Mechanics of Earthquakes and Faulting

28 Jan. 2021

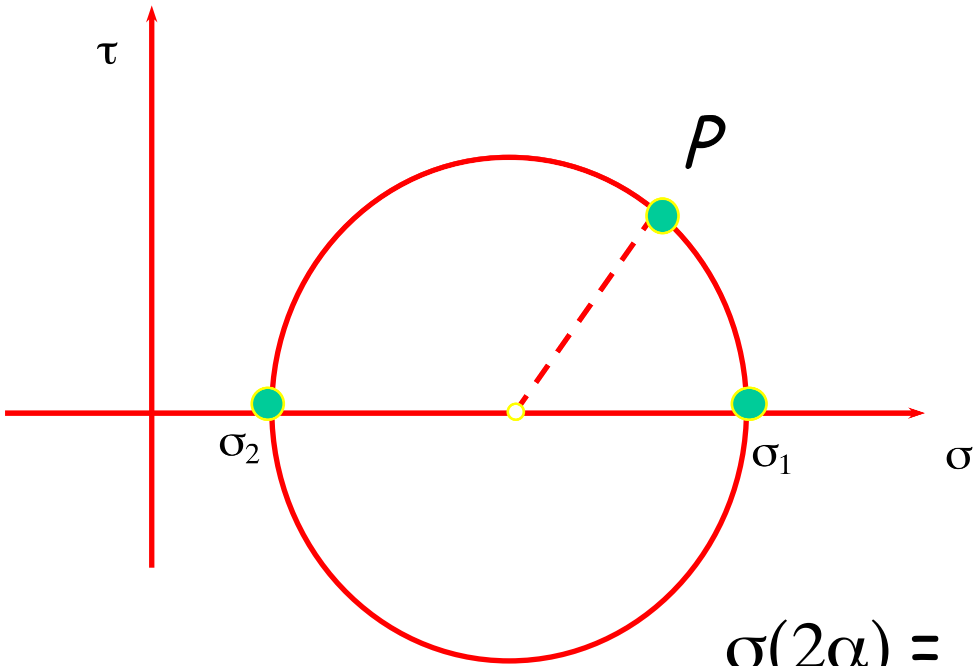
www.geosc.psu.edu/Courses/Geosc508

- Brittle Failure, Stress-Strain curves for some common material behaviors
- Crack mechanics and stress concentrations
- Work of deformation, shear and volume strain

Imagine that you're in a restaurant with some friends. The owner stops by to say hello and after hearing that you're a geophysicist she challenges you to write down the Shear and Normal Stress on a Plane of Arbitrary Orientation given the principal stresses.

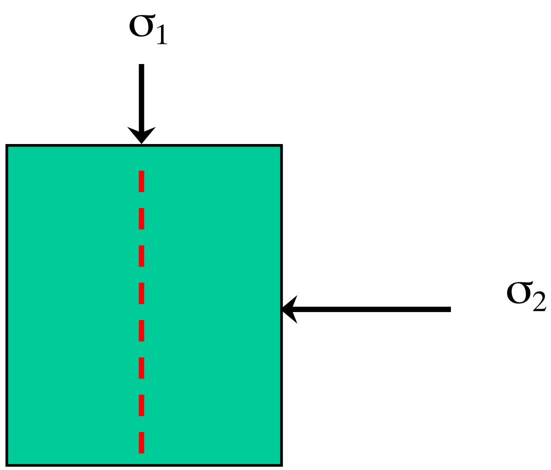
She calls the waiter over and he gives you a couple extra napkins and a pencil and says, don't worry about the third dimension because that's always in the fault plane for simple (Andersonian) faulting. So you know that you can just use two principal stresses. The maximum and minimum stress. Go ahead and call them σ_1 and σ_2

