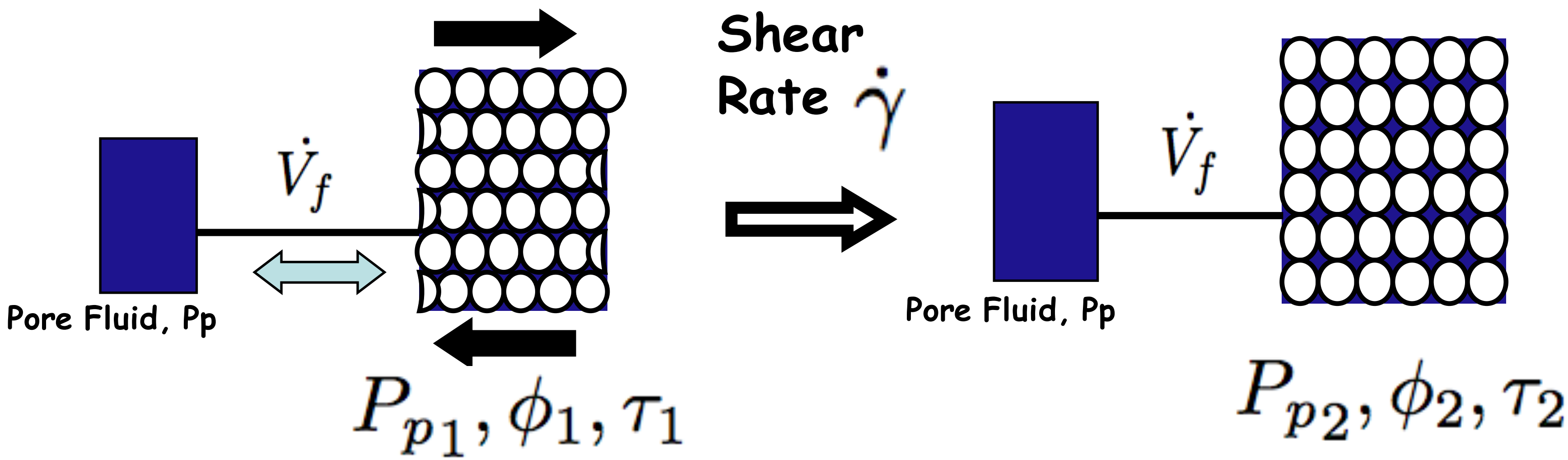
Effective Stress Law

Coupled Effects





Dilatancy

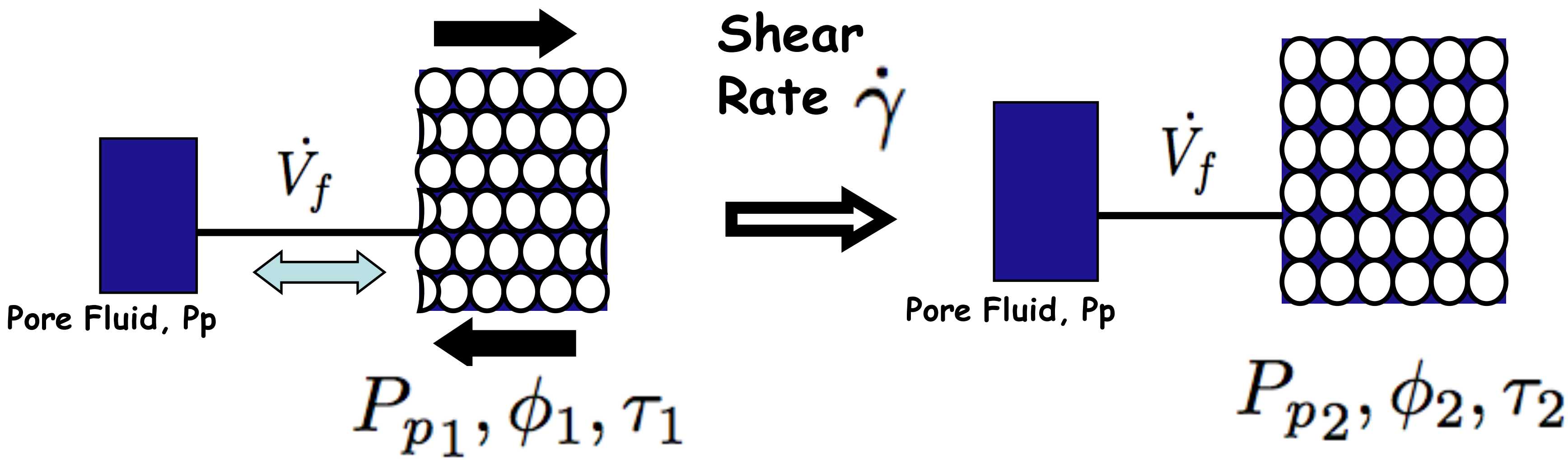


Dilatancy: $\phi_1 \neq \phi_2$

Volumetric Strain: $$d\theta = \frac{(V_2 - V_1)}{V}$$

Assume no change in solid volume

Dilatancy Rate: $$\beta = \frac{d\theta}{d\gamma}$$



Dilatancy: $\phi_1 \neq \phi_2;$ $P_{p1} \neq P_{p2}$ Undrained loading

Volumetric Strain:

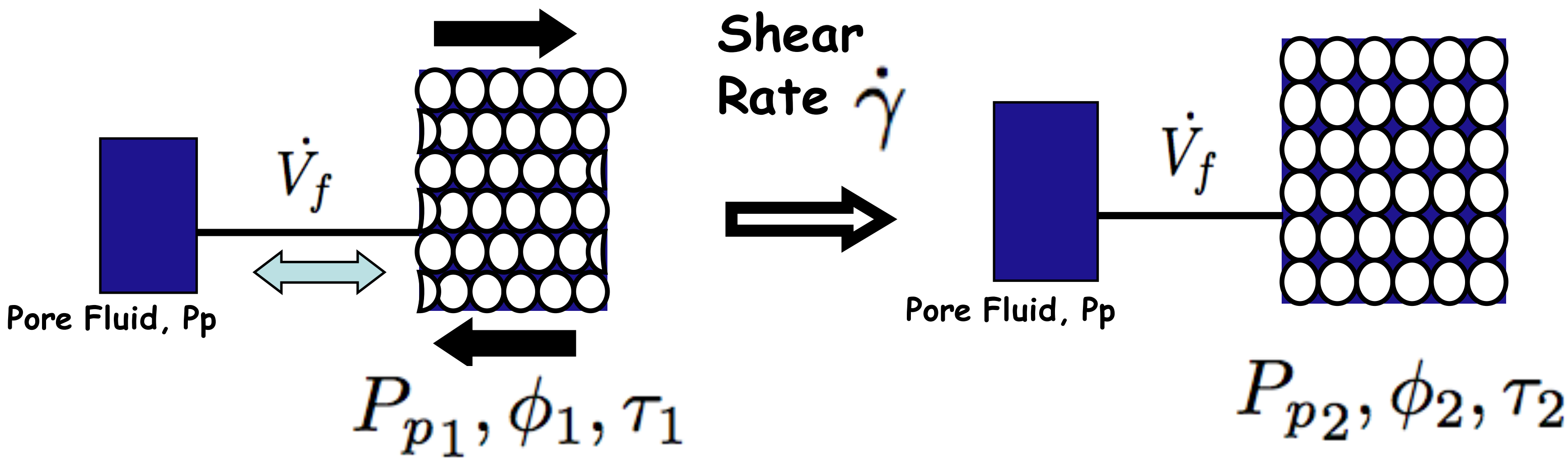
$$d\theta = \frac{(V_2 - V_1)}{V}$$

Assume no change in solid volume

Dilatancy Rate:

