Mechanics of Earthquakes and Faulting

Groundhog Day 2021

www.geosc.psu.edu/Courses/Geosc508

- Stress concentrations and crack mechanics
- Work of deformation, shear and volume strain
- Importance of volume change and dilatancy rate (rate of volume strain with shear strain)
- Effective stress and energy budget for shear, Dilatancy

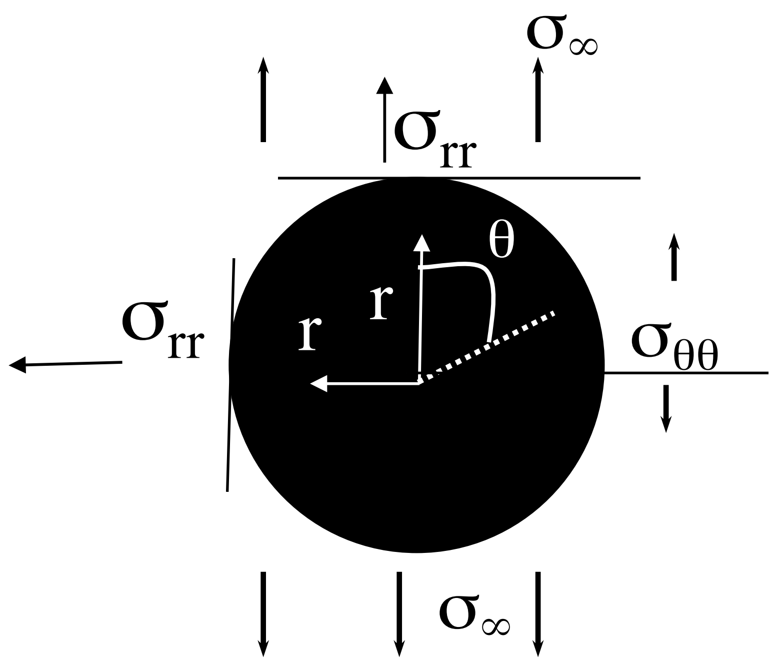
- For next time, come prepared to discuss
- Mead, W. J., The geologic role of dilatancy. *Jour. Geol. 33*, 685-698, 1925.
- Frank, F. C., On dilatancy in relation to seismic sources. *Rev. Geophys. 3*, 485-503, 1965

Full solution for a circular hole of radius r=a

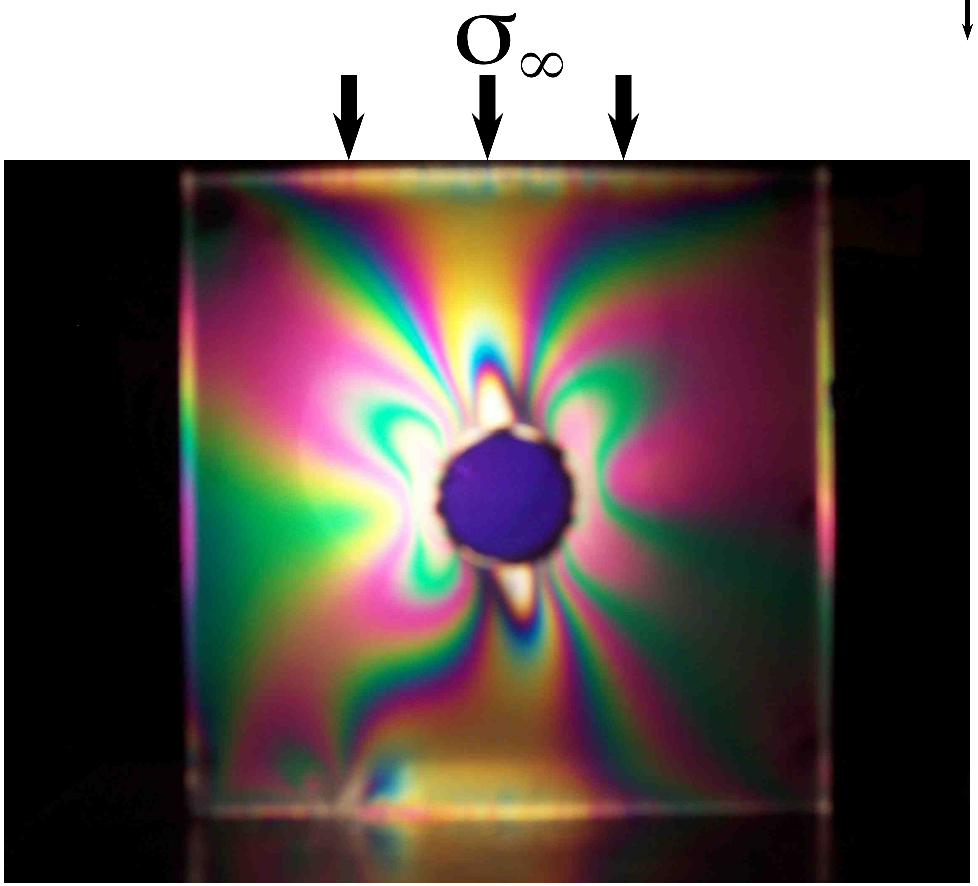
$$\sigma_r = \frac{\sigma_\infty}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_\infty}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma_\infty}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_\infty}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = -\frac{\sigma_\infty}{2}\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$



hole of radius a



Full solution for a circular hole of radius r=a

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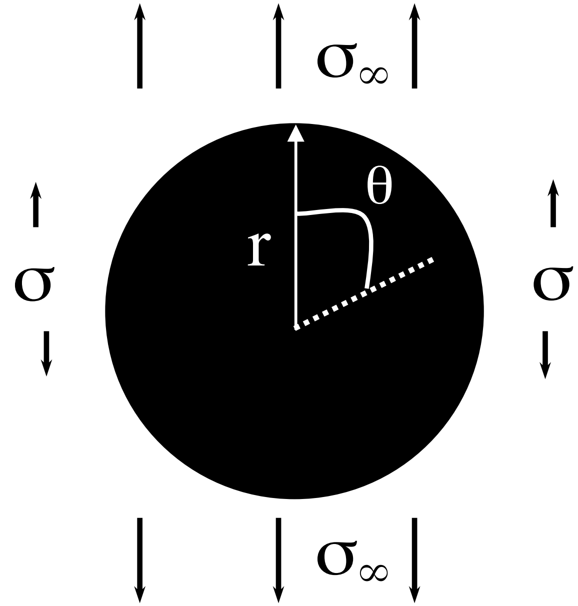


At the top, θ=0 and r=a
then $\sigma_{rr} = 0$ (why?)
and $\sigma_{\theta\theta} = -\sigma_\infty$

At the side, θ=90 and r=a
$\sigma_{rr} = 0$
$\sigma_{\theta\theta} = 3\sigma_\infty$

1. **Elasticity and Stress. [25 pts]** Two specimens of a granite are to be studied in the laboratory. The cross-sectional area of each sample is 1 cm^2. One sample is 11 cm in length. By experimenting, you find that a uniaxial load of 375 MPa causes a deformation of 508 microns on the 11-cm long sample. The second granite sample has stiffness k of 5.08×10^7 N/m.
1.1 What is Youngs modulus E of the granite? Is this a reasonable value, for say Westerly Granite? Please justify your answer using a reference where the elastic properties of Westerly granite have been measured. Summarize the results of this paper in one or two sentences.
1.2 The stiffness k can be expressed in units of force per displacement. Please give an equation for stiffness in terms of E and other variables.
1.3 How long is the second granite sample?

We know that E = σ/ε and we are told that a stress σ of 375 MPa causes a strain ε of (508e-6m/0.11m)
Therefore E = 3.75e8Pa/(508e-6m/0.11m) = 81.2 GPa; Yes, this is a reasonable value (see Simons and Brace, 1965 or Brace, 1965)
The stiffness in force per displacement can be expressed as k=EA/L.
For the second sample k is 5.08×10^{12} N/m. How long is it?
k = EA/L
Therefore, L= (3.75e8Pa/(508e-6m/0.11m)) * 1e-4m^2)/5.08e7N/m = 15.98 cm.

Simmons, G., & Brace, W. F. (1965). Comparison of static and dynamic measurements of compressibility of rocks. *Journal of Geophysical Research*, *70*(22), 5649-5656.
Brace, W. F. (1965). Some new measurements of linear compressibility of rocks. *Journal of geophysical research*, *70*(2), 391-398.