Ok, get to work! You've got to finish before he brings the drinks



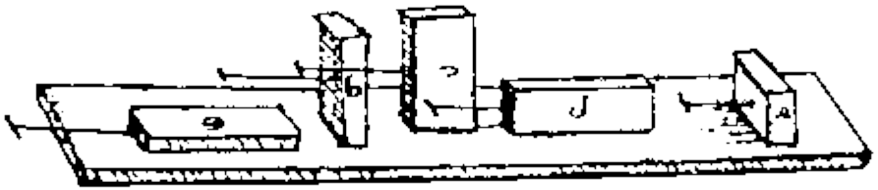
$\sigma(2\alpha) =$

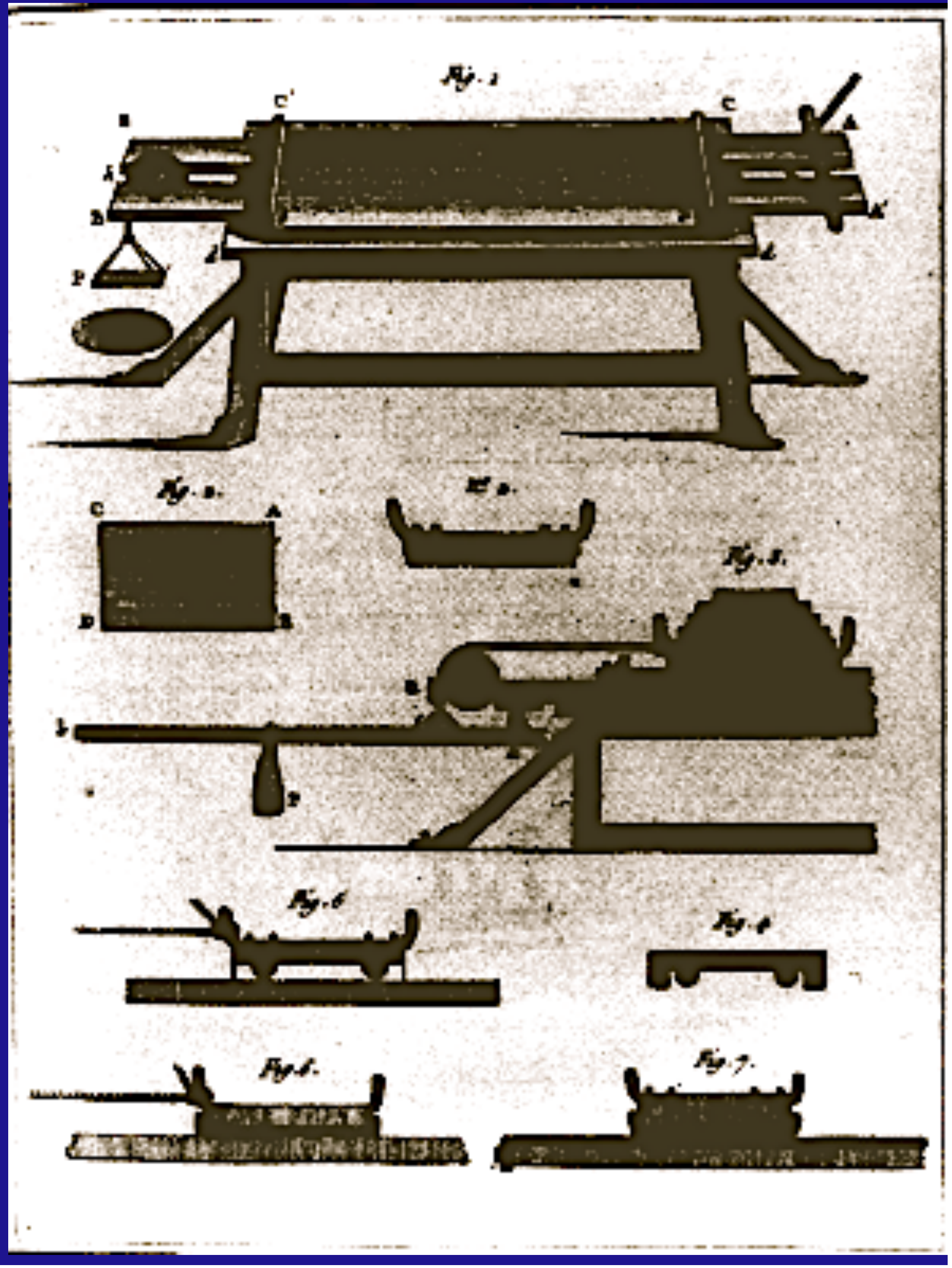
$\tau(2\alpha) =$



Sketch in plane P

C. A. Coulomb (1736-1806)



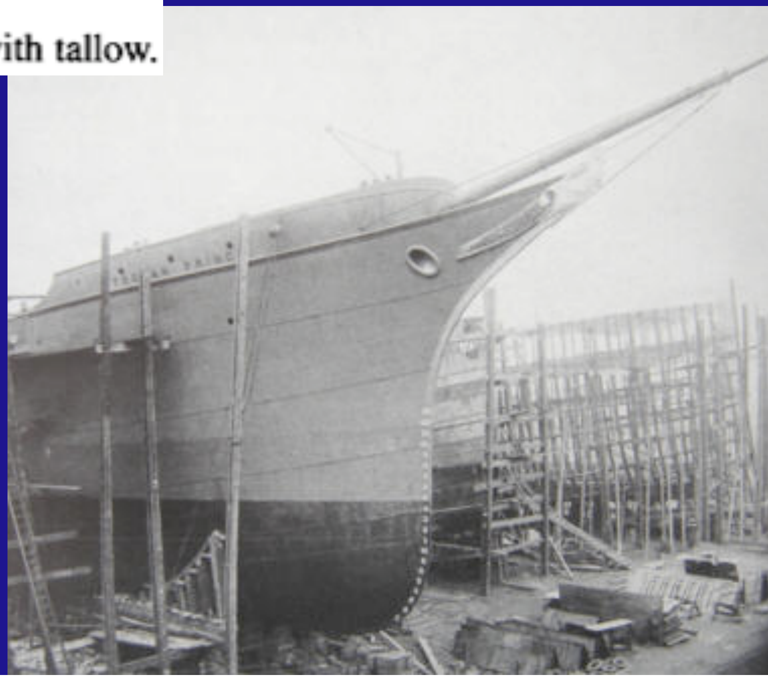


Brittle Failure and Time dependence of “static” friction

Table 9.1

	T (time of repose, min)	$A+mT^{\mu}$ (static friction force, lbf)
Ière observation	0	$A=502$
IIe	2	790
IIIe	4	866
IVe	9	925
Ve	26	1,036
VIe	60	1,186
VIIe	960	1,535

