Mechanics of Earthquakes and Faulting

Lecture 5, 4 Feb 2021

www.geosc.psu.edu/Courses/Geosc508

- Work of shear deformation with volume strain
- Importance of volume change and dilatancy rate (rate of volume strain with shear strain)
- some basic elasticity
- Friction theory
- Amonton's laws
- Chemical effects
- Hydrolytic weakening
- Basic observations of: time-dependent static friction
- velocity-dependent sliding friction
- Adhesive theory of friction
- Hertian contact
- ploughing

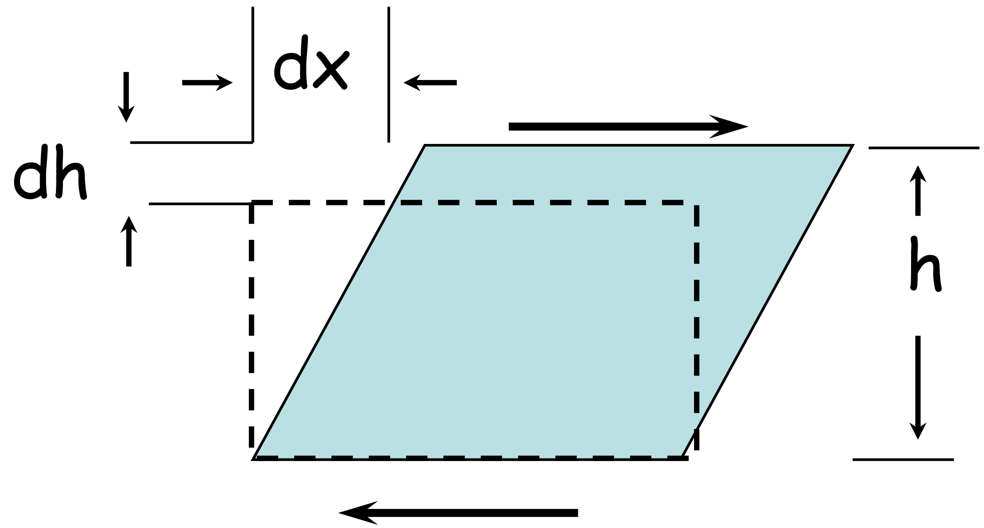
- Read *Rabinowicz*, 1951 & 1956 (we will discuss these next week on Feb 11)
- Read Chapter 2 of Scholz (and look ahead at other chapters)

Work per unit volume is given by of an intrinsically frictional term ($\tau_p\, d\gamma$) and a term involving volume change ($\sigma\, d\theta$)

We can replace τ_p with μ_p and then write W the measured shear stress τ for shear deformation. Note that $\tau \neq \tau_p$

$$W = \tau_p \; d\gamma + \sigma \; d\theta$$

$$W = \sigma \mu_p d\gamma + \sigma d\theta$$



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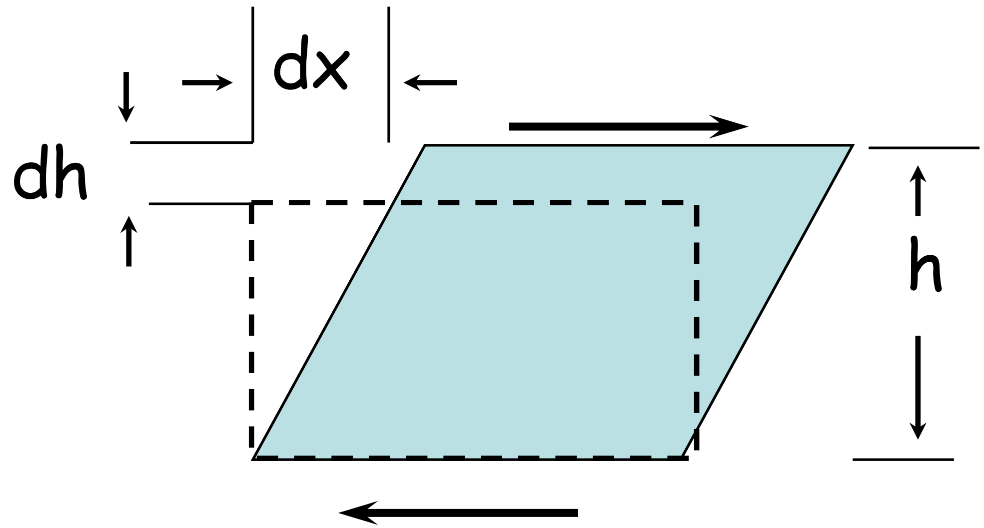
$$\tau d\gamma = \sigma(\mu_p d\gamma + d\theta)$$