2. **Horizontal Stress.** **[25]** For regions far from significant tectonic stresses, the applied horizontal stress in the crust is produced by the applied vertical overburden stress and Poisson expansion, assuming uniaxial strain (e.g., strain occurs in the vertical direction but lateral strain is essentially zero, just enough to generate a horizontal stress). In terms of effective stress, the relevant equations are written: $\sigma_h' = \sigma_v' [\nu/(1-\nu)]$ where ν is Poisson's ratio, σ_h' is effective horizontal stress, and σ_v' is effective vertical stress. Assuming a Poisson ratio of 0.25, a rock density of 2.5 x10^3 kg/m^3, and a pore fluid density of 1.0 x10^3 kg/m^3: calculate

2.1 The vertical effective stress at 10 km

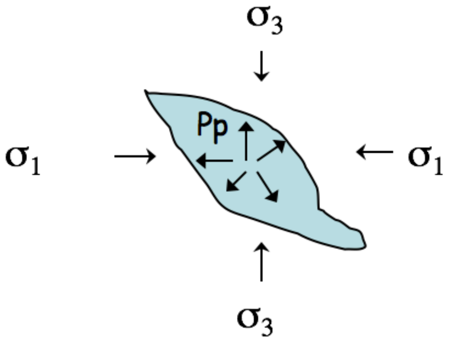
2.2 The horizontal effective stress at 10 km

Effective Stress

$$\sigma'_{ij} = \sigma_{ij} - P_p\delta_{ij}$$

$$\delta_{ij} = 1; i = j$$
$$\delta_{ij} = 0; i \neq j$$

$\sigma_{effective} = \sigma_n - Pp$



Exercise: Follow through the implications of Kronecker's delta to see that pore pressure only influences normal stresses and not shear stresses. Hint: see the equations for stress transformation that led to Mohr's circle.

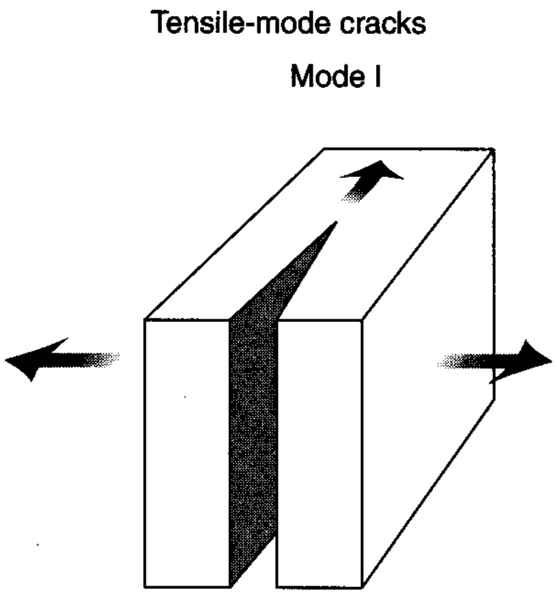
$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2}\cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

$U = (-W + U_e) + U_s$

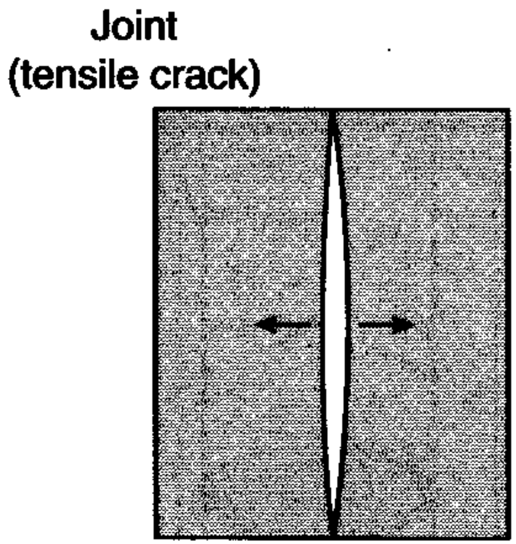
- Crack will extend if $dU/dc < 0$
- System is at equilibrium if $dU/dc = 0$

- The critical stress for crack propagation (failure stress): $\sigma_f = (4E\gamma/\pi c)^{1/2}$



(a)

$$\sigma_\infty = \sqrt{\frac{E\gamma}{4c}}$$



(b)

Taking σ_∞ of 10 MPa, E= 10 GPa and γ of 4×10^{-2} J/m^2, gives a crack half length c of 1 micron.

$U = (-W + U_e) + U_s$

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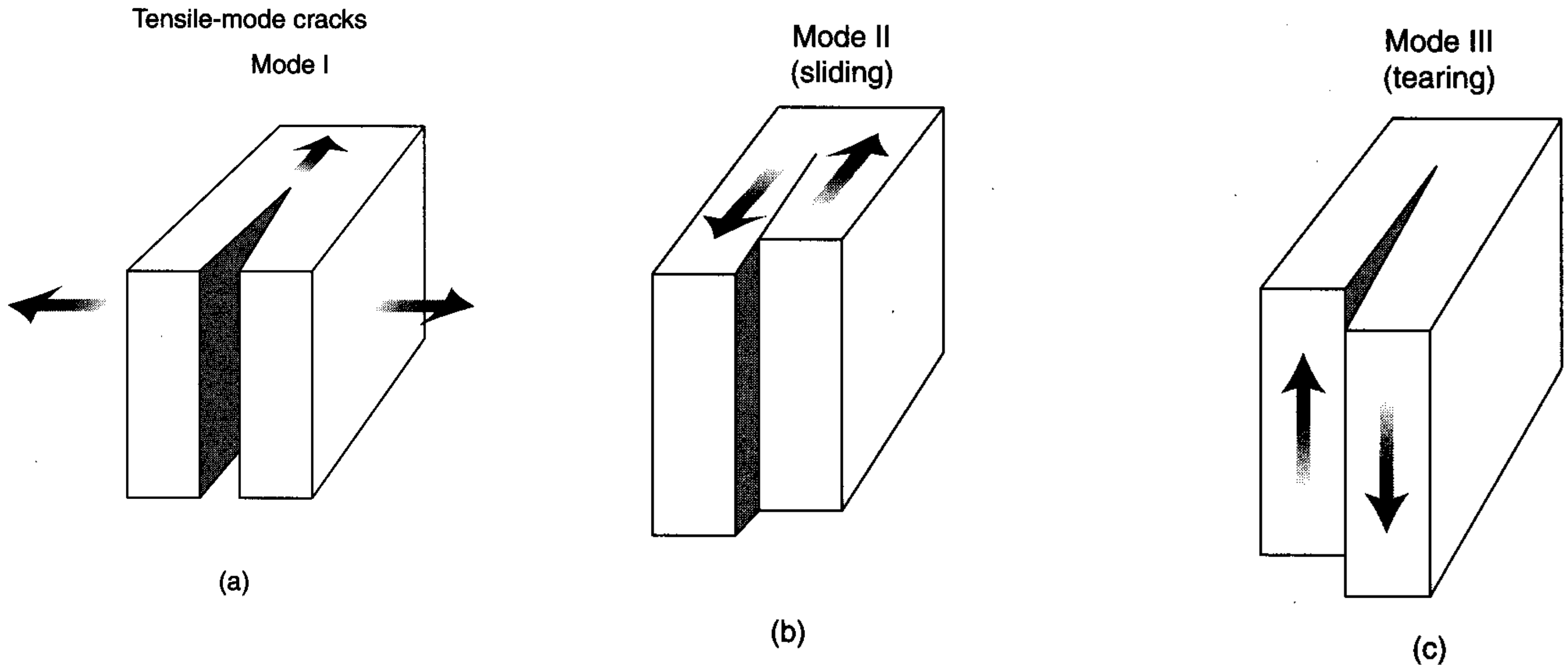
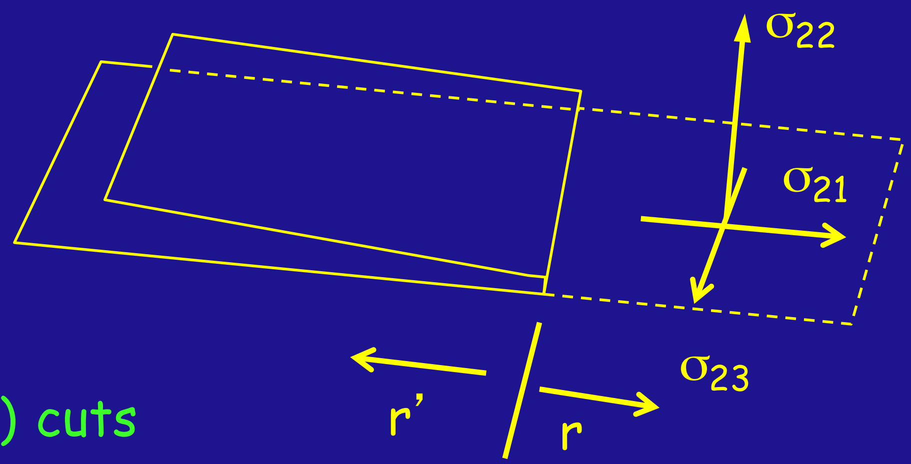
- System is at equilibrium if $dU/dc = 0$