static friction of two pieces of well-worn oak lubricated with tallow.



Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of **Principal Stresses**:

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

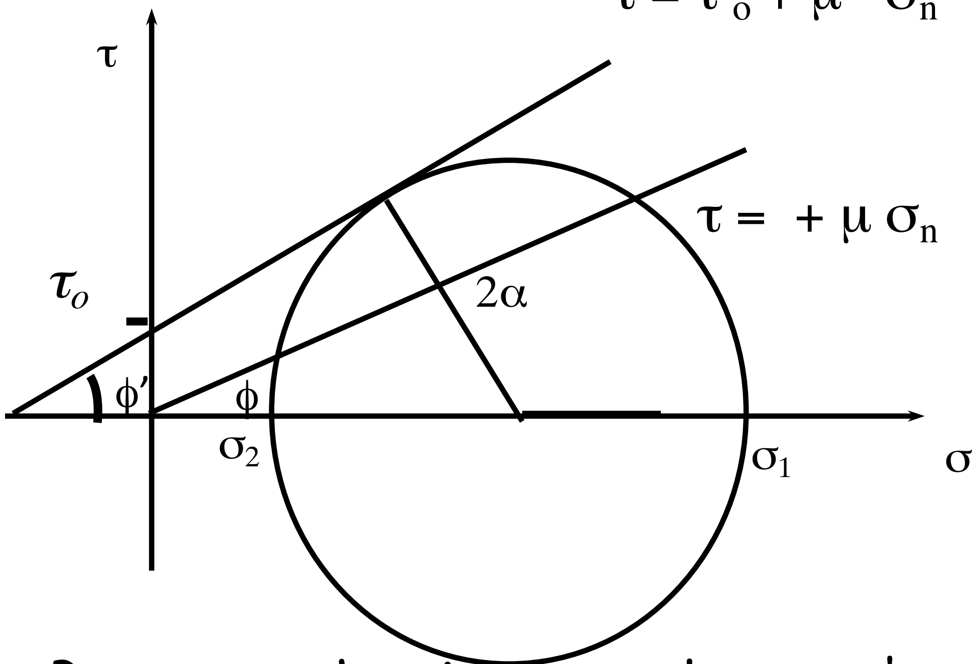
Mohr Circle.