$$\beta = \frac{d\theta}{d\gamma}$$

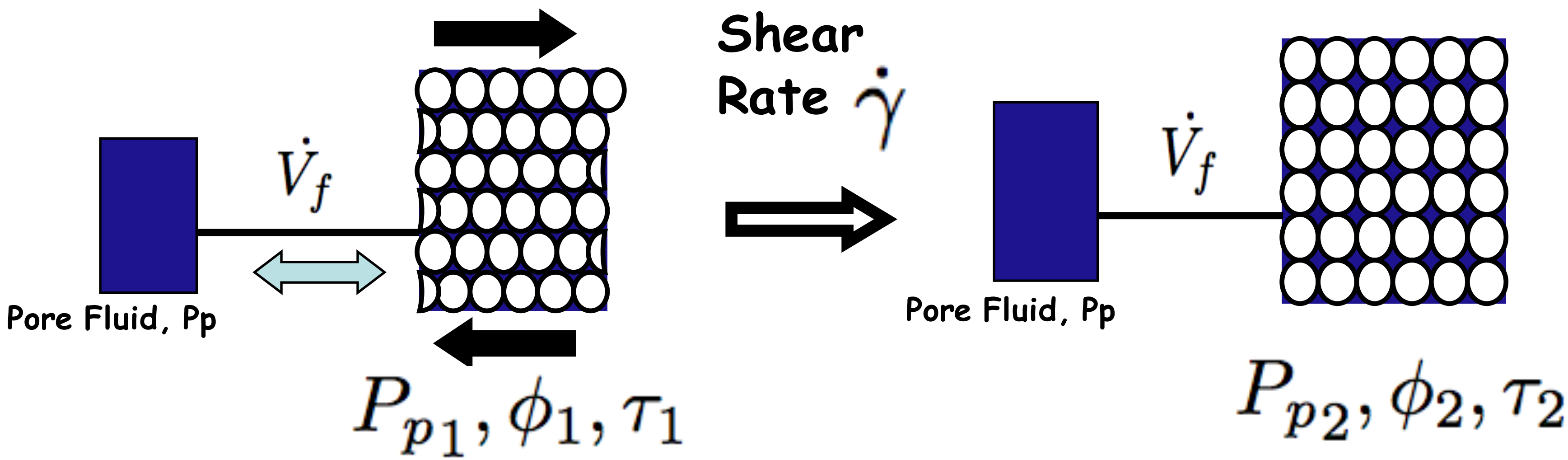
Dilatancy Hardening if $\dot{d\theta} > \dot{V}_f$ or $\dot{V}_f < \dot{\beta}$



Dilatancy Hardening if : $\dot{V}_f < \dot{\beta}$

$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$

$$\tau = C + \mu_i \sigma'$$

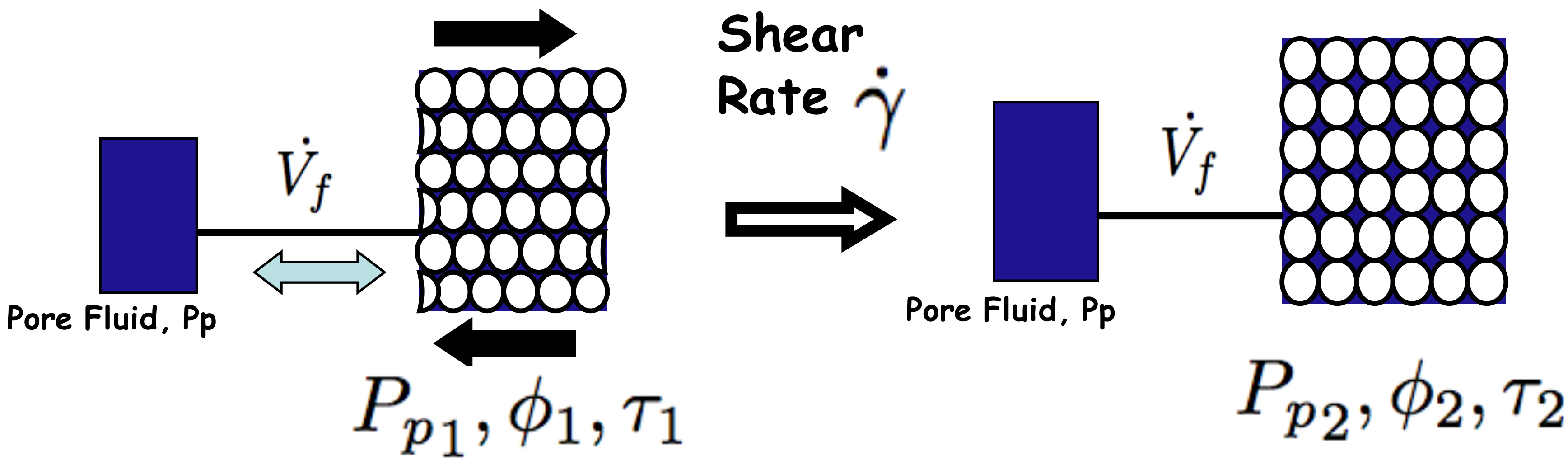


Dilatancy Weakening can occur if: $d\theta < 0 \; and \; |d\dot{\theta}| > \dot{V}_f$

This is shear driven compaction

$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$

$$\tau = C + \mu_i \sigma'$$



• Elasticity:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\theta$$

where λ and μ are Lame's constants, δ is Kronecher's delta ($\delta_{ij} = 1$ for i = j and $\delta_{ij} = 0$ for i≠j) and θ is the volumetric strain.

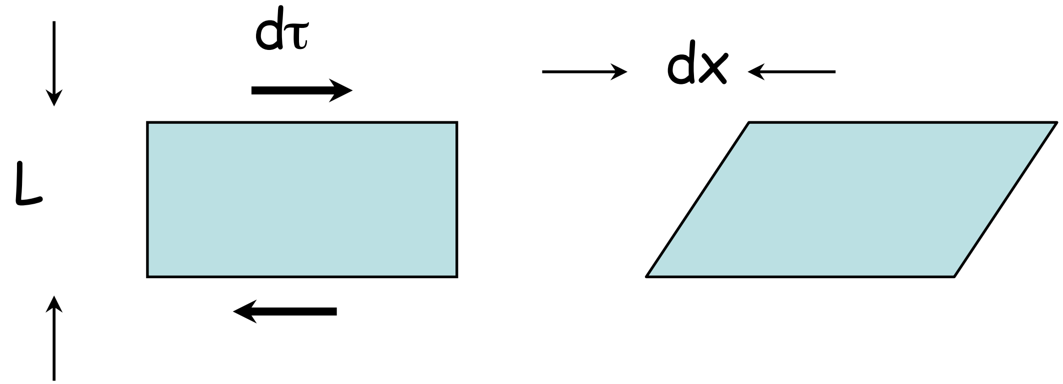
Lame's constants are (can be related to) Elastic moduli

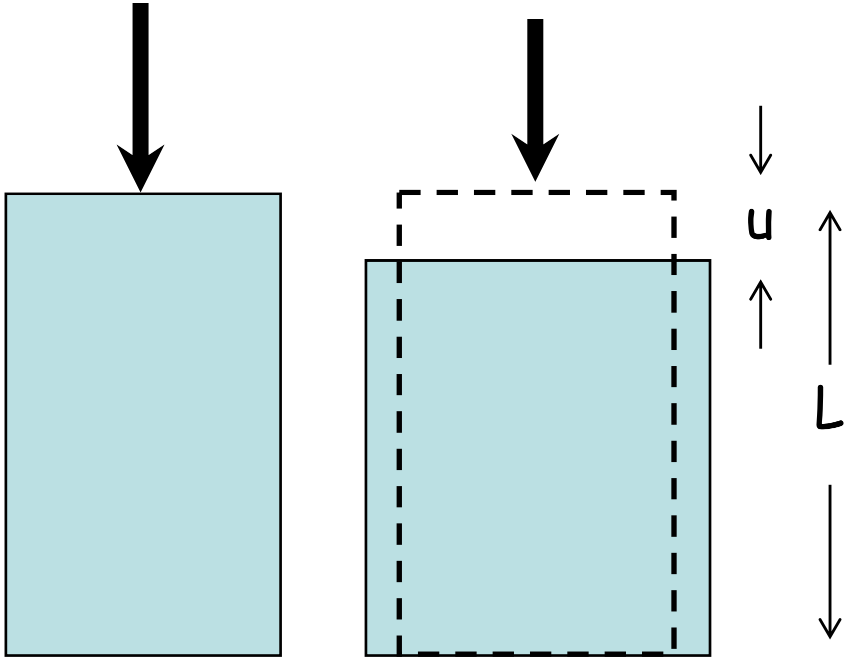
Shear Modulus, Bulk Modulus, Young's Modulus

$$\mu = \frac{E}{2(1 + \upsilon)} = G$$

The shear modulus, *G* or μ, is the shear deformation (normalized by the initial length) for a given change in shear stress