Fracture Mechanics and Stress intensity factors for each mode

K_I, K_{II}, K_{III}

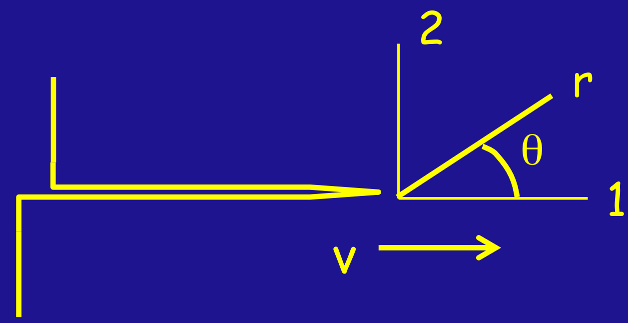


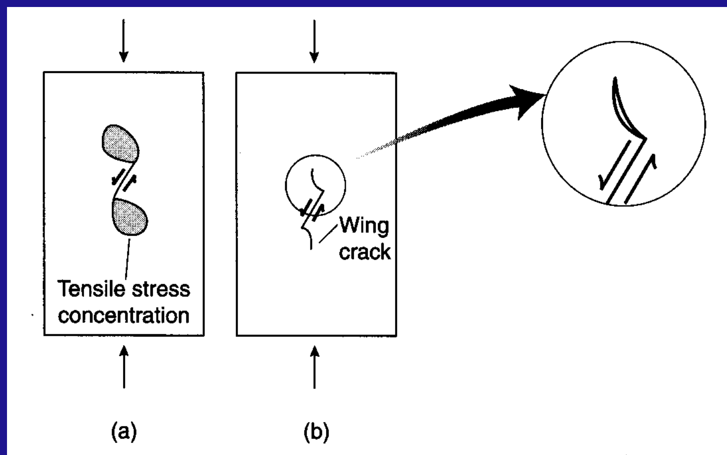
Linear Elastic Fracture Mechanics
- Frictionless cracks
- Planar, perfectly sharp (mathematical) cuts

Crack tip stress field written in a generalized form

$$\sigma_{ij} = K_n \frac{1}{\sqrt{2\pi r}} f_{ij}^n(\theta)$$

$$\begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{tip} \approx \frac{1}{\sqrt{2\pi r}} \begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix}$$





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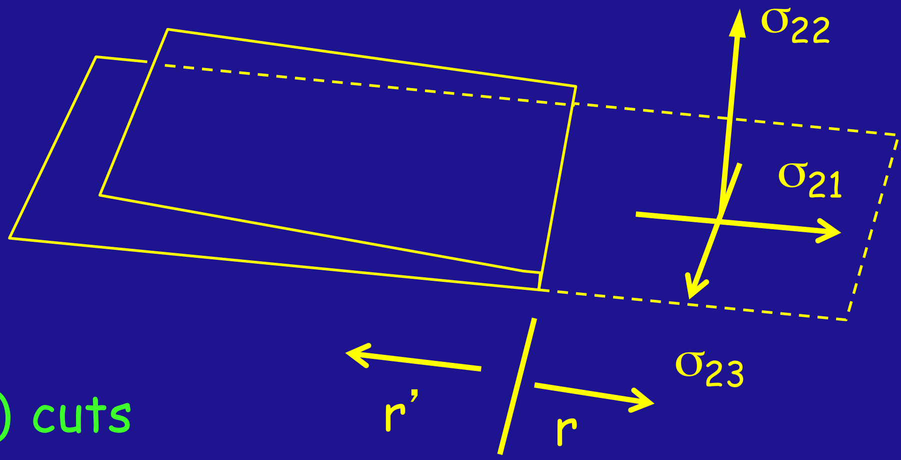

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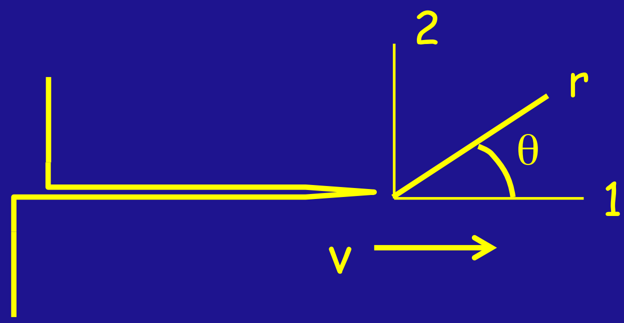

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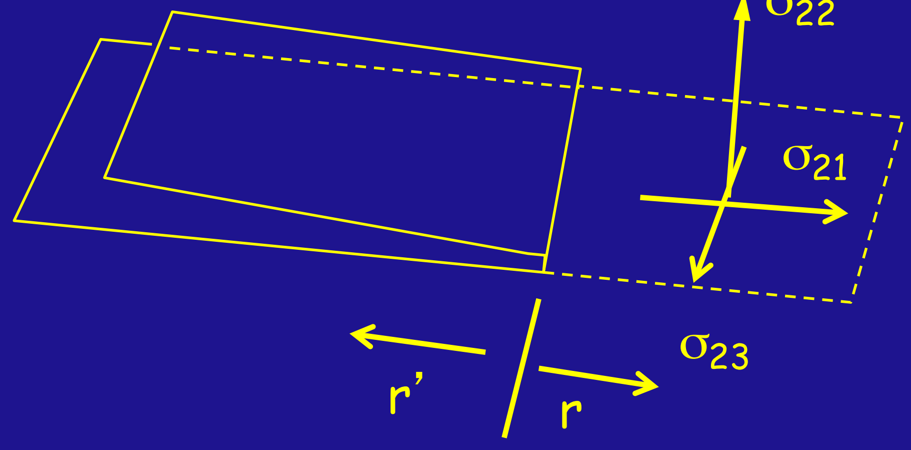
$$\begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{tip} \approx \frac{1}{\sqrt{2\pi r}} \begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix}$$

Note that the functions $f_{ij}^{n}(\theta)$ vary from ± 2, so are not major factors

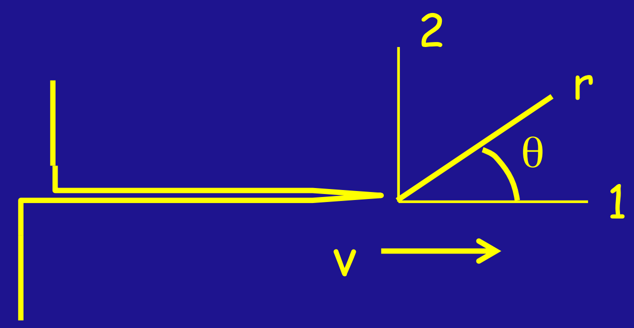
For uniform remote loading of a crack of length 2c:

$$\begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix} = \sqrt{\pi c} \begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{applied}$$



$$\begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{tip} \approx \frac{1}{\sqrt{2\pi r}} \begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix}$$

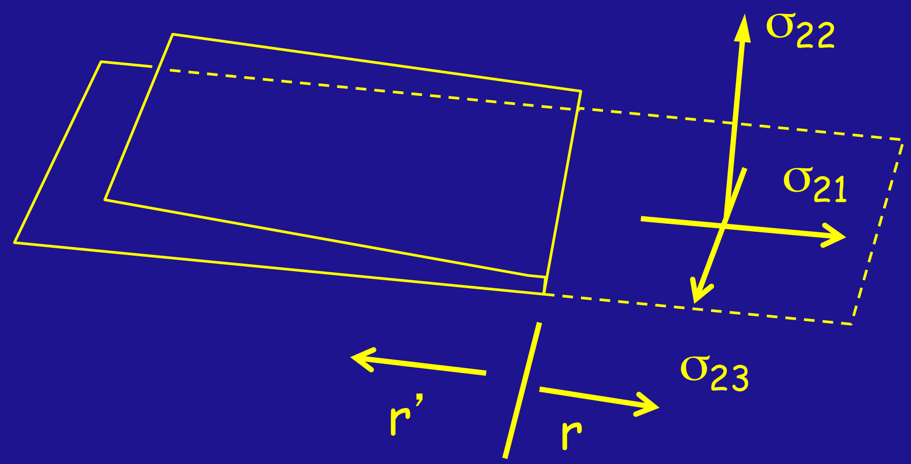
$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}} K_n$$