FRACTURE

$\tau = \tau_o + \mu' \sigma_n$

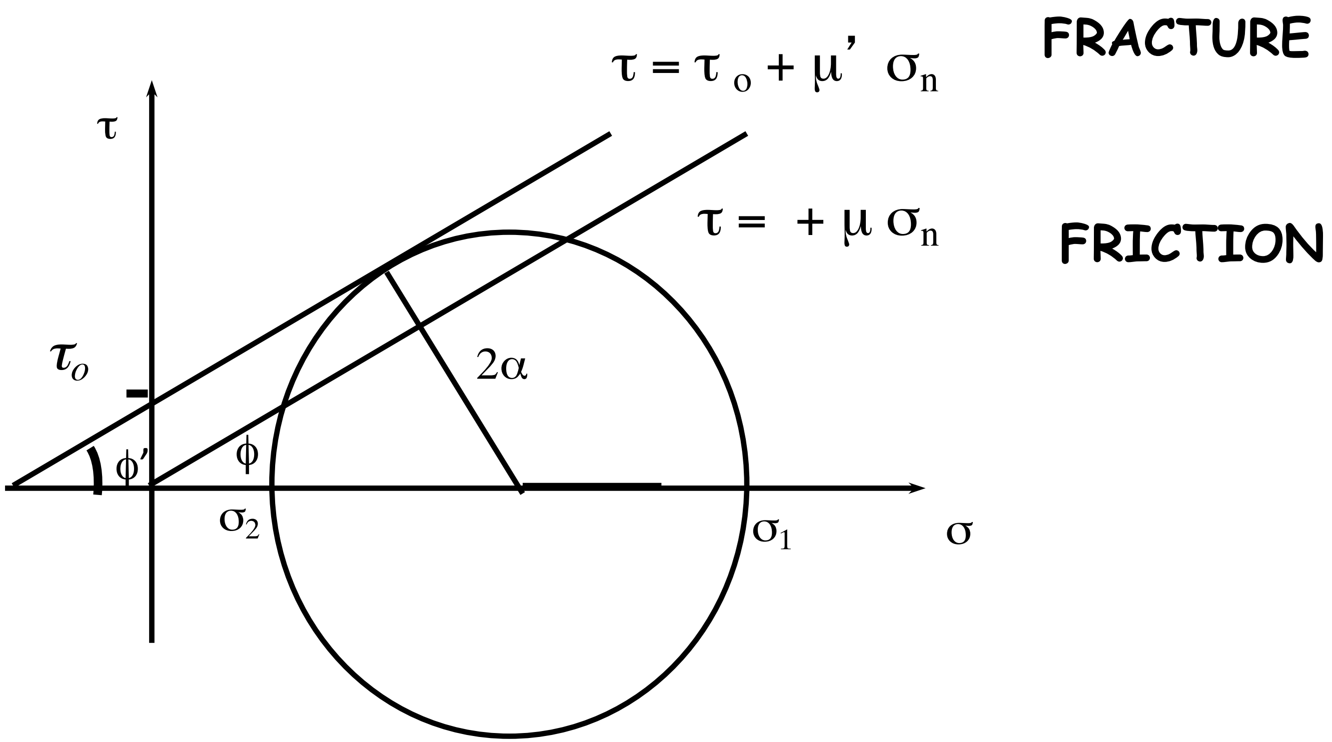
FRICTION

$\tau = + \mu \sigma_n$

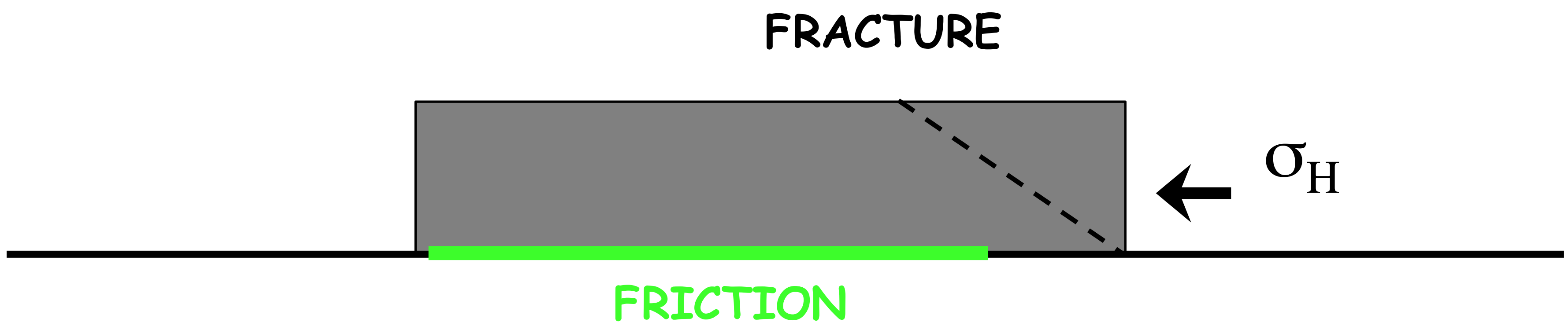


But, note that in general: $\mu \neq \mu'$ and $\phi \neq \phi'$

That is, the coefficient of sliding friction is not necessarily equal to the coefficient of internal friction.



What is the longest block that can be slid in frictional contact?
Fracture will occur first for very long blocks



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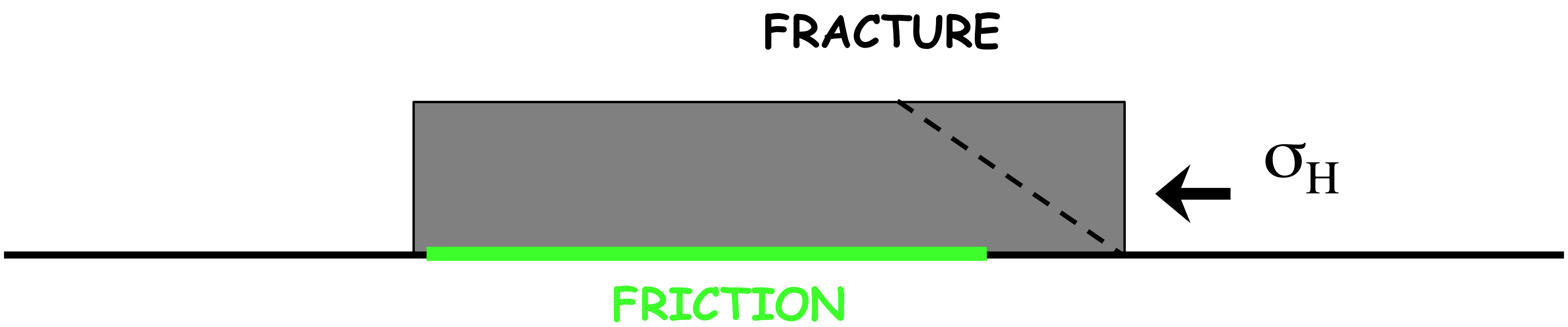