G = dτ/(dx/L) = dτ/dγ





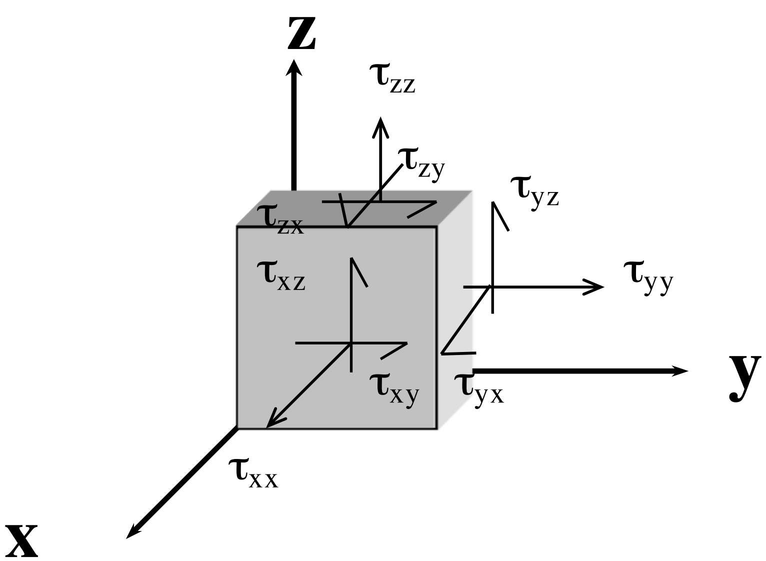
ε = u/L, linear strain

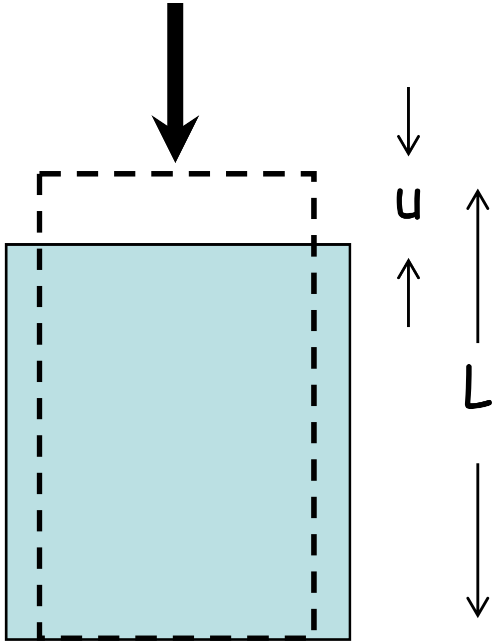
σ = E ε, where **E is Young's Modulus.**

Note that Modulus has units of stress (Pa)

Young's Modulus is important in many problems.

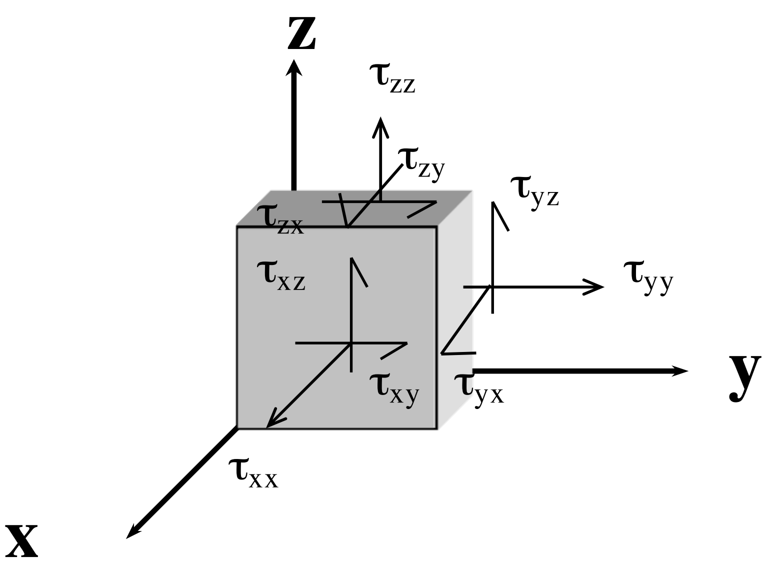
- Think of it as a generalized (i.e., complex) spring constant
- As in Hooke's law, which relates force and displacement through a spring constant, the modulus relates stress and strain.





There are nine components of the strain tensor

$$\varepsilon_{xx}, \varepsilon_{xy}, \varepsilon_{xz}$$
$$\varepsilon_{yx}, \varepsilon_{yy}, \varepsilon_{yz}$$
$$\varepsilon_{zx}, \varepsilon_{zy}, \varepsilon_{zz}$$



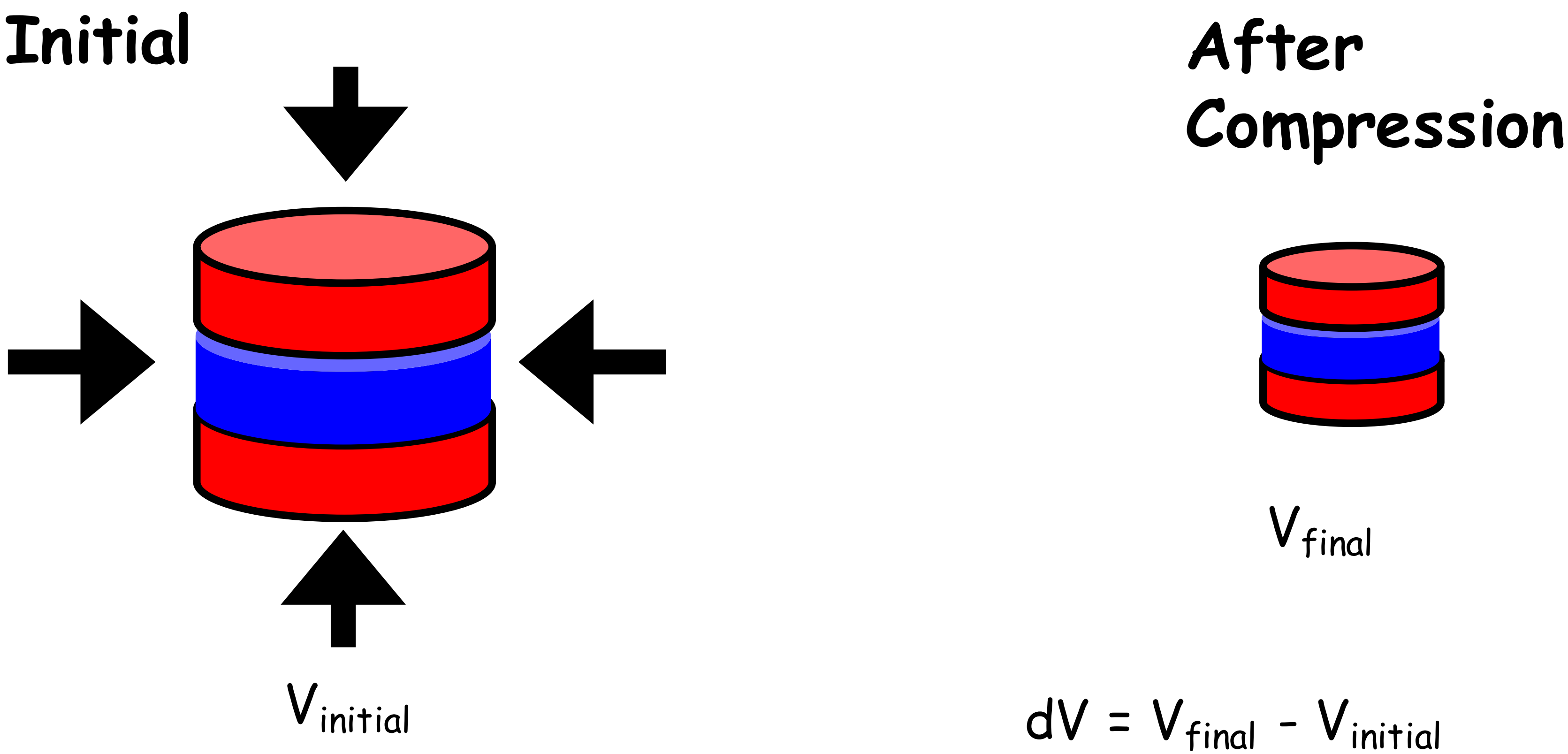
Poisson's ratio ν is an elastic parameter that describes the lateral expansion due to an axial deformation:

$$\nu = -\varepsilon_{zz}/\varepsilon_{xx}$$

FYI: Poisson's ratio is 0.5 for water and 0.25 for a typical granite.

$$K = \lambda + \frac{2}{3}\mu$$

K is the bulk modulus. The bulk modulus is the change in volume (normalized by the initial volume) for a given change in hydrostatic pressure: $K = dP/(dv/V_i) = dP/d\theta$



$dV = V_{final} - V_{initial}$

- Elasticity:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\theta$$

where λ and μ are Lame's constants, δ is Kronecher's delta (δ_{ij} = 1 for i = j and δ_{ij} = 0 for $i \neq j$) and θ is the volumetric strain.