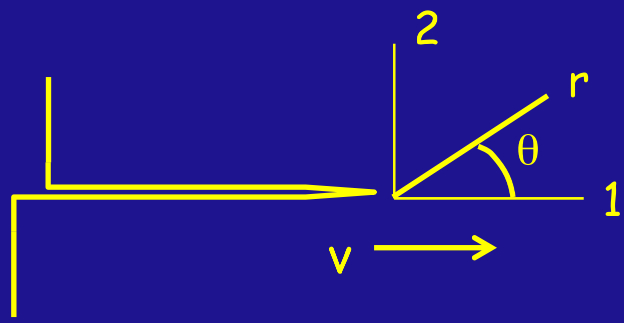
$$\sigma_{22}^{tip} = \frac{\sqrt{c}}{\sqrt{2r}} \sigma_{22}$$

$$\begin{Bmatrix} \Delta u_2 \\ \Delta u_1 \\ \Delta u_3 \end{Bmatrix} \approx \frac{4(1-\eta)}{\mu} \sqrt{\frac{r'}{2\pi}} \begin{Bmatrix} K_I \\ K_{II} \\ \dfrac{K_{III}}{(1-\eta)} \end{Bmatrix}$$

Static vs. dynamic fracture mechanics, relativistic effects



$$\begin{Bmatrix} \Delta u_2 \\ \Delta u_1 \\ \Delta u_3 \end{Bmatrix} \approx \frac{4(1-\eta)}{\mu} \sqrt{\frac{r'}{2\pi}} \begin{Bmatrix} g_I(v)\, K_I \\ g_{II}(v)\, K_{II} \\ g_{III}(v)\, \dfrac{K_{III}}{(1-\eta)} \end{Bmatrix}$$



$$g_I(0) = g_{II}(0) = g_{III}(0) = 1$$

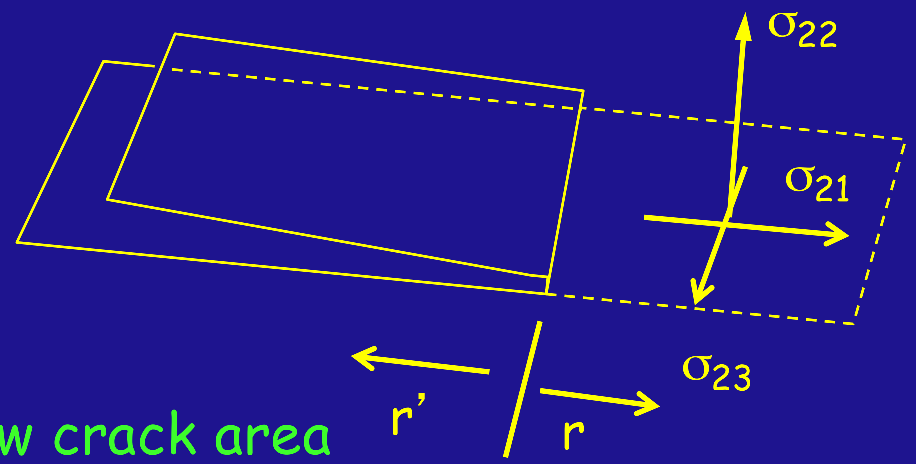
Static

Dynamic crack propagation

$$g_I(v) \rightarrow \infty \quad and = g_{II}(v) \rightarrow \infty, \quad as \quad v \rightarrow C_R$$

$$g_{III}(v) = \frac{1}{\sqrt{1-\eta^2 / C_s^2}} \rightarrow \infty, \quad as \quad v \rightarrow C_s$$

$$\begin{Bmatrix} \Delta u_2 \\ \Delta u_1 \\ \Delta u_3 \end{Bmatrix} \approx \frac{4(1-\eta)}{\mu}\sqrt{\frac{r'}{2\pi}} \begin{Bmatrix} g_I(v)\ K_I \\ g_{II}(v)\ K_{II} \\ g_{III}(v)\ \dfrac{K_{III}}{(1-\eta)} \end{Bmatrix}$$



G is Energy flow to crack tip per unit new crack area

$$G = \frac{(1-\eta)}{2\mu}\left[g_I(v)K_I^2 + g_{II}(v)K_{II}^2 \right] + \frac{1}{2\mu} g_{III}(v)K_{III}^2$$

$$G = G_{critical} = 2\gamma$$

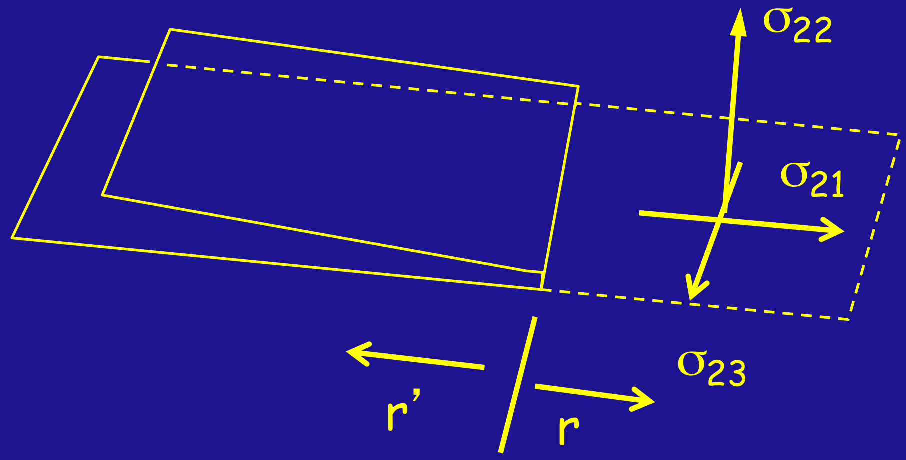
Critical energy release rate

G_{crit} is a material property --the "fracture energy"

$G_{crit} = K_c^2 / E = 2\gamma$, where K_c is the critical stress intensity factor (also known as the fracture toughness).

$$\begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{tip} \approx \frac{1}{\sqrt{2\pi r}} \begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix}$$

$$K_I = \sqrt{\pi c}\ \sigma_\infty$$

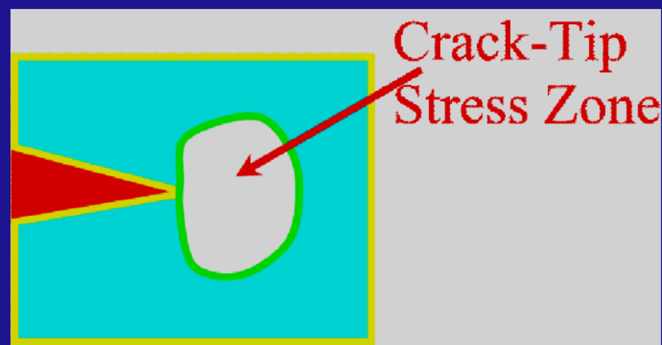


Stress field is singular at the crack tip.

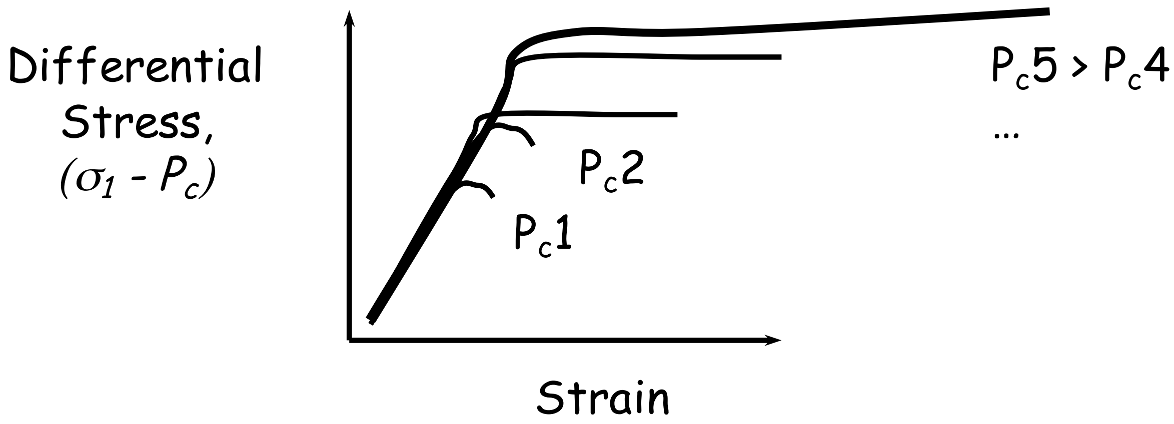
- because we assumed perfectly sharp crack
- but real materials cannot support infinite stress

Process zone (Irwin) to account for non-linear zone of plastic flow and cracking

- Size of this zone will depend upon crack velocity, material properties and crack geometry
- Energy dissipation in the crack tip region helps to limit the stresses there (why?)