$$\tau = \sigma\left(\mu_p + d\theta / d\gamma\right)$$

$$\tau = \sigma\left(\mu_p + dh / dx\right)$$



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V = hA; dv=dhA

$d\gamma = dx/h$

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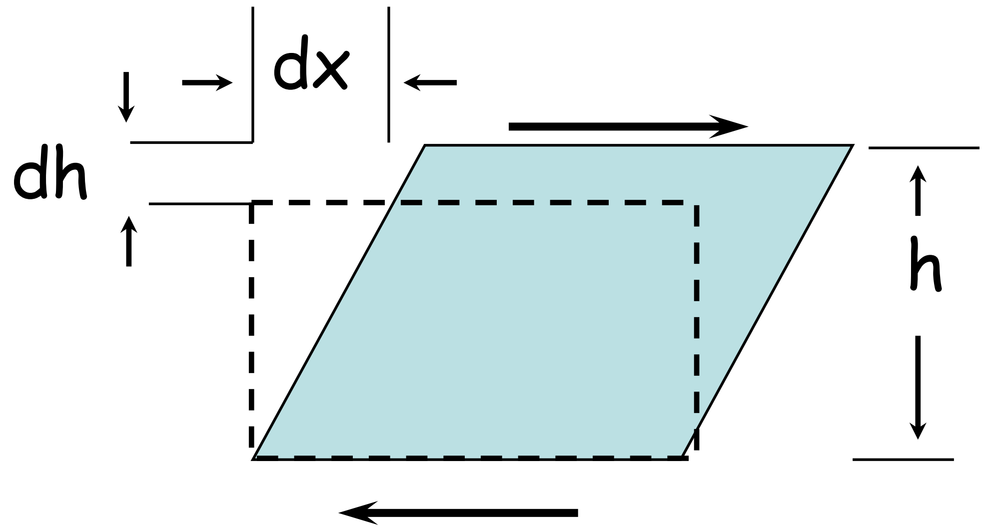
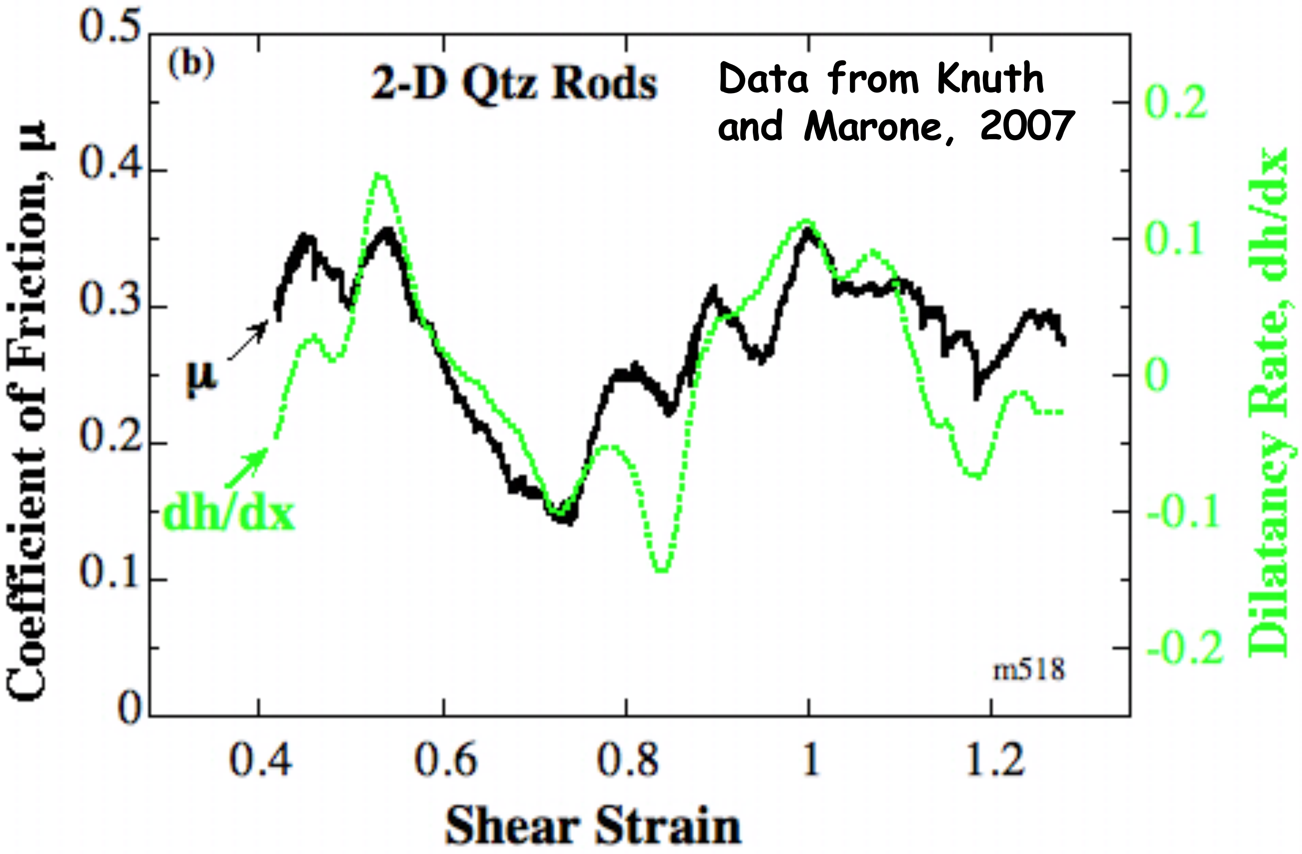
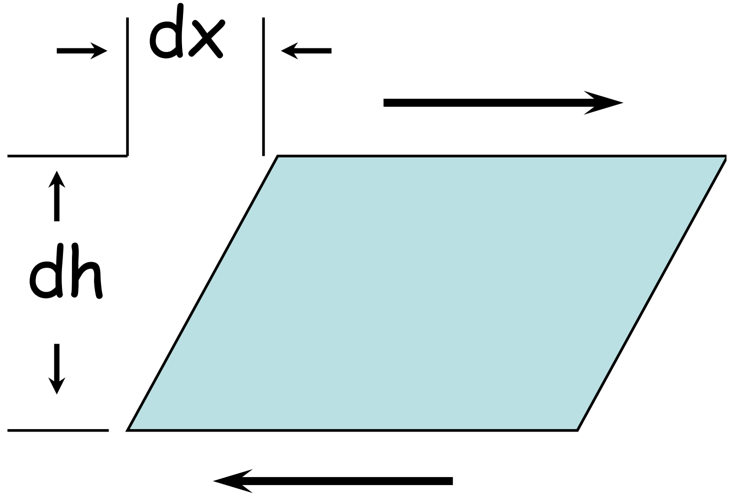