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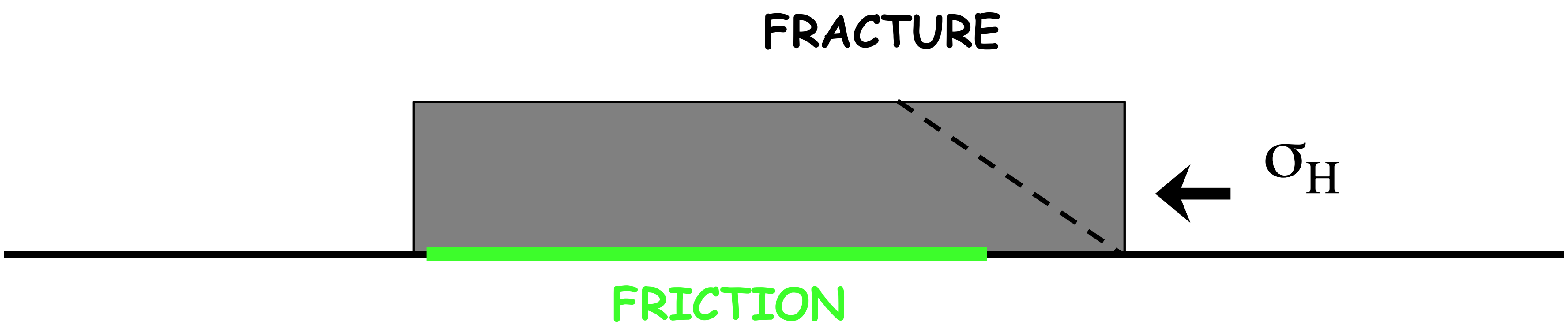