Stress-strain-failure curves

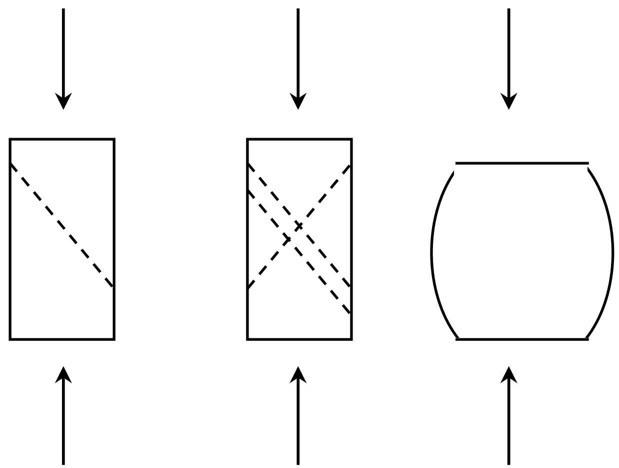


These styles can be loosely related to Brittle and Ductile deformation, respectively.

Brittle refers to pressure sensitive deformation

Brittle Failure: If we draw the stress-strain-failure curves for a range of confining pressures, we'll get a range of yield strengths, showing that σ_y is proportional to P_c.

With increasing confining pressure there is a transition from localized to more broadly distributed deformation.



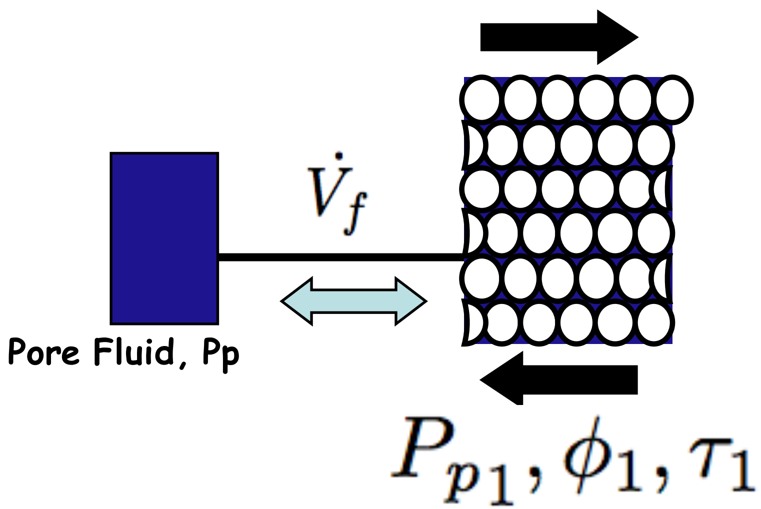
Consider the implications of dilatancy and volume change for the total work (per unit volume) of shearing, W

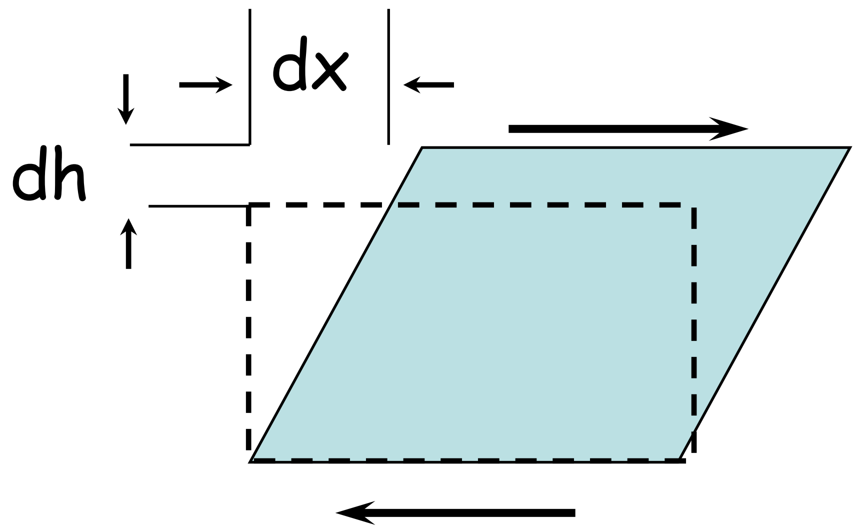
$W' = f\,dx$
$W = W'/V$
$W = f dx/AL$
$W_{ijkl} = \sigma_{ij}\,\varepsilon_{kl}$

$$W = \tau_p\ d\gamma + \sigma\ d\theta$$

W is total work of shearing

$W = \tau\,d\gamma = \sigma\,\mu\,d\gamma$





$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$

$\sigma_{effective} = \sigma_n - Pp$

$$\delta_{ij} = 1; i = j$$
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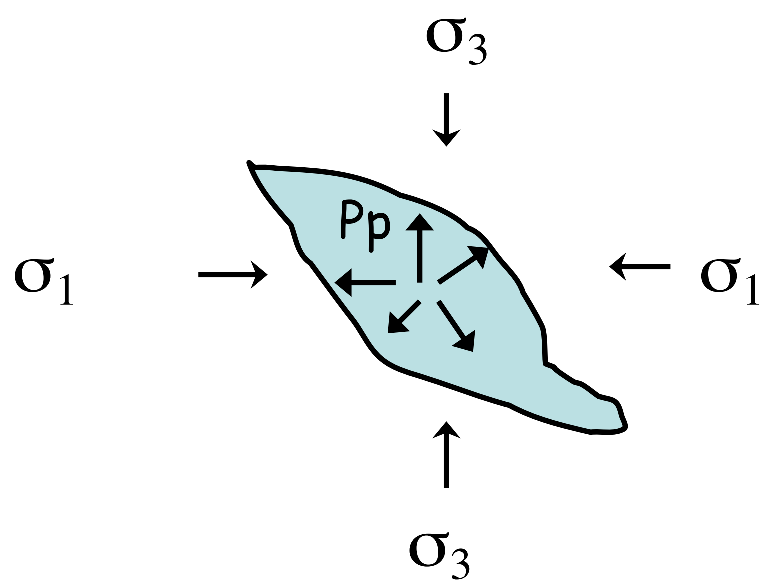


Exercise: Follow through the implications of Kronecker's delta to see that pore pressure only influences normal stresses and not shear stresses. Hint: see the equations for stress transformation that led to Mohr's circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

Consider the implications of dilatancy and volume change for the total work of shearing, W

Geochemistry Geophysics Geosystems G3 KNUTH AND MARONE: SHEARED GRANULAR LAYERS 10.1029/2006GC001327