Friction mechanics of 2-D particles





work per unit volue

$$W = \tau_p \ d\gamma + \sigma \ d\theta$$

$$\tau = \sigma\left(\mu_p + d\theta / d\gamma\right)$$

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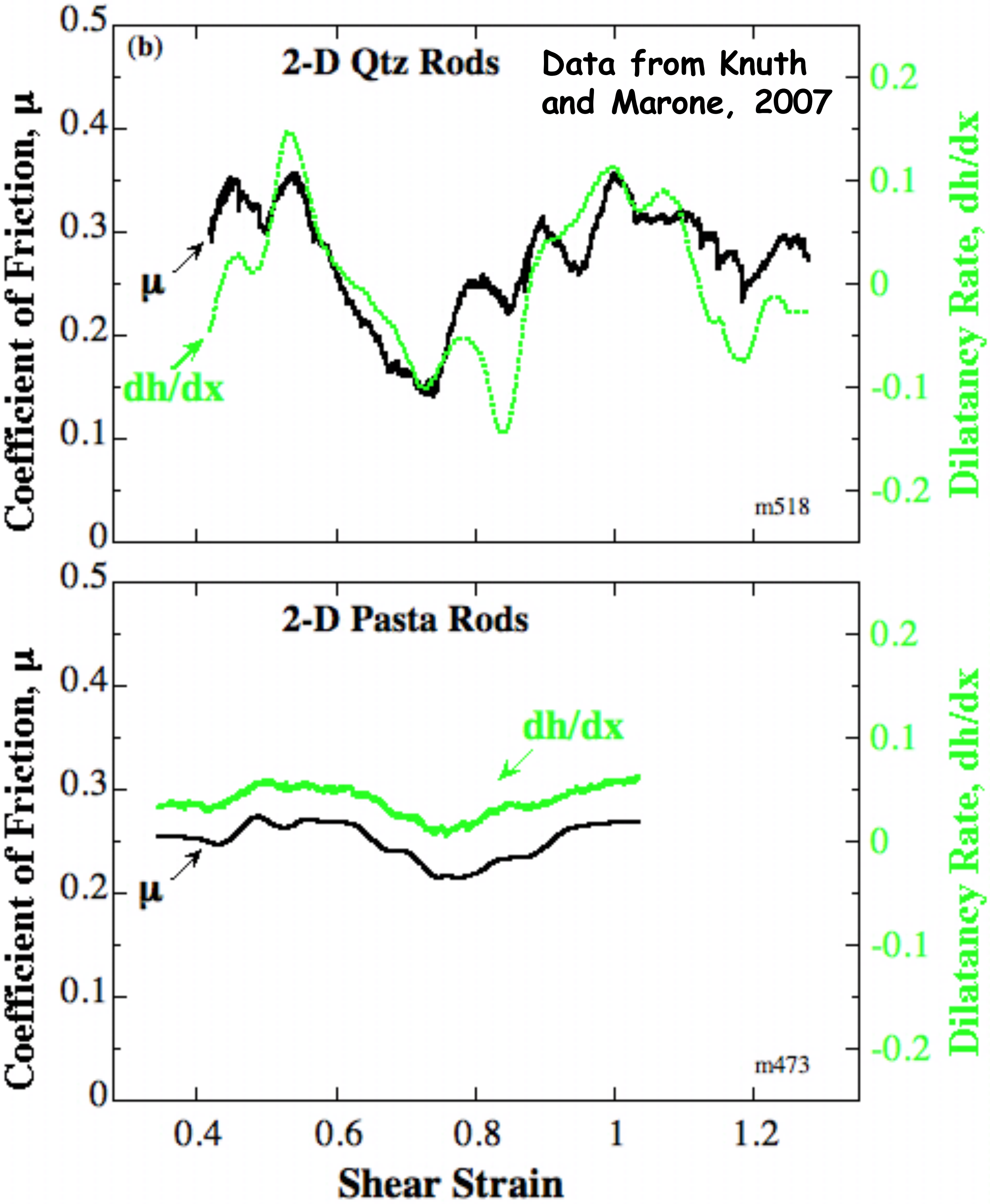
- Dilatancy rate plays an important role in setting the frictional strength

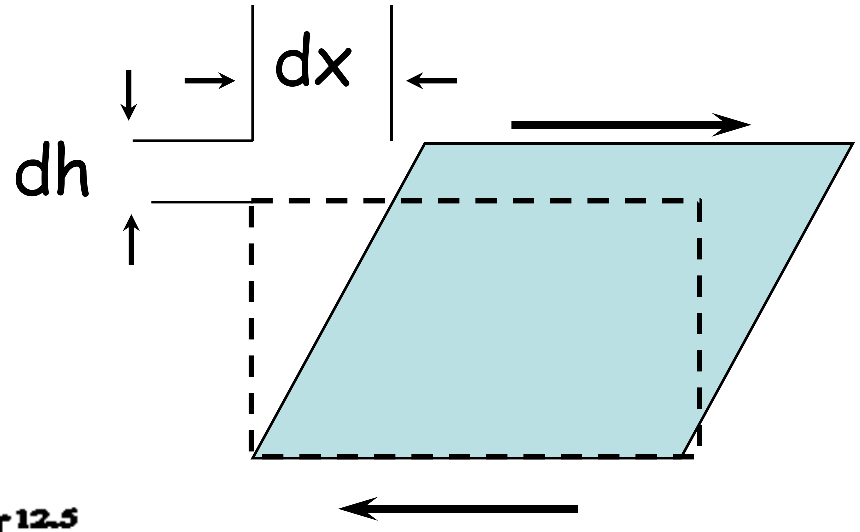


- Macroscopic variations in friction are due to variations in dilatancy rate.
- Smaller amplitude fluctuations in dilatancy rate produce smaller amplitude friction fluctuations.