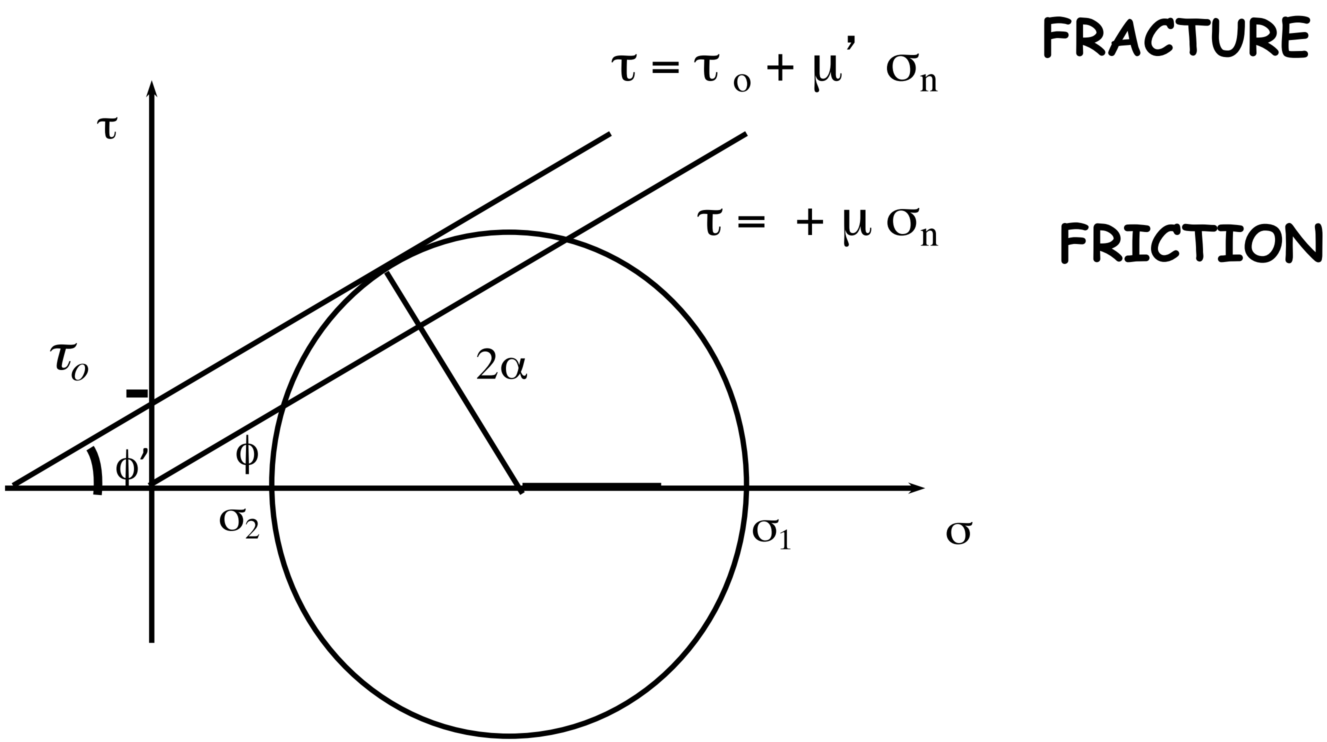

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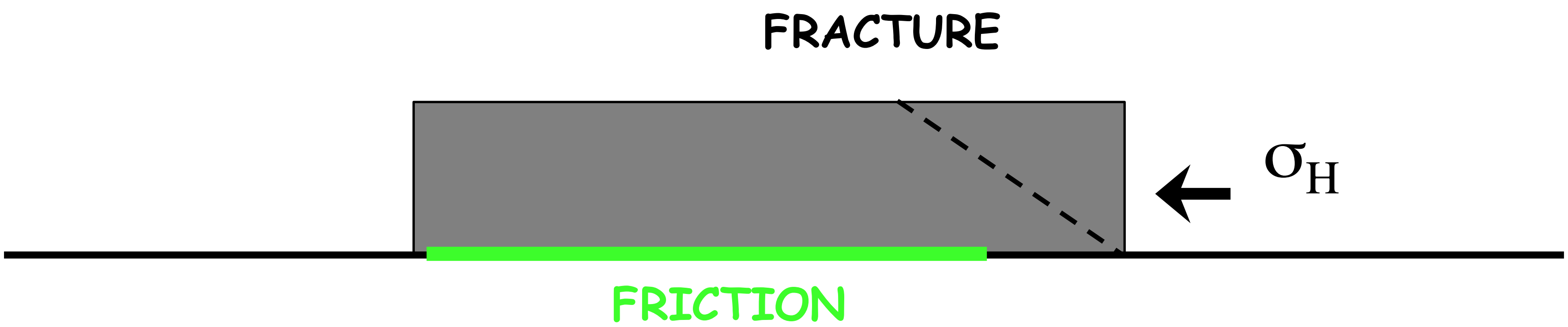
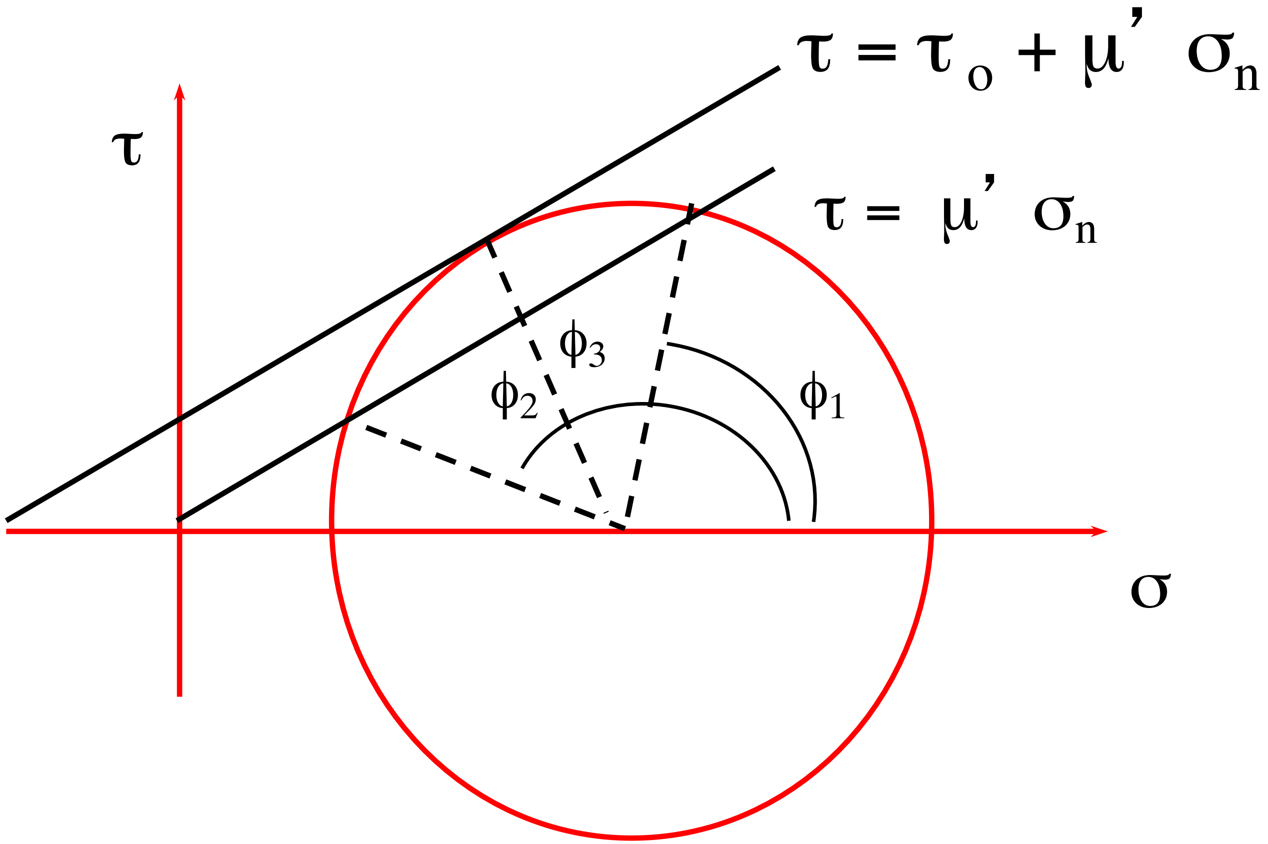