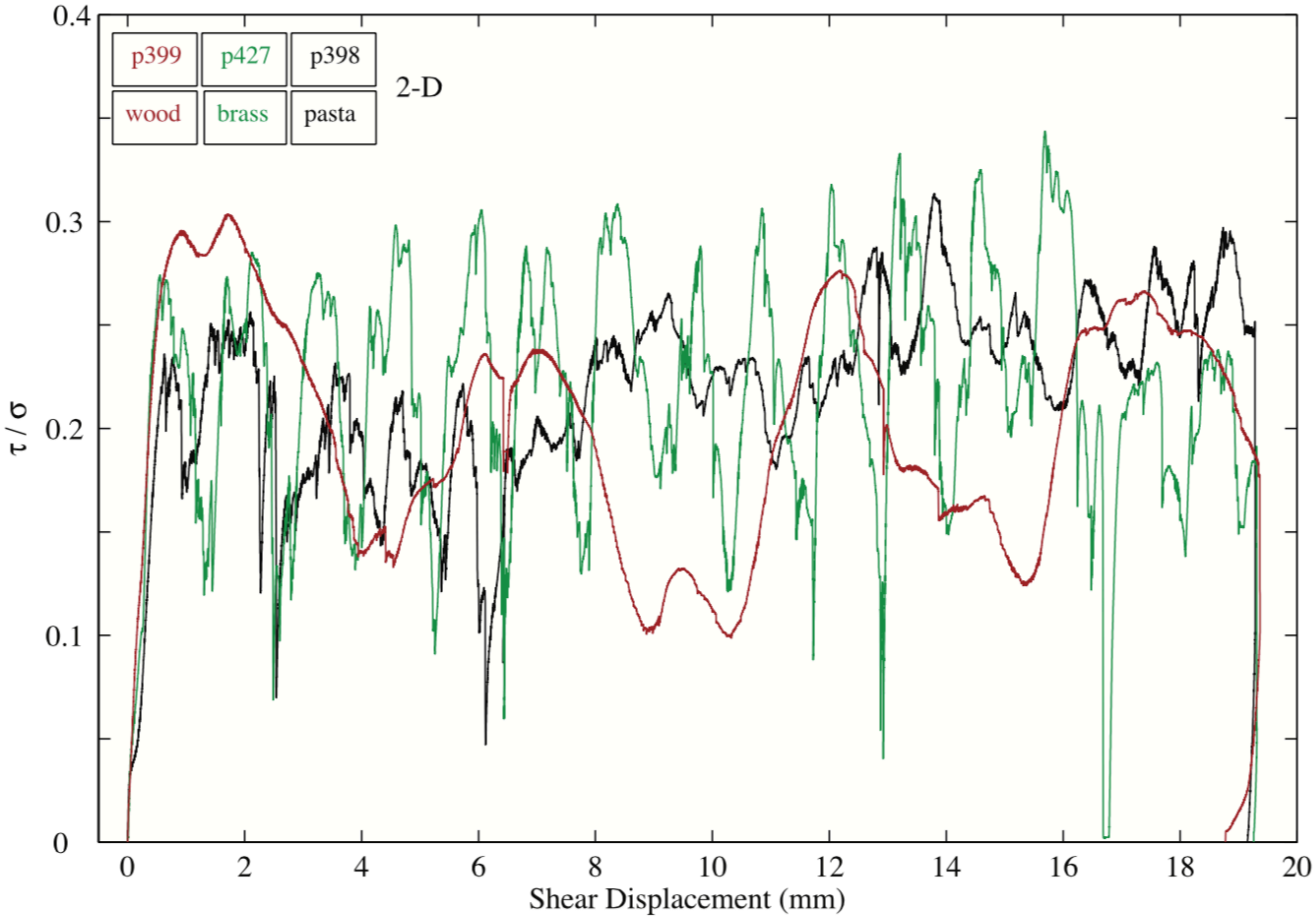
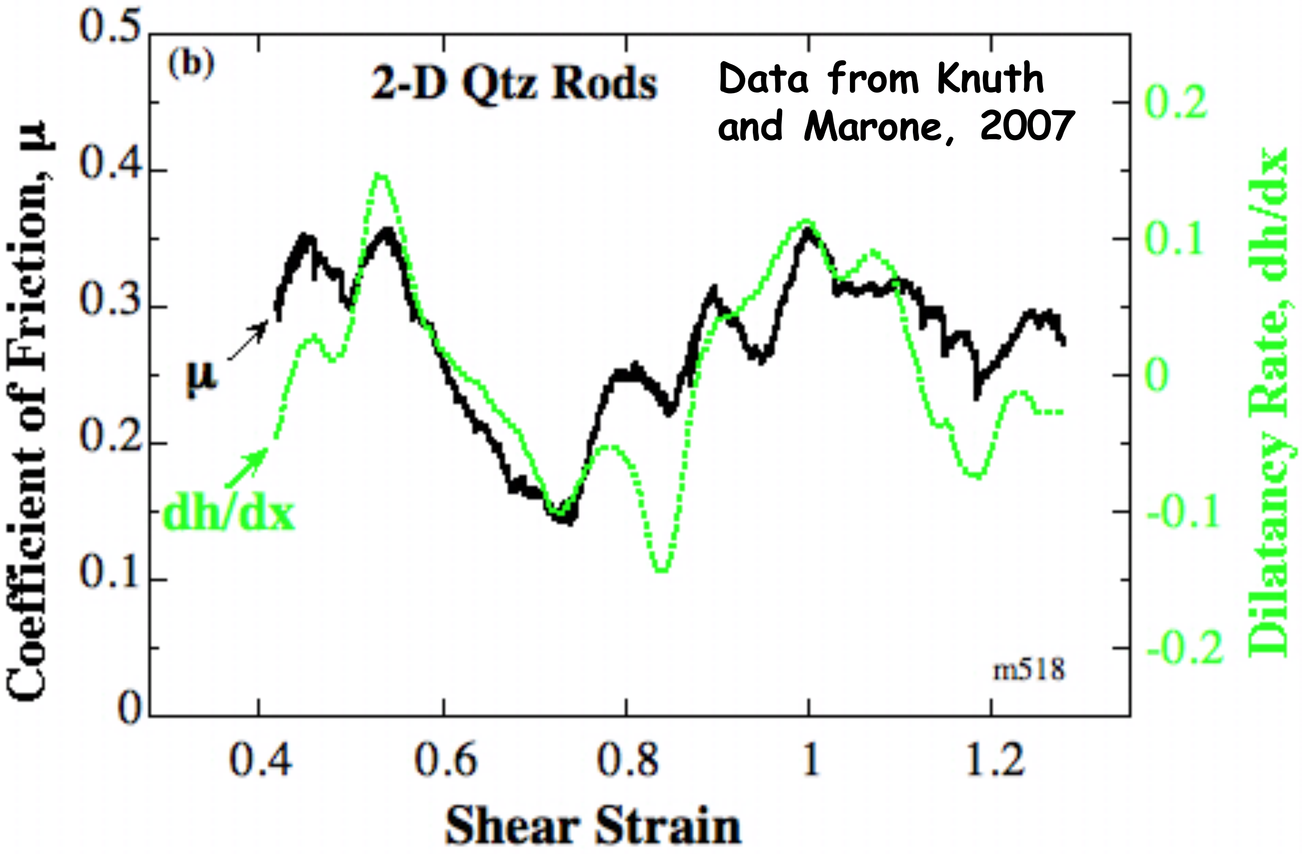
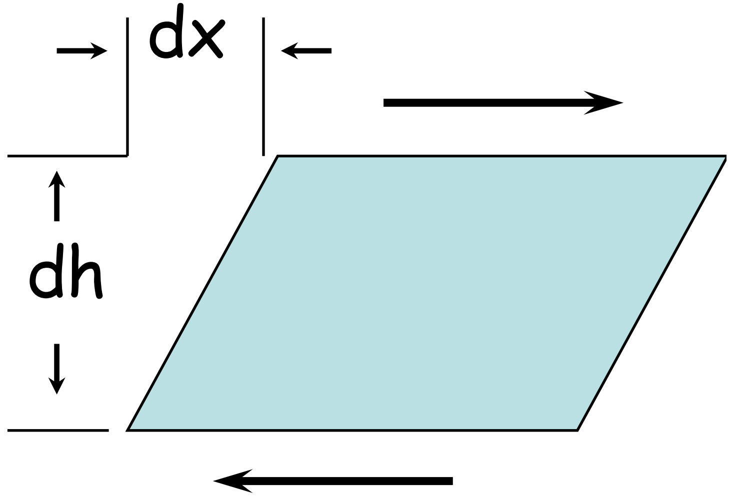


Figure 8. Comparison of granular materials in the 2-D configuration. Wood layers exhibit long wavelength, smooth variations. Pasta and brass exhibit higher-frequency variation. In all cases, sliding is stable and without abrupt, audible stick-slips.