$$W = \tau_p \; d\gamma + \sigma \; d\theta$$

W is total work of shearing

$$W = \tau \, d\gamma = \sigma \, \mu \, d\gamma$$



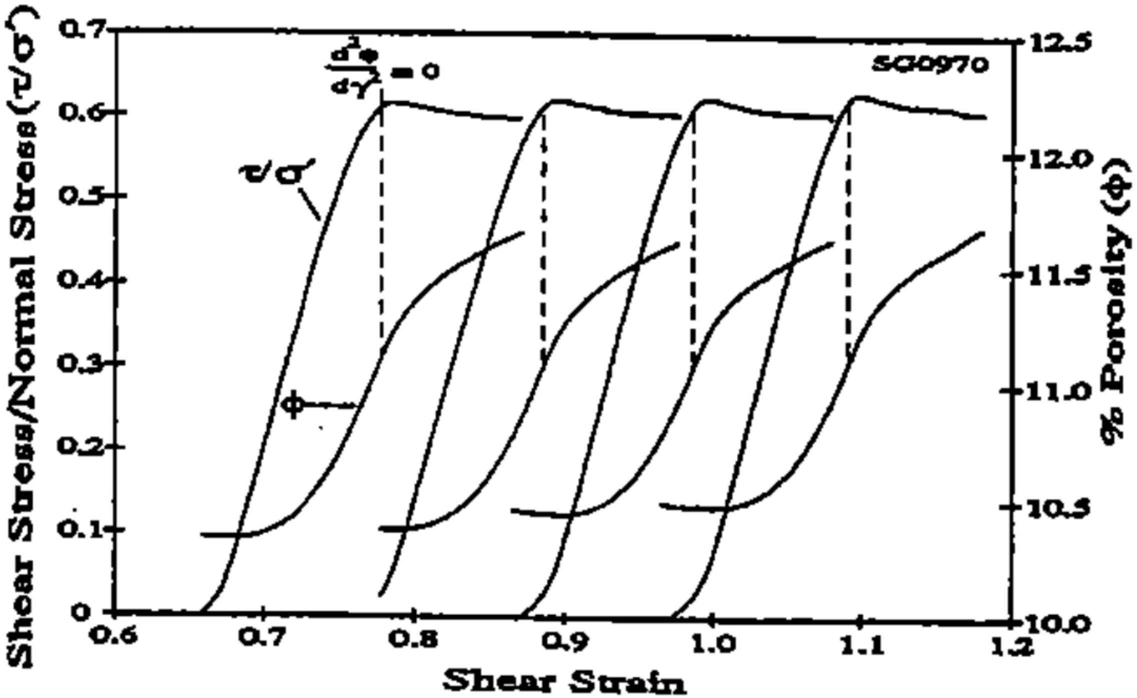


Fig. 8. An enlargement of the later stage of Figure 3. The curves have been truncated at the onset of unloading for clarity. Unlike the behavior at small strain (shown in Figure 7), neither $d\phi/d\gamma_L$ nor the form of the porosity-strain curves varied with progressive cycling. Dashed lines show the point at which $d^2\phi/d\gamma^2 = 0$.

Marone et al., 1990

Mead, 1925
(Geologic Role of Dilatancy)