The Coulomb and Frictional failure criteria may be considered together, on a Mohr diagram

This shows that pre-existing planes of weakness, of orientations from ϕ_1 to ϕ_2, will fail by frictional slip prior to a new fracture forming at orientation ϕ_3.

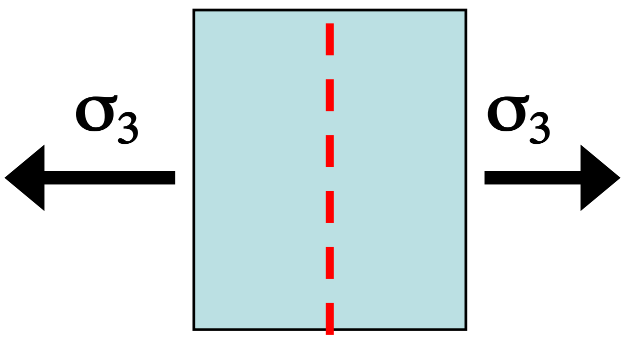


Macroscopic Failure Criteria: Faulting, Fracture, Friction

These are not in general rheologic laws, but rather relationships between principal stresses (or applied stresses) at failure.

In tension, we have failure at σ_3 = -To, where To is the tensile strength.

How does tensile strength depend on pressure, applied stresses, temperature?