Friction mechanics of 2-D particles

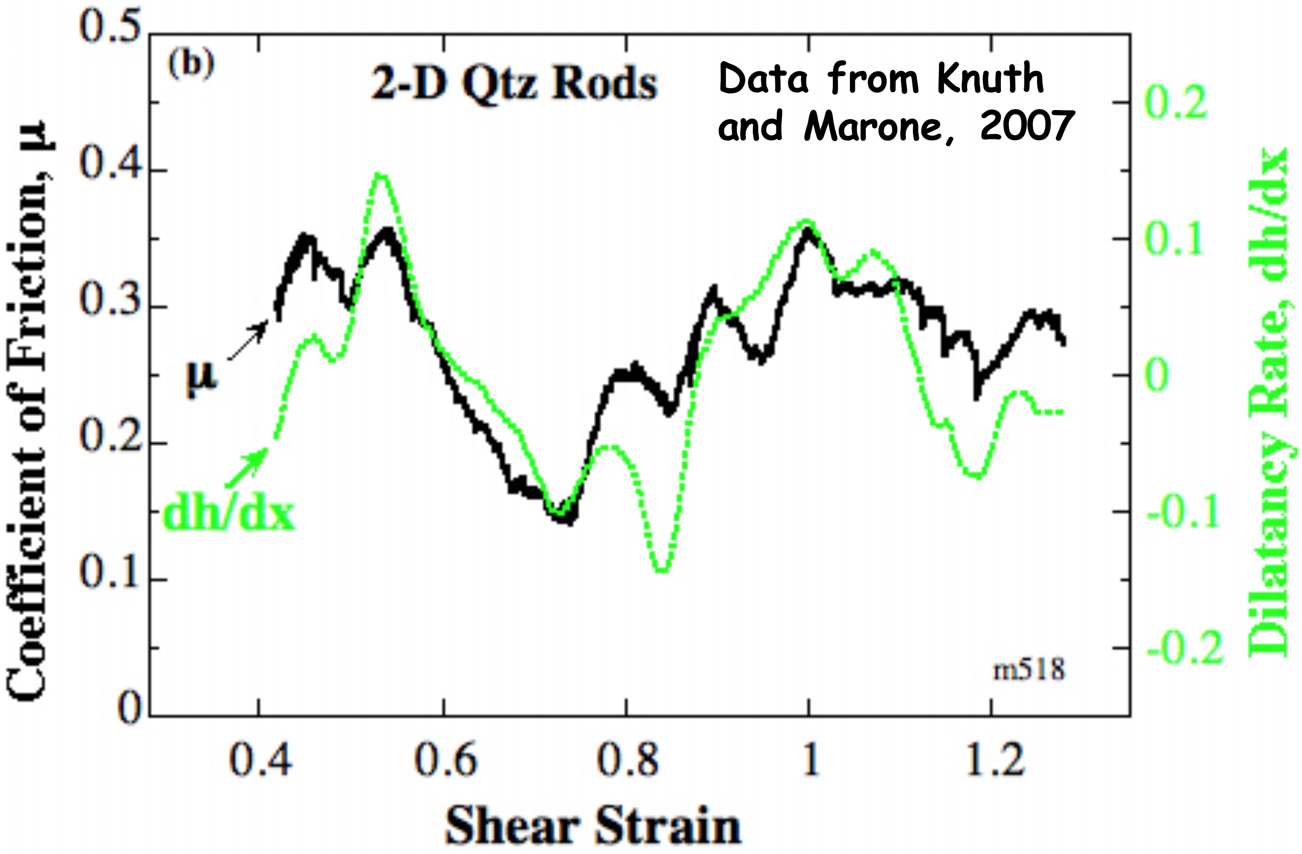


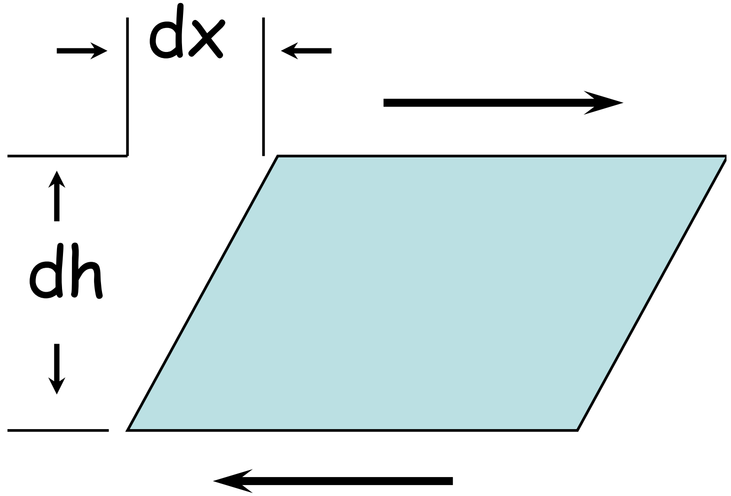
$$W = \tau_p \, d\gamma + \sigma \, d\theta$$

$$\tau = \sigma\left(\mu_p + d\theta / d\gamma\right)$$

$$d\theta = dV / V \; ; \; d\gamma = dx / h$$

Friction mechanics of 2-D particles



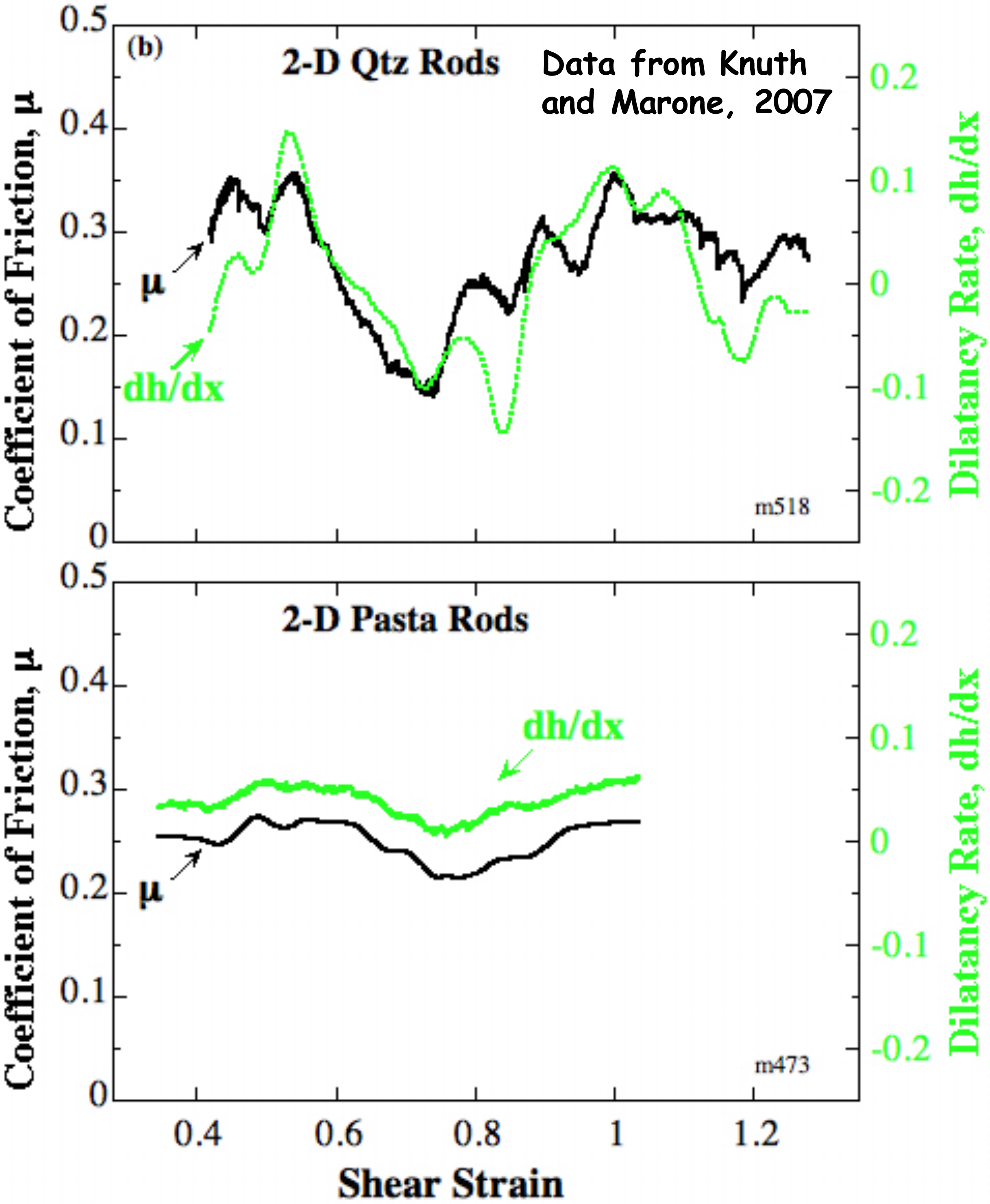


$$W = \tau_p \, d\gamma + \sigma \, d\theta$$

$$\tau = \sigma\left(\mu_p + d\theta / d\gamma\right)$$

$$\tau = \sigma\left(\mu_p + dh / dx\right)$$

- Dilatancy rate plays an important role in setting the frictional strength

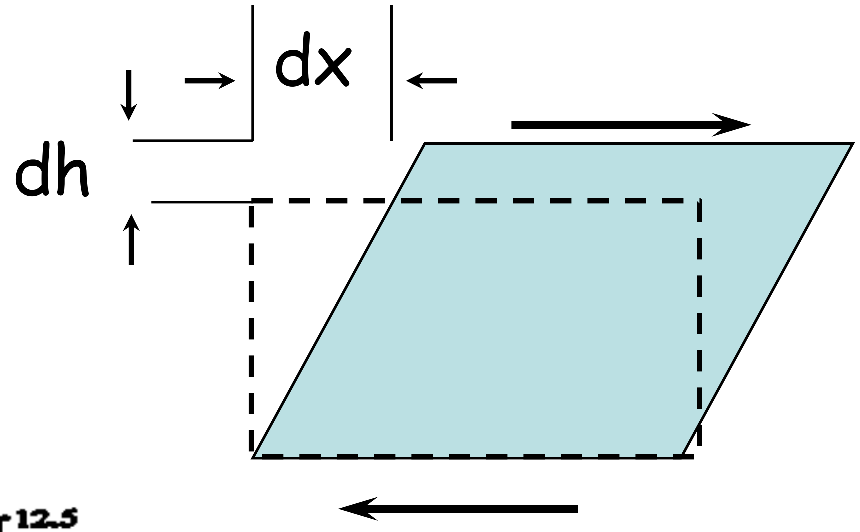


- Macroscopic variations in friction are due to variations in dilatancy rate.
- Smaller amplitude fluctuations in dilatancy rate produce smaller amplitude friction fluctuations.