Lame's constants are (can be related to) Elastic moduli

$$\lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)}$$

λ can be related to E and υ

$$\mu = \frac{E}{2(1+\upsilon)} = G$$

The shear modulus, G or μ, is the shear deformation (normalized by the initial length) for a given change in shear stress
$G = d\tau/(dx/L) = d\tau/d\gamma$

$$K = \lambda + \frac{2}{3}\mu$$

K is the bulk modulus. The bulk modulus is the change in volume (normalized by the initial volume) for a given change in hydrostatic pressure: $K = dP/(dv/V) = dP/d\theta$

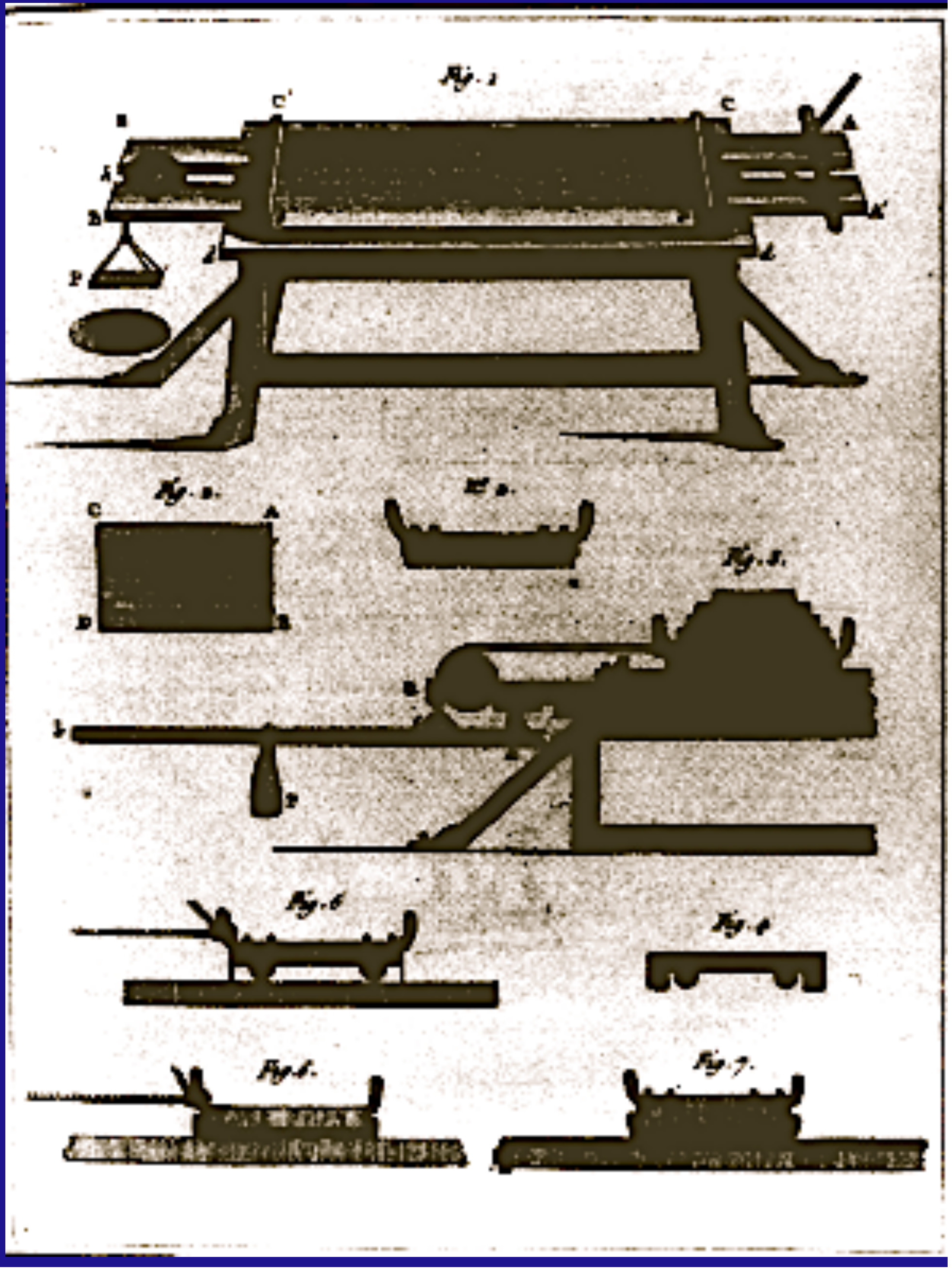
Friction

Galileo
Amonton
Coulomb
Others

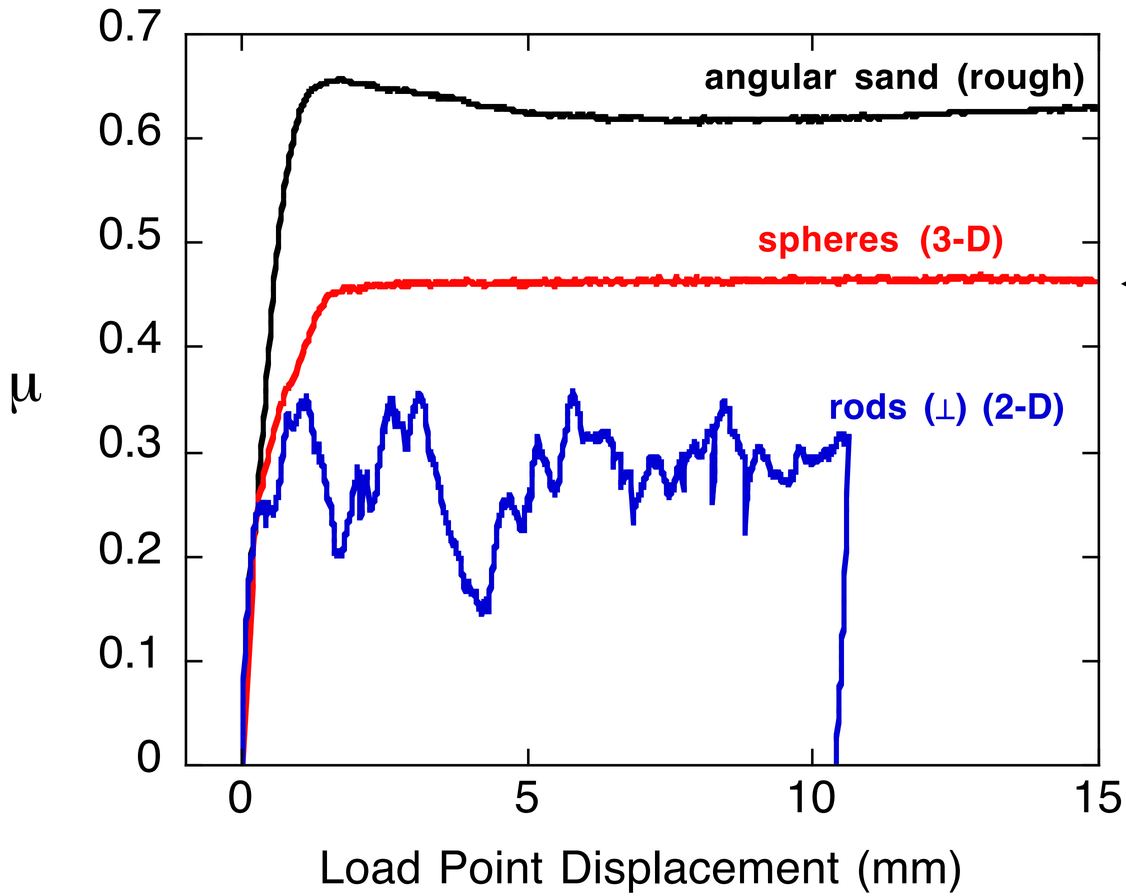
These are generally called *laws but they are not Laws. They are or historical interest*

- Amontons' First Law: The force of friction is independent of the apparent area of contact.
- Amontons' Second Law:
- The force of friction is directly proportional to the applied load.

- Coulomb's Law of Friction: Kinetic friction is independent of the sliding velocity.



Base Friction vs. 2nd order variations



Base Friction, μ_o

μ_o

For metals: μ_o ~ 1/3

For rocks: μ_o ~ 2/3

(Frye and Marone, GRL 2002)

Amonton's Laws (1699)

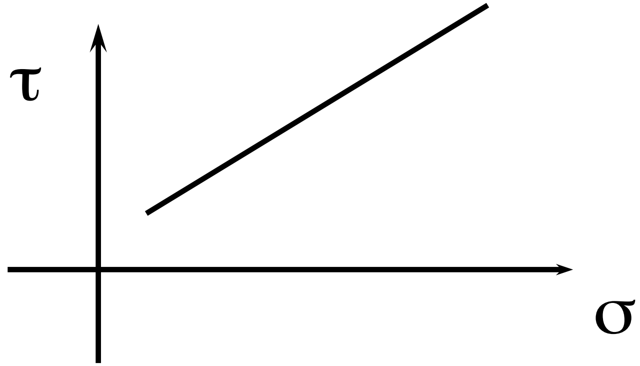
(Both apply to base friction, μ_o)