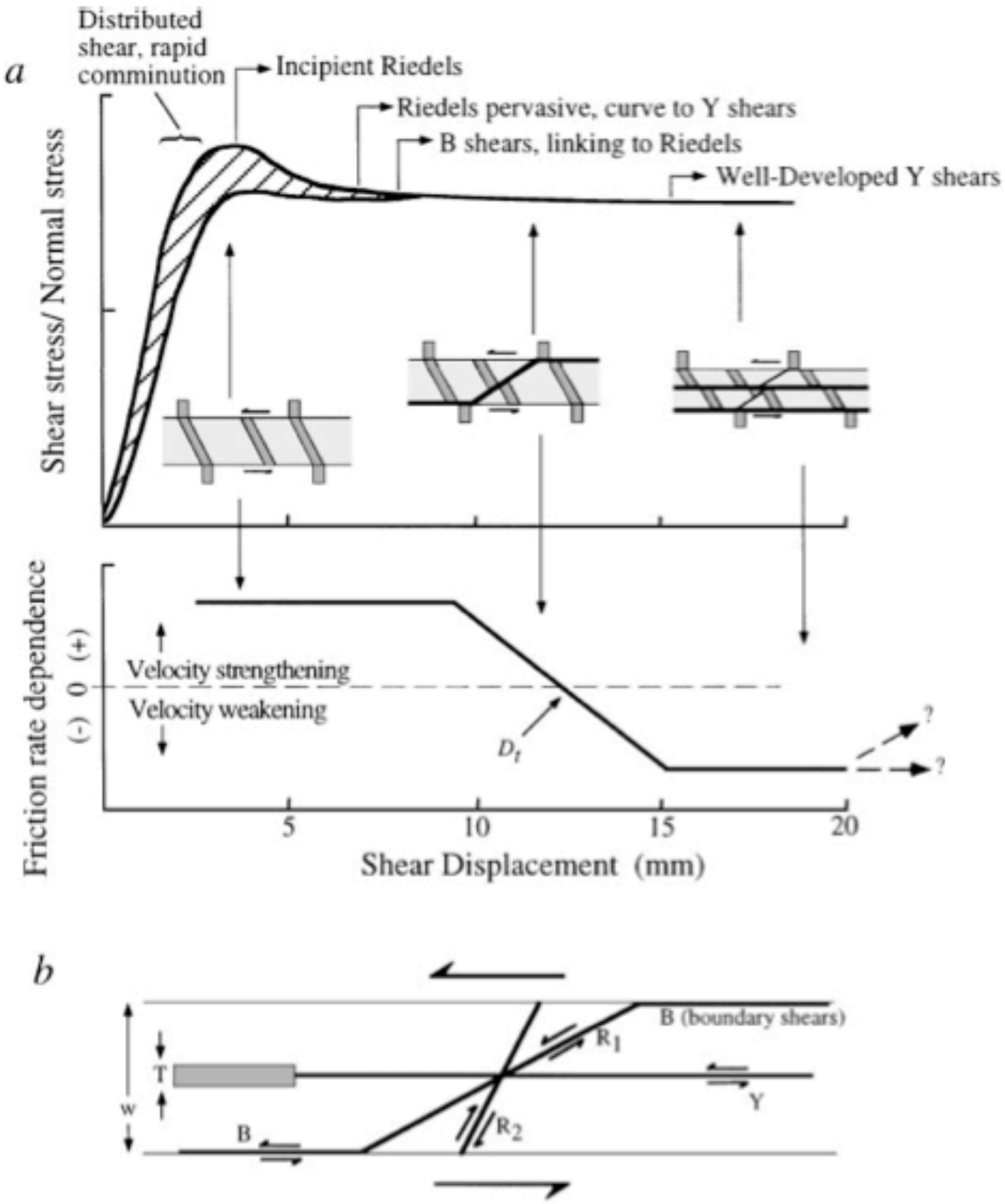
Shear Localization

Strain homogeneity depends on whether dilatancy is restricted

- Homogeneous strain if dilatancy is not opposed
- Strain localization if deformed under finite confining pressure

Shear Bands Form if:

$$\frac{\partial \theta^{SB}}{\partial \gamma} < \frac{\partial \theta^{homo.}}{\partial \gamma}$$



Marone, 1998

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Strain homogeneity depends on whether dilatancy is restricted

Shear strength depends on friction and dilatancy rate

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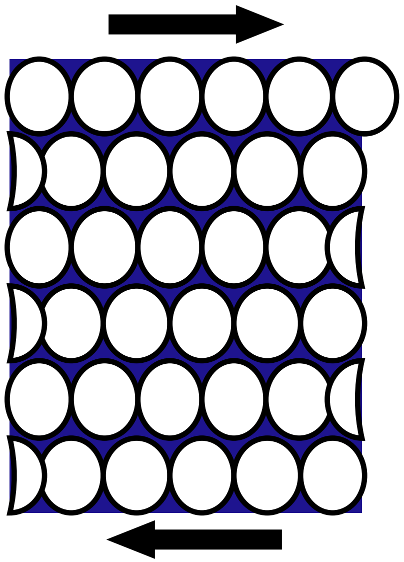

Deformation mode (degree of strain localization) minimizes dilatancy rate

Mead, 1925
(Geologic Role of Dilatancy)