$$W = \tau_p \; d\gamma + \sigma \; d\theta$$

W is total work of shearing

$$W = \tau \, d\gamma = \sigma \, \mu \, d\gamma$$



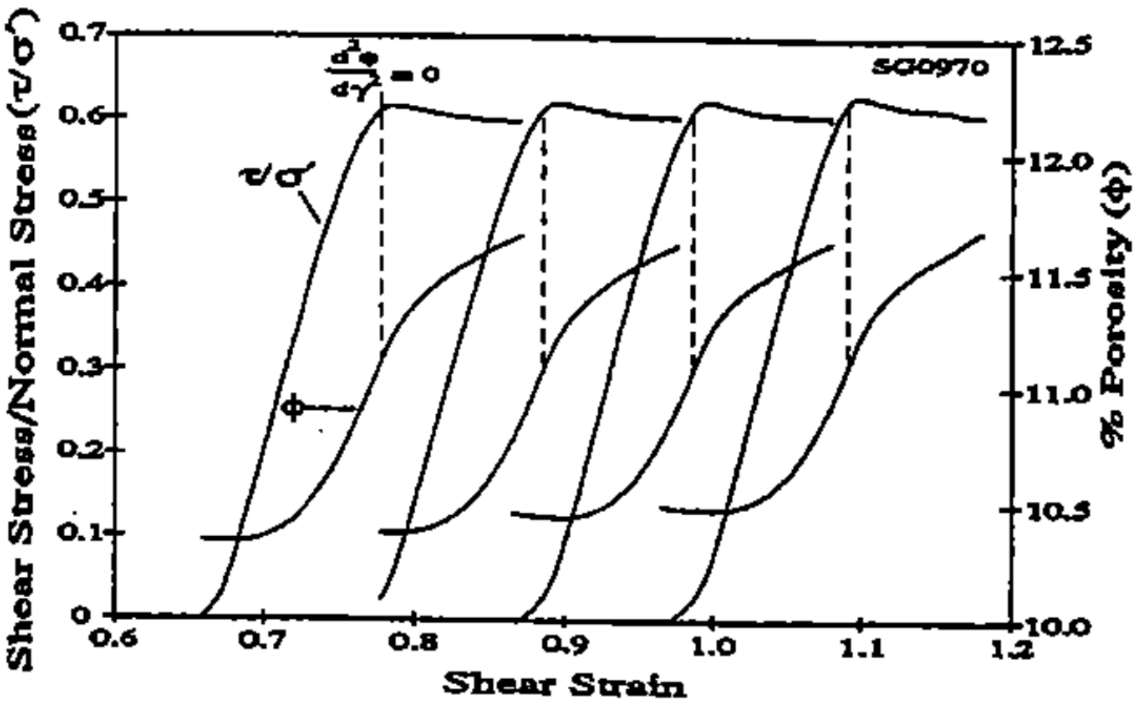


Fig. 8. An enlargement of the later stage of Figure 3. The curves have been truncated at the onset of unloading for clarity. Unlike the behavior at small strain (shown in Figure 7), neither $d\phi/d\gamma L$ nor the form of the porosity-strain curves varied with progressive cycling. Dashed lines show the point at which $d^2\phi/d\gamma^2 = 0$.

Marone et al., 1990

Mead, 1925
(Geologic Role of Dilatancy)

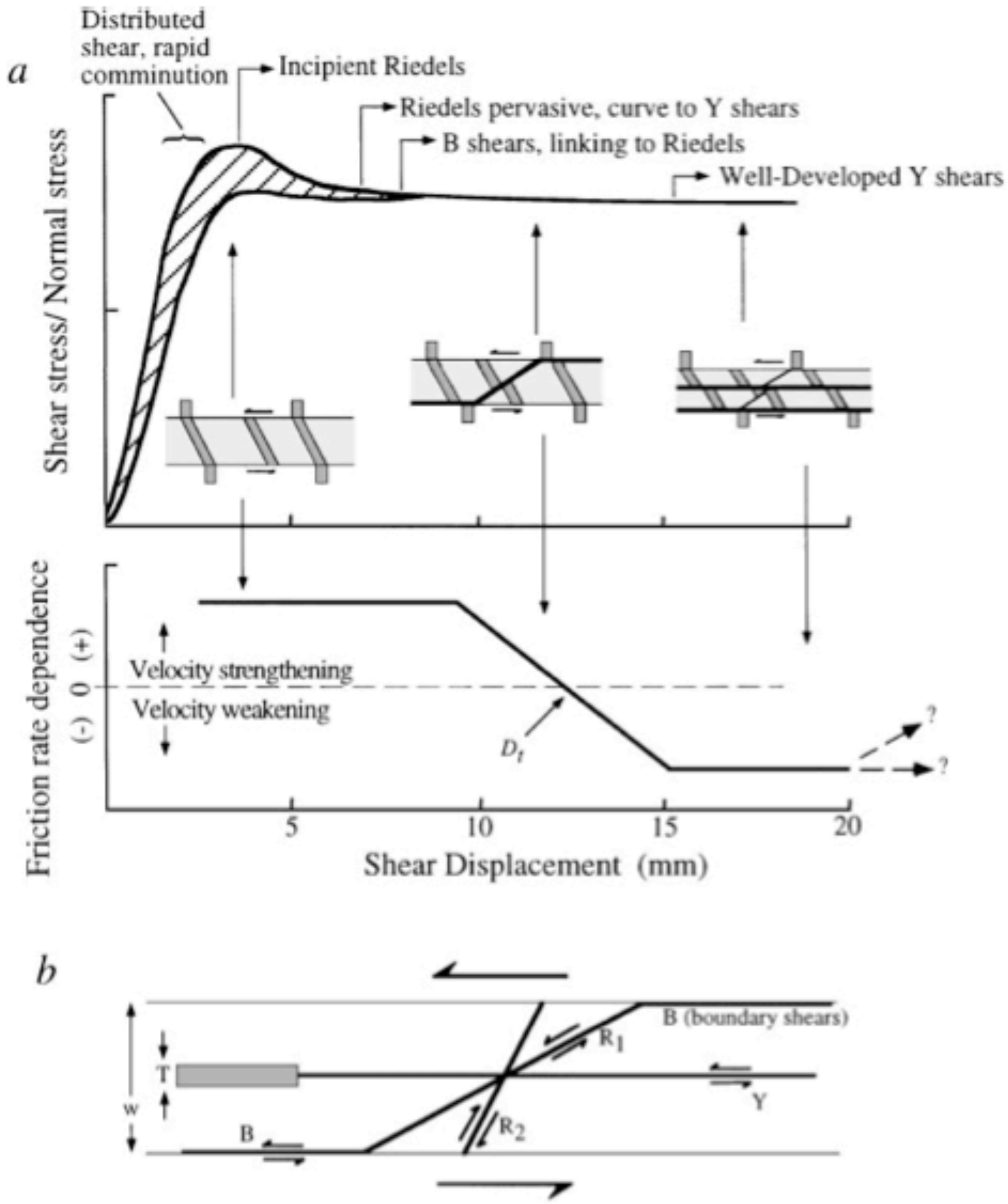
Shear Localization

Strain homogeneity depends on whether dilatancy is restricted

- Homogeneous strain if dilatancy is not opposed
- Strain localization if deformed under finite confining pressure

Shear Bands Form if:

$$\frac{\partial \theta^{SB}}{\partial \gamma} < \frac{\partial \theta^{homo.}}{\partial \gamma}$$



Marone, 1998

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Shear Localization

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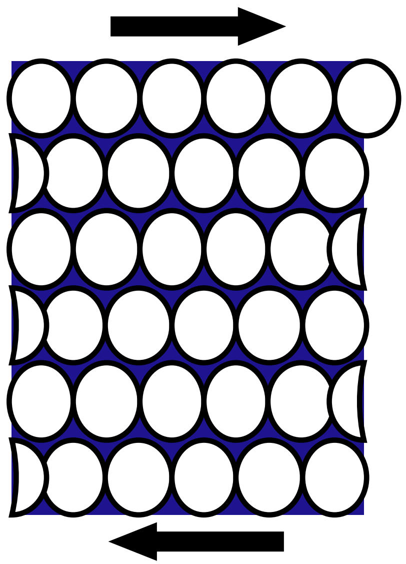
$$W = \tau_p \; d\gamma + \sigma \; d\theta$$

$$\tau = \sigma\left(\mu_p + d\theta / d\gamma\right)$$

Shear strength depends on friction and dilatancy rate

Shear Bands Form if:

$$\frac{\partial \theta^{SB}}{\partial \gamma} < \frac{\partial \theta^{homo.}}{\partial \gamma}$$



Deformation mode (degree of strain localization) minimizes dilatancy rate