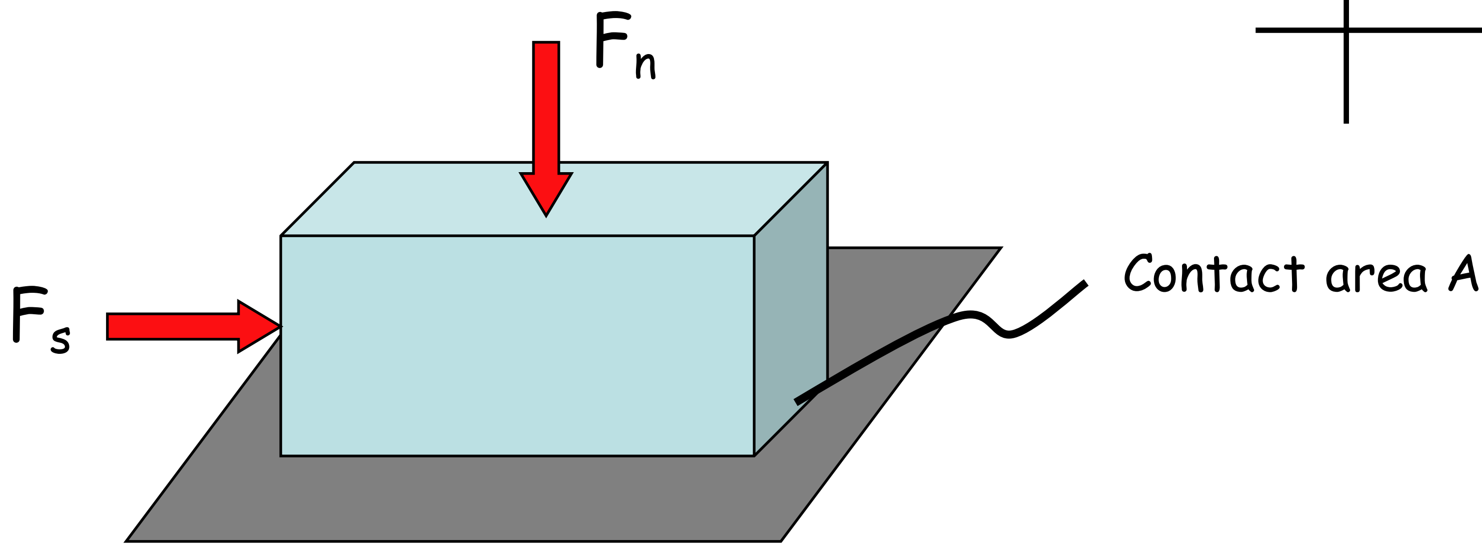
1st Friction force independent of the size of surface contact dimension A

$$\tau_o \neq \tau_o(A) \qquad\qquad \mu_o \neq \mu_o(A)$$

2nd Friction force is proportional to normal load

$$\tau_o = \mu_o \sigma$$





Amonton's Laws (1699)

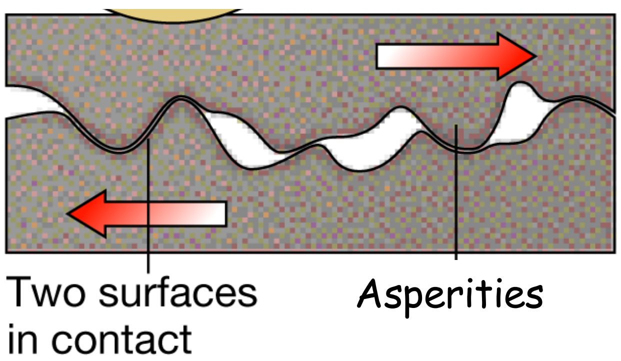
Friction force is the same for objects small and large as long as is σ ~ equal

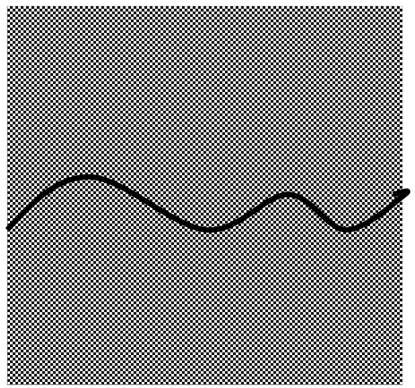
μ_o ~ 1/3 regardless of surface or material for a wide range of metals and technological materials, excluding lubricated surfaces and modern polymers such as teflon

Why does it hold?

Friction is a contact problem. Therefore base friction is primarily a surface property and not a material property (we' ll have to relax this a bit when we talk about 2nd order variations in friction

Friction ~ independent of surface roughness for low normal loads and unmated surfaces





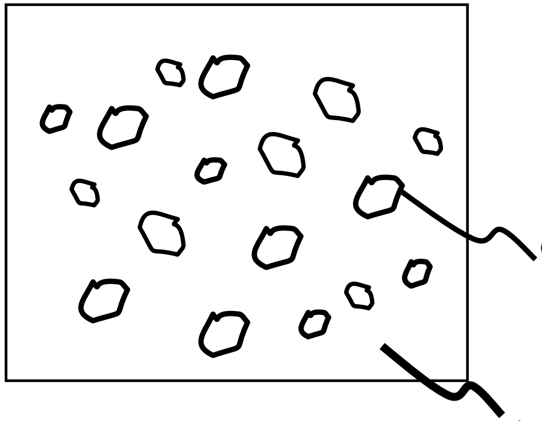
mated joint

Adhesive Theory of Friction

1st Friction force independent of the size of surface contact dimension A

Why does it hold?

Solution to Amonton's Problem: Asperities and contact junctions

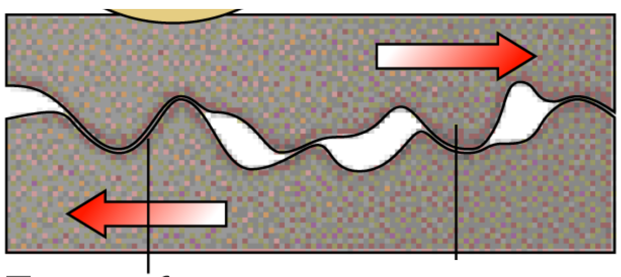


a_i

$$A_r = \Sigma a_i$$

$$A_r \neq A$$

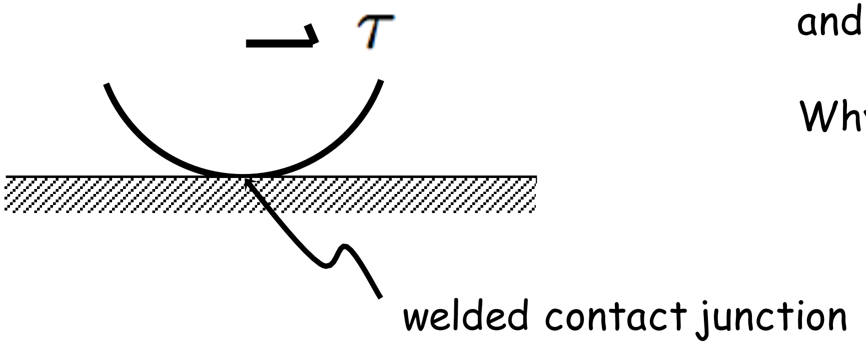
Real area of contact ~ 10% A for unmated rough surfaces --doesn't apply for very light loads, mirror-smooth surfaces or lubricated surfaces



But we still have the problem of $A_r \; \alpha \; \sigma$
and μ_o ~ independent of material

Why is this a problem?

Adhesive Theory of Friction



But we still have the problem of $A_r \; \alpha \; \sigma$
and μ_o ~ independent of material

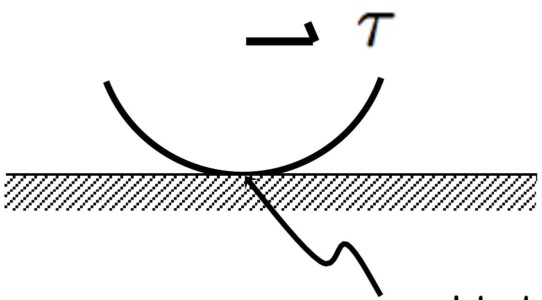
Why is this a problem?

consider a hemispherical contact against a flat, under a shear load

Hertzian contact predicts $A_r \; \alpha \; \sigma^{\frac{2}{3}}$

ah, hmmm, but what about Coulomb or Amonton? They said that 'friction force' scales linearly with normal stress $\tau = \tau_o + \mu \sigma$

Adhesive Theory of Friction



But we still have the problem of $A_r \; \alpha \; \sigma$
and μ_o ~ independent of material

Why is this a problem?

consider a hemispherical contact against a flat, under a shear load

(Bowden & Tabor, 1950)

Two assumptions:

1) Yielding at asperities is just sufficient to support normal load

$$\sigma = p \; A_r$$ where, p is penetration hardness

2) Slip involves shearing of adhesive contacts and/or asperities

$$\tau = s \; A_r$$ where, s is shear strength

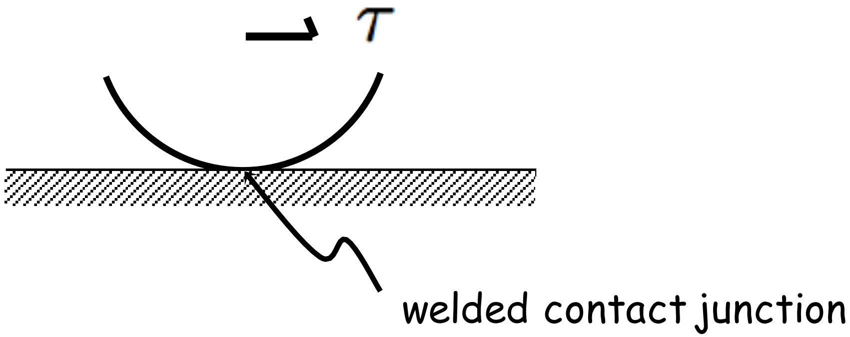
combing these equations shows why μ_o ~ independent of material

$$\mu_o = \frac{\tau}{\sigma} = \frac{sA_r}{pA_r} = \frac{s}{p}$$

friction is the ratio of two material properties

Adhesive Theory of Friction

(Bowden & Tabor, 1950)



$$\mu_o = \frac{\tau}{\sigma} = \frac{sA_r}{pA_r} = \frac{s}{p}$$

friction is the ratio of two material properties

Generally see that p ~ 3 σ_y compressive yield strength and s ~ σ_y /2
This gives μ_o = 1/6 --but recall that observation is that μ_o ~ 1/3.

--difference due to unaccounted effects, such as ploughing, **wear** and surface production, interlocking, dilational work, etc.

But we still have the problem of linearity between τ_o and σ

Hertzian contact predicts $A_r \ \alpha \ \sigma^{\frac{2}{3}}$

but, this is dealt with by realistic descriptions of surface roughness: asperities have asperities on them…. Archard (1957), Greenwood and Williamson (1966)

Friction: Observations & Geophysical Experimental Studies

See Scholz Fig 2.5 for common experimental configurations

Rock Mechanics Lab Studies

- Experiments designed to investigate mechanisms and processes, not scale model experiments
- Application of friction/fracture studies to earthquakes/fault behavior
- Scaling problem.
  Lab: cm-sized samples, Field: earthquake source dimensions 10's to 100's km
- Friction is scale invariant to 1st order (Amonton) --i.e. μ is a dimensionless constant. But will this extend to 2nd order characteristics of friction that control slip stability

Byerlee's Law (Byerlee, 1967, 1978)

Base Friction is:
~ independent of rock type and normal stress
~ the same for bare, *ground* surfaces and gouge

$\tau = 0.85\ \sigma_n$ for $\sigma_n < 200$ MPa
$\tau = 50 + 0.6\ \sigma_n$ for $\sigma_n > 200$ MPa

This applies (only) to ground surfaces, primarily Westerly granite

For granular materials, powders, and fault gouge: $\tau = 0.6\ \sigma_n$

Note that Byerlee's law is just Coulomb Failure. It's simply a statement about brittle (pressure sensitive) deformation and failure.

Byerlee's Law (Byerlee, 1967, 1978)

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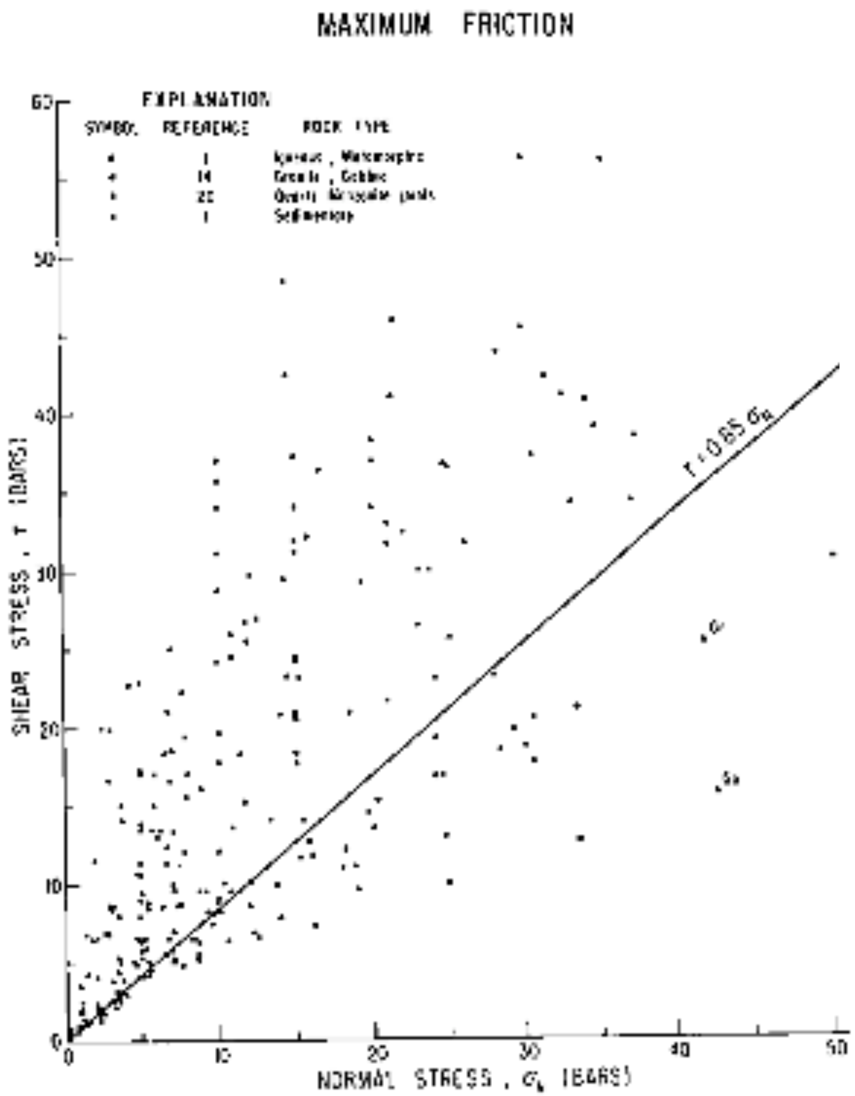
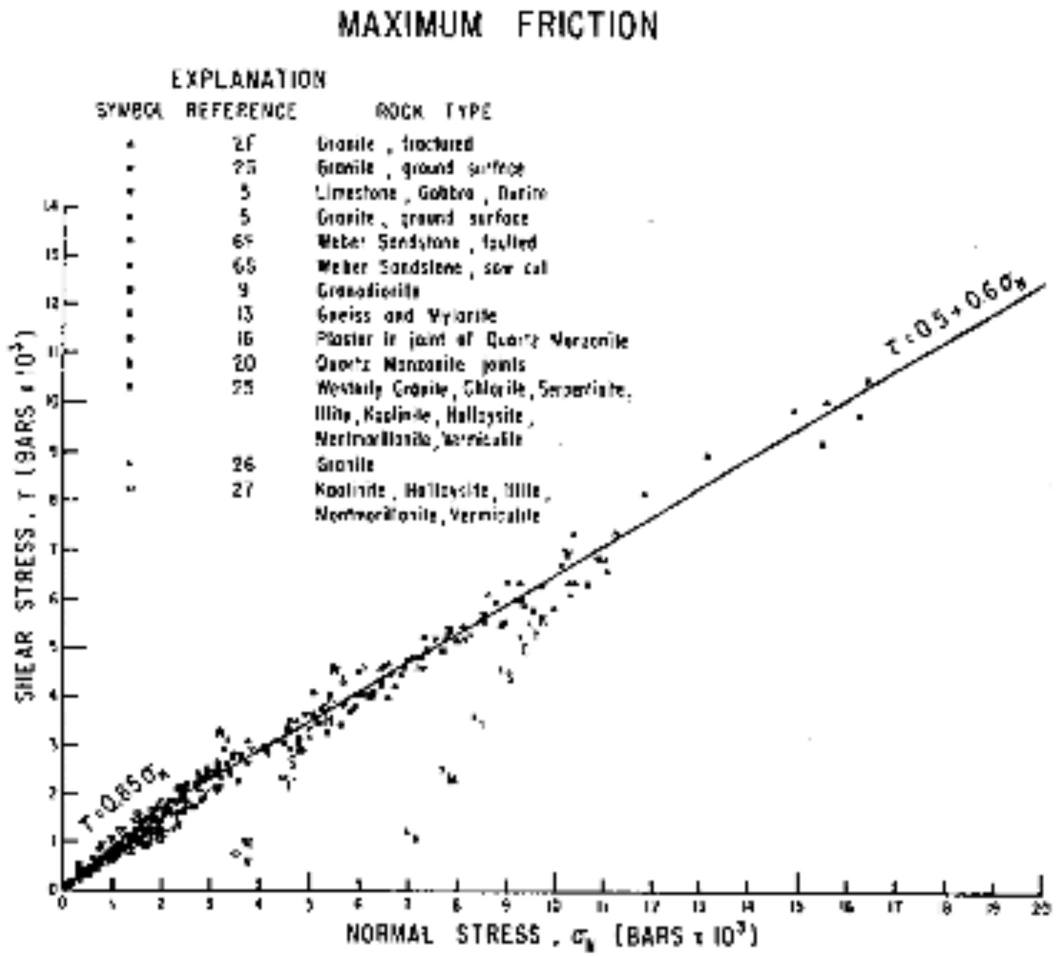
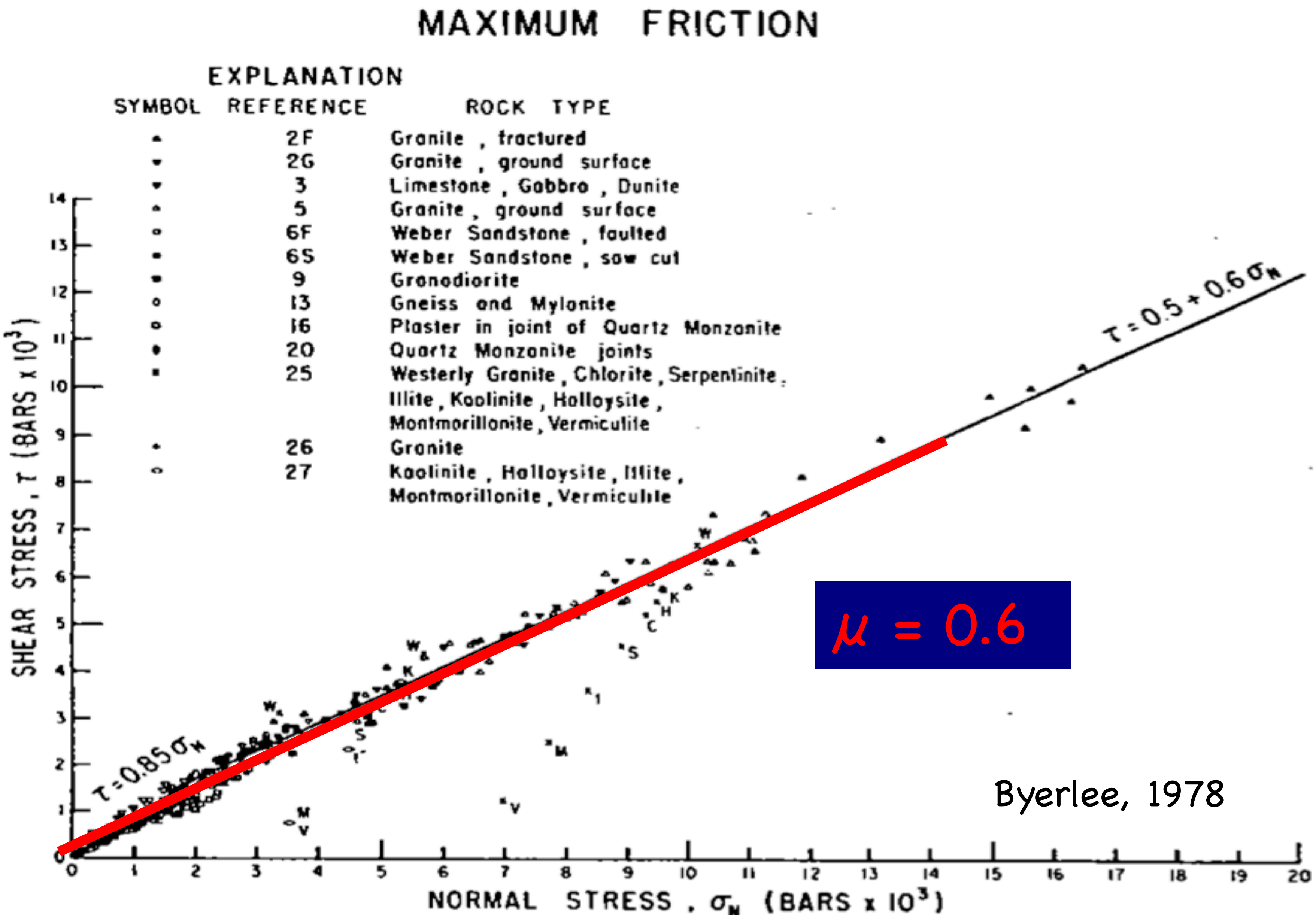