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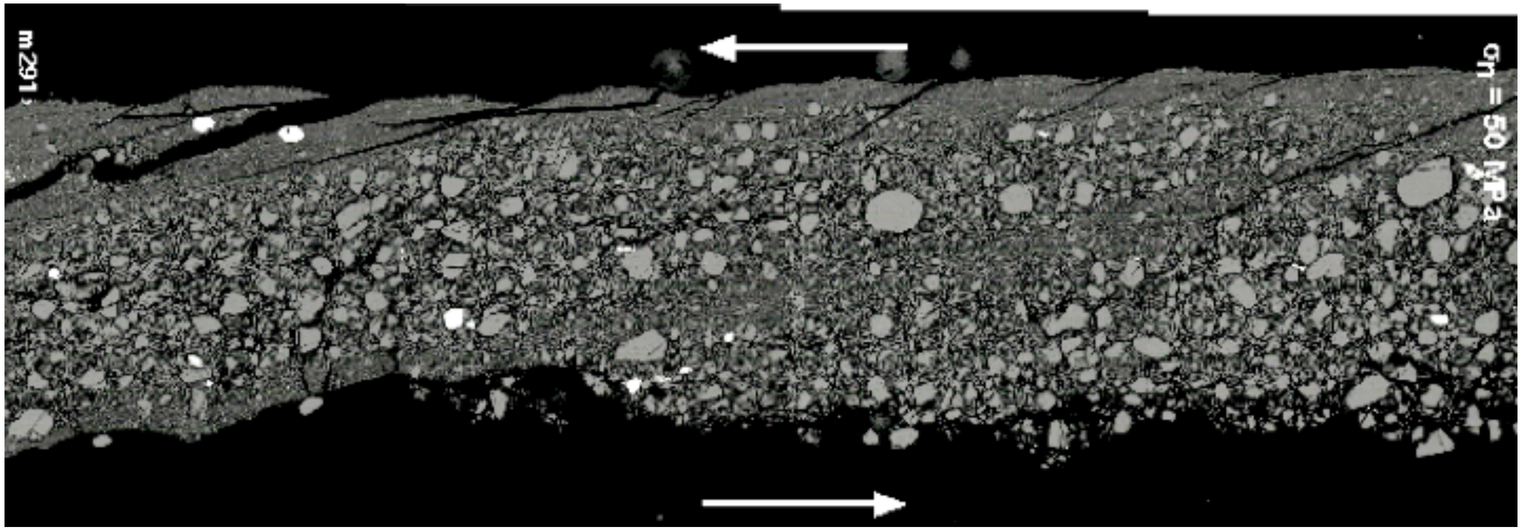
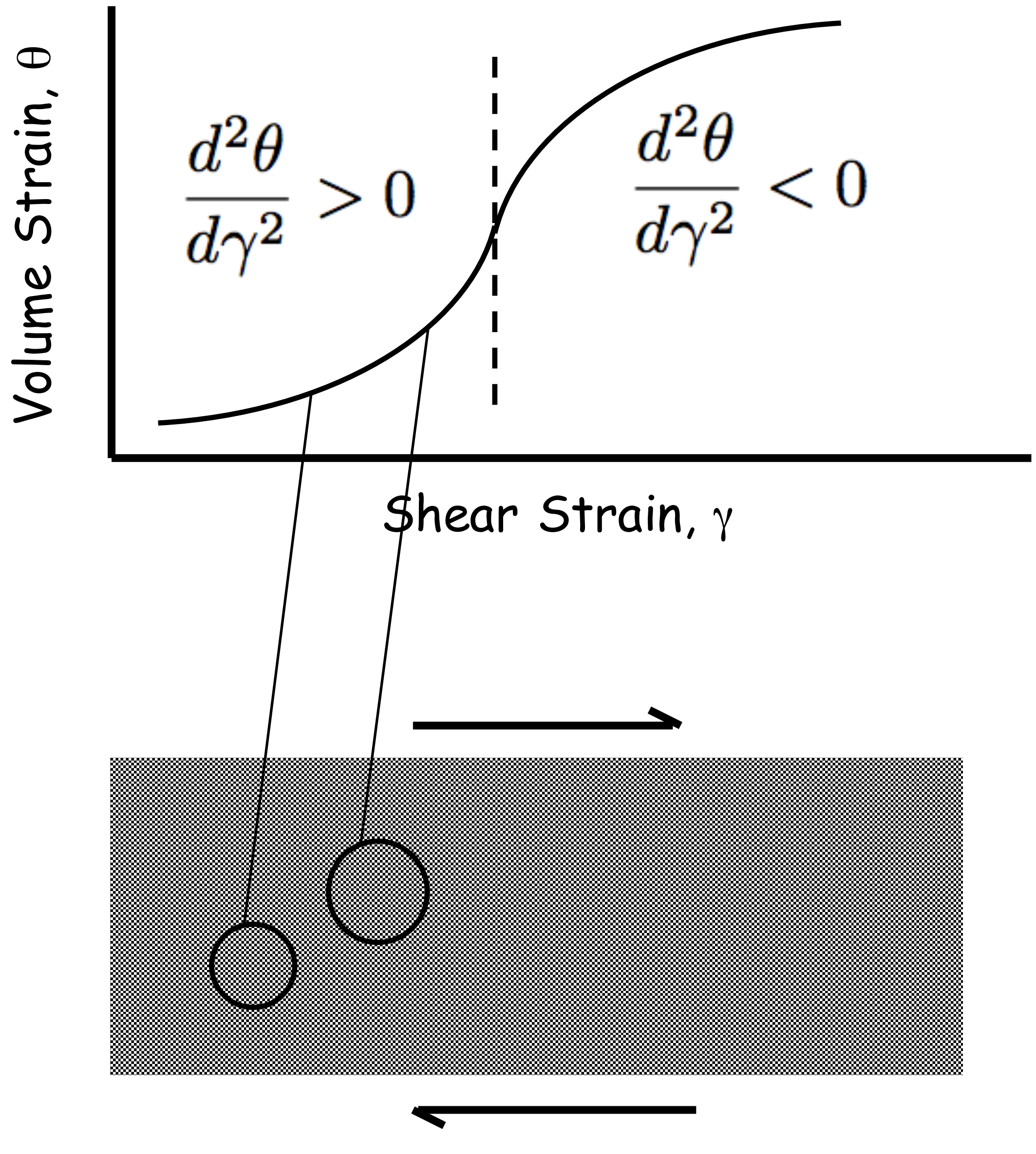


Frank, 1965. Volumetric work, shear localization and stability.
Applies to Friction and Fracture

$$W = W_f + \sigma d\theta$$

$$W = \tau d\gamma$$

$$\tau = \tau_f + \sigma \frac{d\theta}{d\gamma}$$



Strain hardening when: $\frac{d^2\theta}{d\gamma^2} > 0$

Strain softening when: $\frac{d^2\theta}{d\gamma^2} < 0$

$$\frac{d\tau}{d\gamma} = \frac{d\tau_f}{d\gamma} + \sigma \frac{\frac{d\theta}{d\gamma}}{d\gamma}$$

$$W = W_f + \sigma d\theta$$

$$W = \tau d\gamma$$

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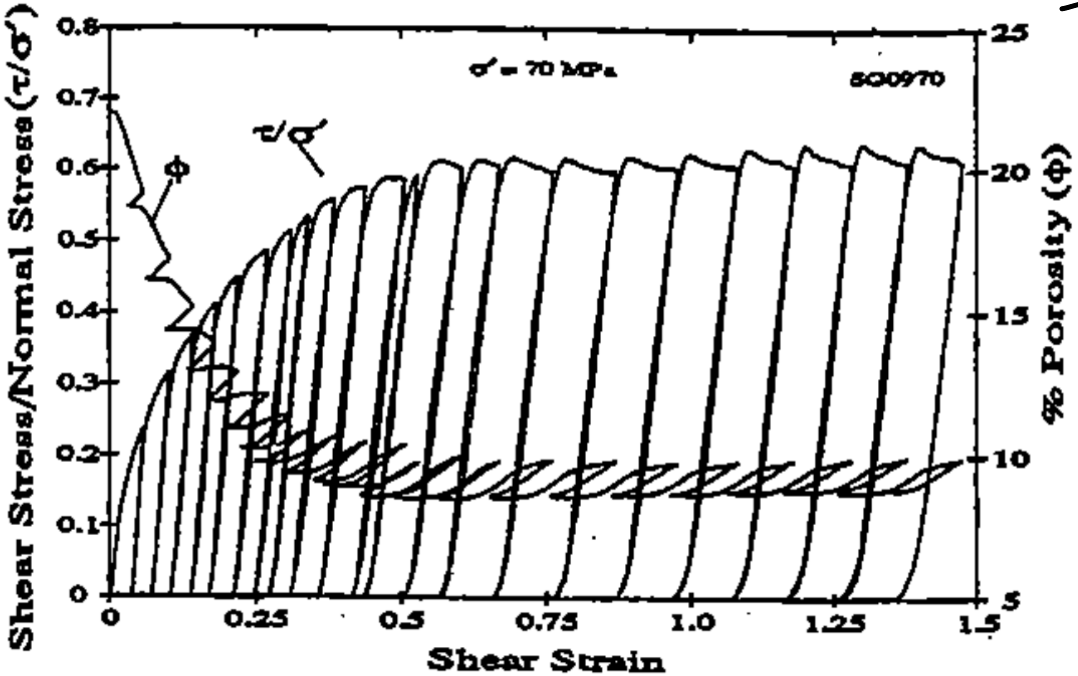
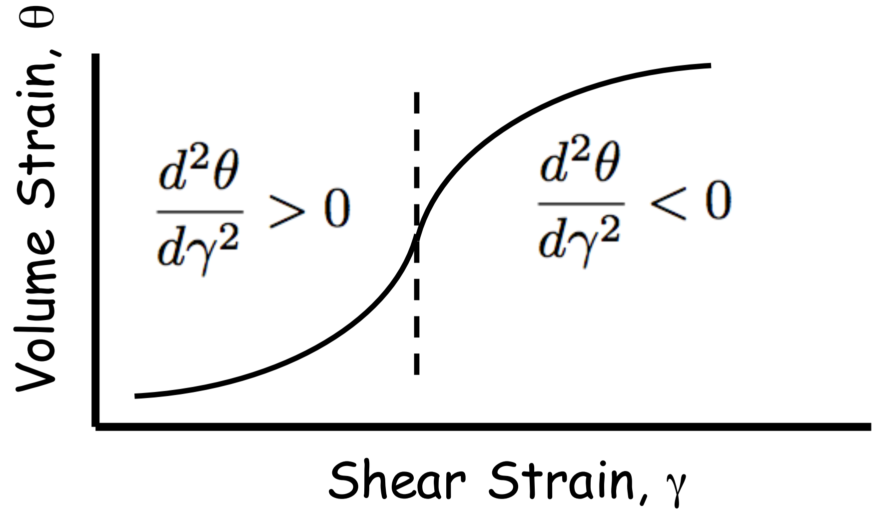


Fig. 3. Normalized shear stress τ/σ' and porosity versus shear strain for a 4.0-mm-thick gouge layer sheared within rough steel surfaces. Effective normal stress was constant during shear loading in all experiments. We refer to each increase and decrease in τ/σ' as a load cycle. Overall compaction occurred during the initial increments of strain, after which shear occurred at constant shear stress and porosity maintained a roughly constant level as measured at the end of each cycle.



Marone, Raleigh & Scholz, 1990

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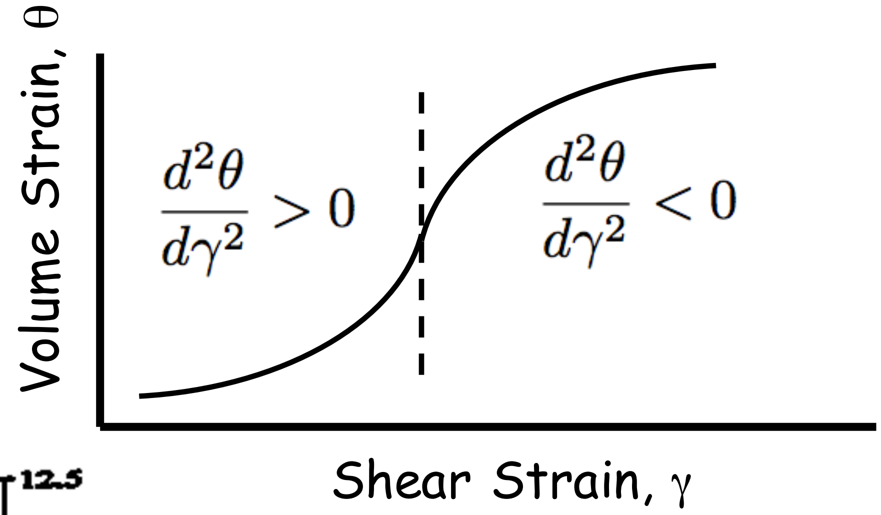