For granular materials, powders, and fault gouge: $\tau = 0.6\ \sigma_n$

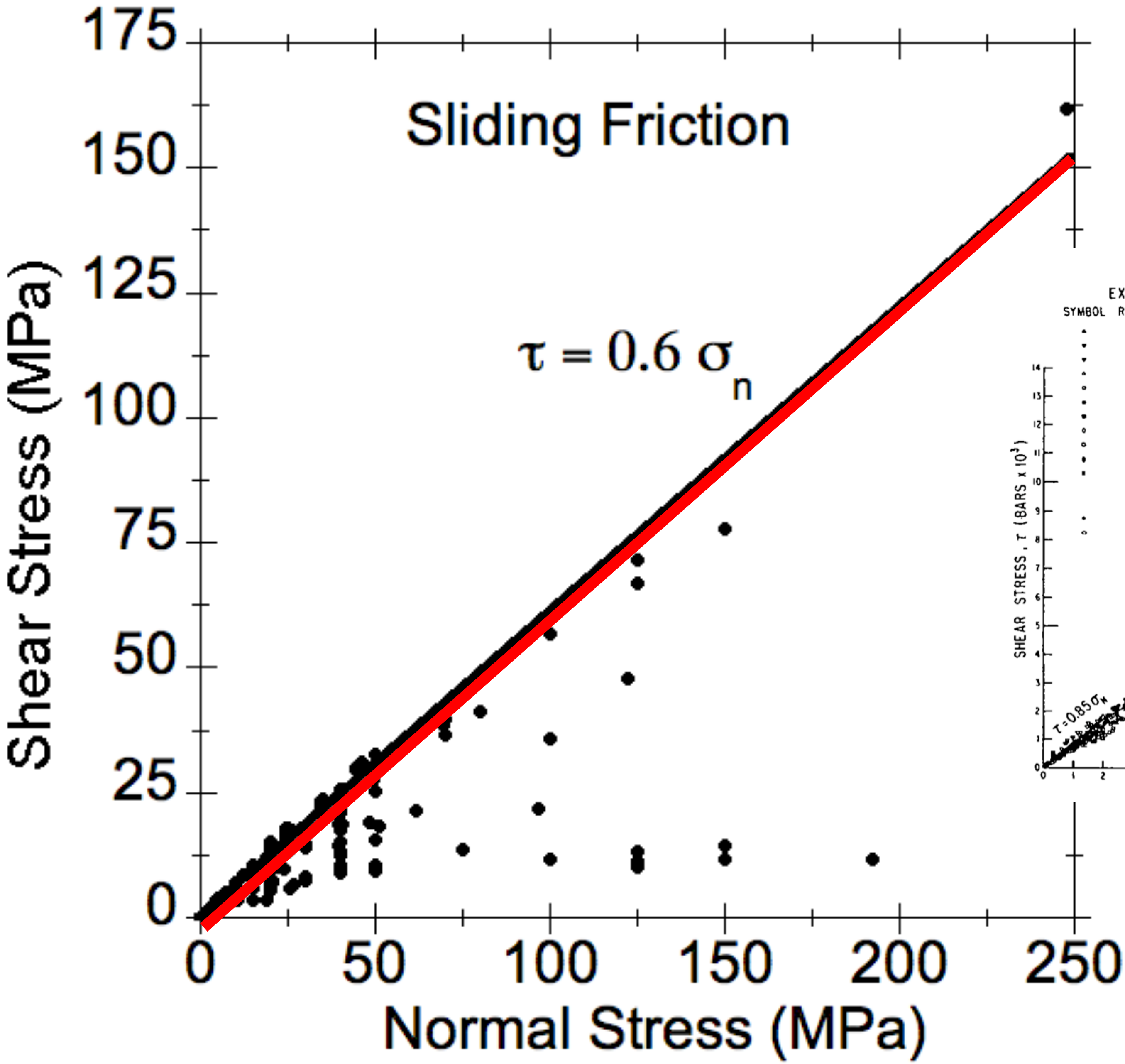
Byerlee's Law for Rock Friction (Coulomb's Criterion)