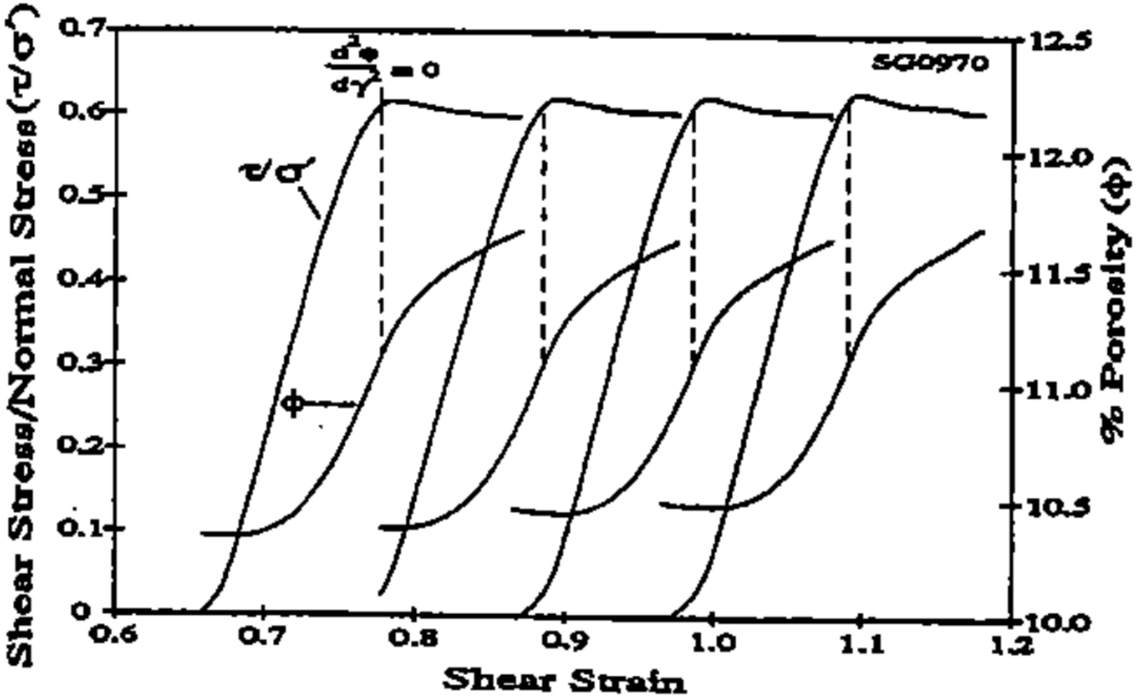

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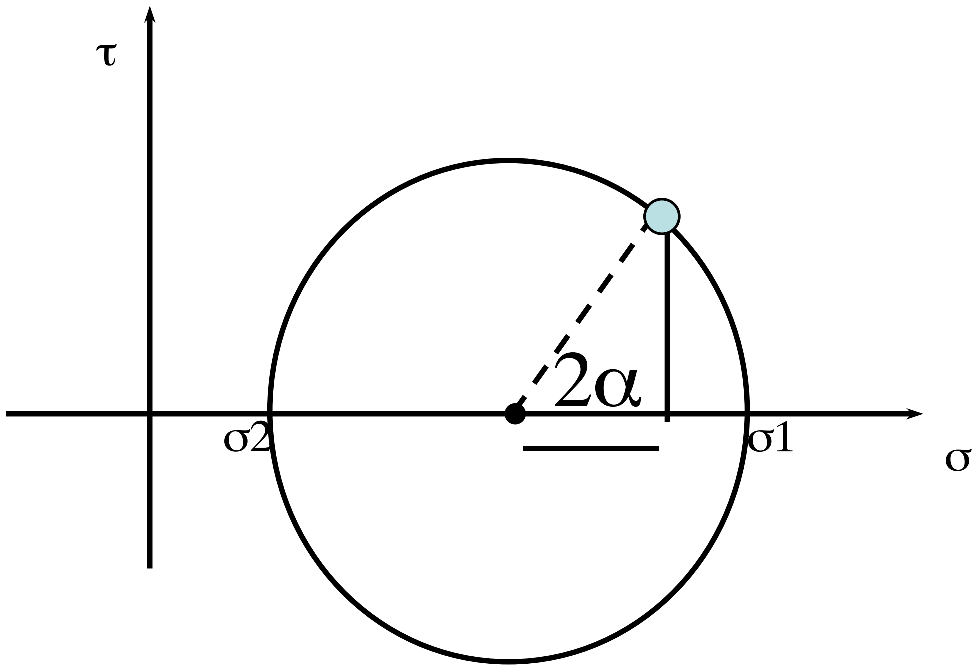
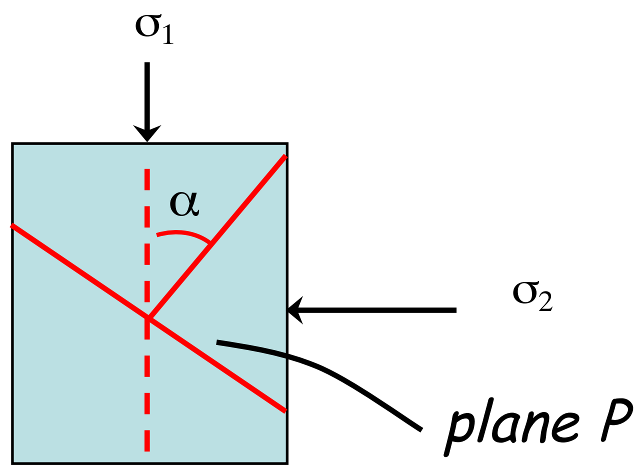
Marone, Raleigh & Scholz, 1990

$$\sigma 1 - (\sigma 1 - \sigma 2)/2 + (\sigma 1 - \sigma 2)/2 \quad \cos 2\alpha$$

$$\sigma(\alpha) = \qquad \sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

$$\tau(\alpha) = (\sigma 1 - \sigma 2)/2 \quad \sin 2\alpha$$





Pop Quiz:

Derive the relations for shear and normal stress on a plane of arbitrary orientation in terms of principal stresses σ_1 and σ_2

Hint, use the Mohr Circle.

Fluids: Consider the affect on shear strength

- Mechanical Effects
- Chemical Effects

Mechanical Effects: Effective Stress Law

$$\sigma_{effective} = \sigma_n - Pp$$