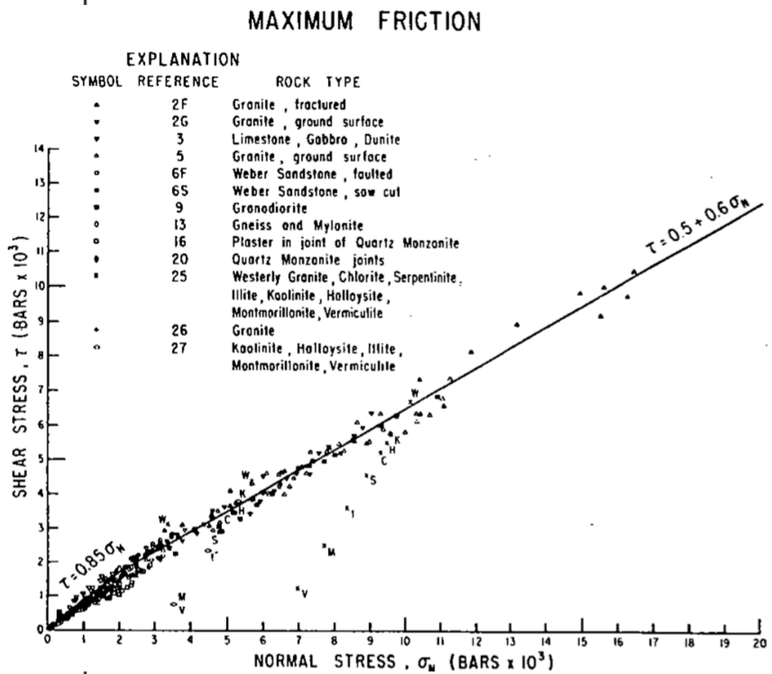


Byerlee, 1978

Friction of Fault Zones

Penn State Lab, ~ 2000 samples



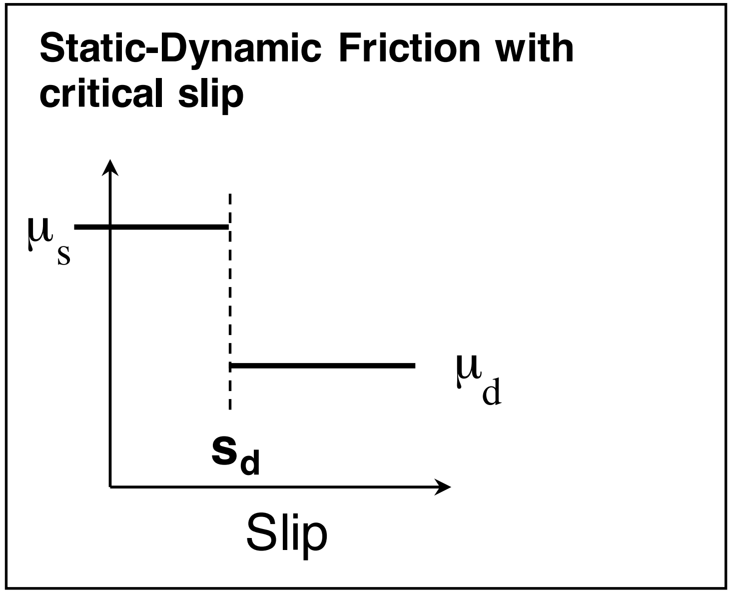


Friction: 2nd order variations, slick-slip and stability of sliding

Rabinowicz 1951, 1956,. 1958
Static vs. dynamic friction & state dependence

$$\left.\begin{aligned}\mu &= \mu_s \ \ (s = 0)\\ \mu &= \mu_d \ \ (s > 0)\end{aligned}\right\}$$ Classical view

Rabinowicz recognized that finite slip was necessary to achieve fully dynamic slip



$$\mu = \mu_s \ \ (s < s_d)$$

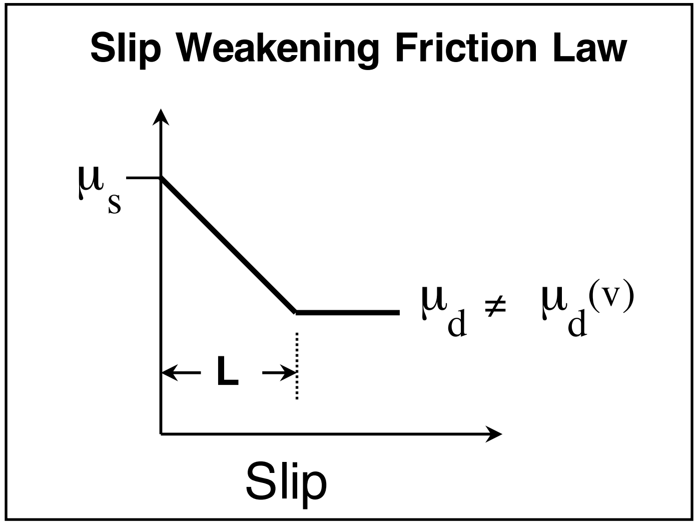
$$\mu = \mu_d \ \ (s > s_d)$$

s_d is the critical slip distance

Rabinowicz experiments showed state, memory effects and that μ_d varied with slip velocity.

Friction: 2nd order variations, slick-slip and stability of sliding

Rabinowicz' s work solved a major problem with friction theory: he introduced a way to deal with the singularity in going from μ_s to μ_d



$$\mu(x) = \mu_s - \frac{x}{L}\Delta\mu \qquad (for\ L > x > 0)$$

$$\mu(x) = \mu_s - \Delta\mu \qquad (for\ x > L)$$

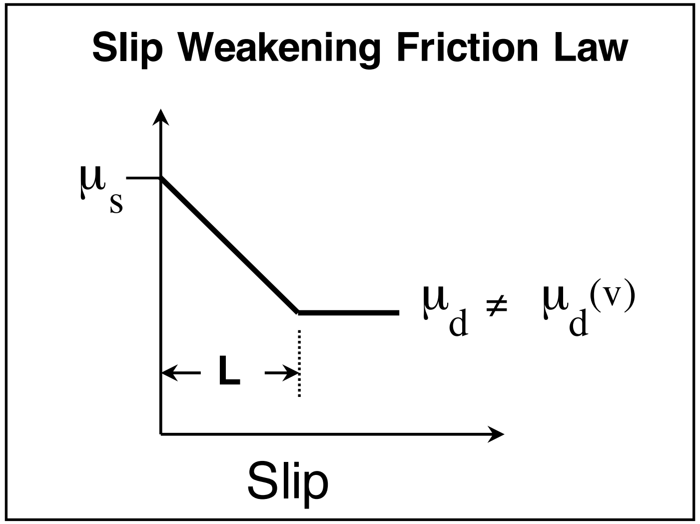
Palmer and Rice, 1973; Ide, 1972; Rice, 1980

For solid surfaces in contact (without wear materials), the slip distance L represents the slip necessary to break down adhesive contact junctions formed during 'static' contact.

The slip weakening distance is also known as the critical slip or the breakdown slip

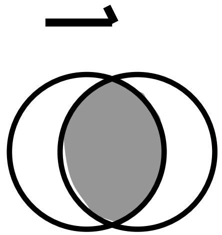
This slip distance helps with the stress singularity at propagating crack tips, because the stress concentration is smeared out over the region with slip < L.

Friction: 2nd order variations, slick-slip and stability of sliding



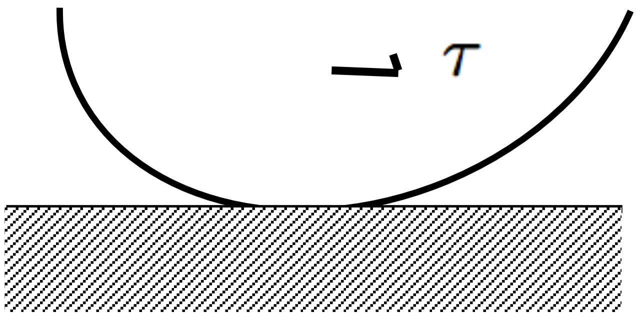
$$\mu(x) = \mu_s - \frac{x}{L}\Delta\mu \qquad (for\ L > x > 0)$$

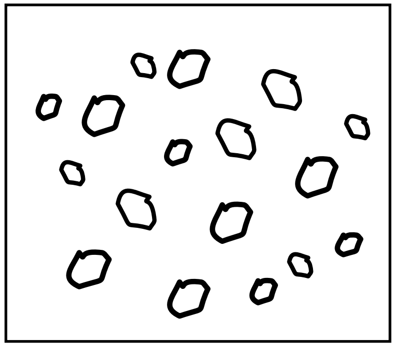
$$\mu(x) = \mu_s - \Delta\mu \qquad (for\ x > L)$$



Critical friction distance represents slip necessary to erase existing contact

Adhesive Theory of Friction





For a surface with a distribution of contact junction sizes, L, will be proportional to the average contact dimension.

Critical friction distance scales with surface roughness