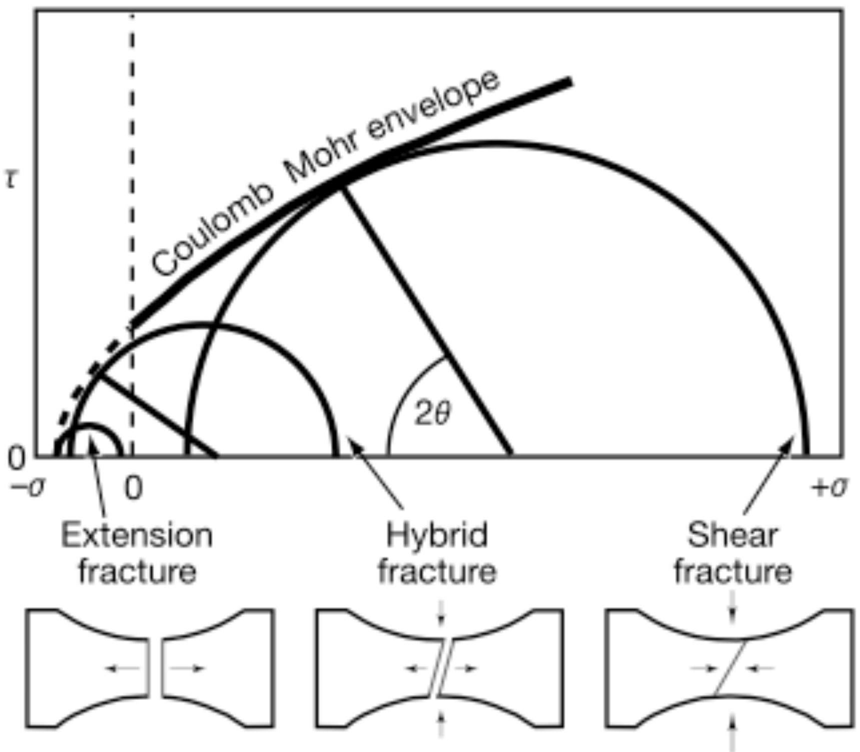


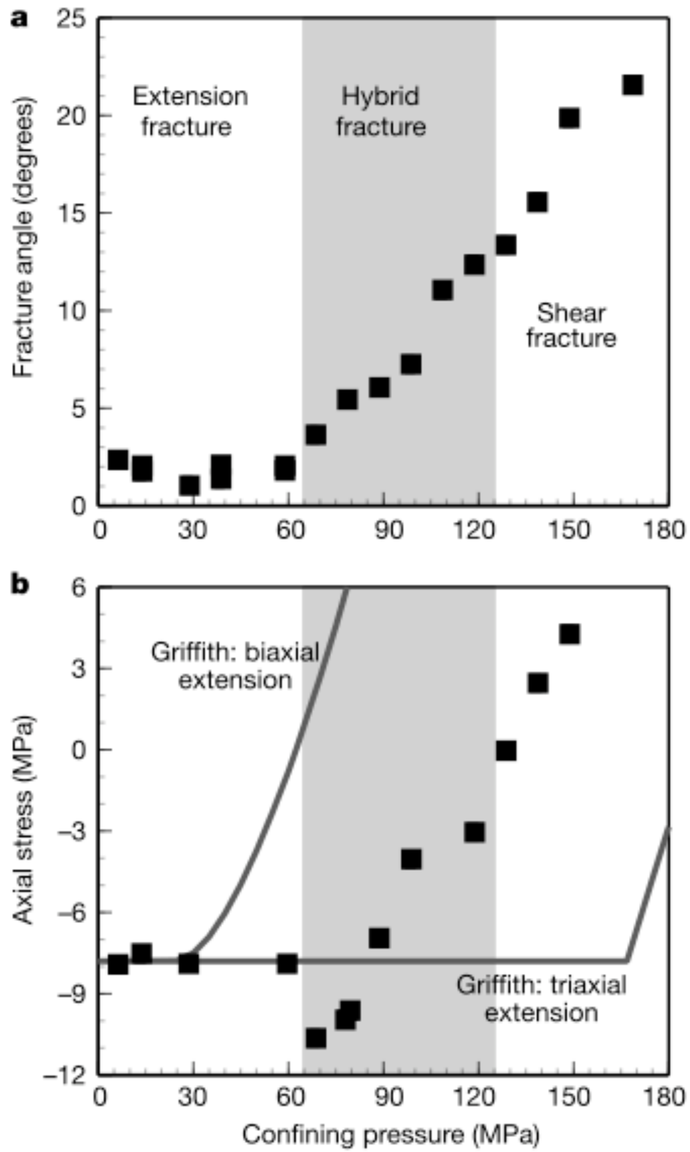
What about under compression?

- Pressure-dependent brittle failure
- Failure stress is higher under higher normal stress.

Handin, J. (1969), *On the Coulomb-Mohr failure criterion*, JGR. *74*, 5343-5348

Griffith, Modified Griffith: to explain curvature and transitional (hydrid) fractures. Based on stress concentrations at crack tips.



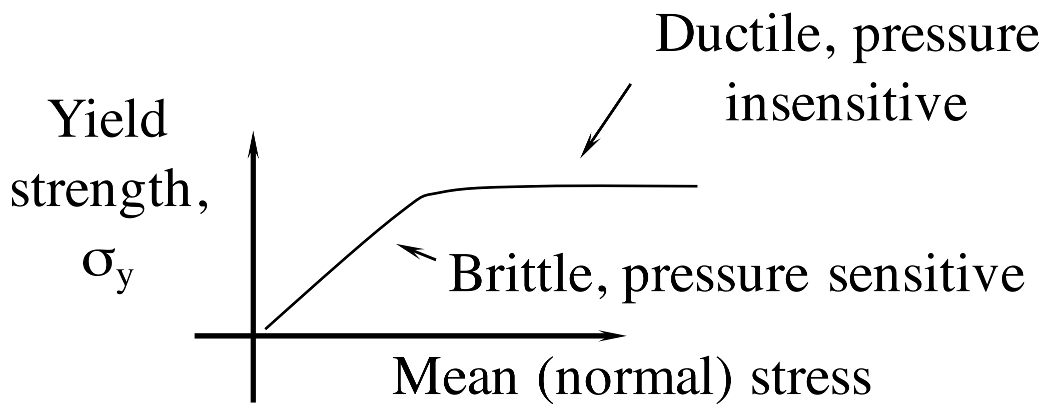


Ramsey & Chester: Hybrid fracture and the transition from extension fracture to shear fracture, *Nature* **428**, 63-66 (4 March 2004)

Rheology and Deformation. Definitions.

The terms brittle and ductile can be defined in a number of ways. One def. is given by the Coulomb Failure Criterion.

Another important operational definition involves the stress-strain characteristics and the dependence of strength on mean or normal stress.

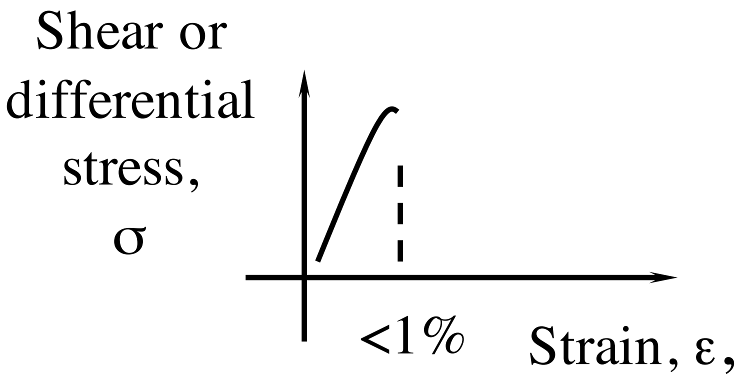


Brittle and Ductile (or plastic) deformation can be distinguished on the basis of whether the yield strength depends on pressure (mean stress or normal stress).

Why would yield strength depend on mean stress?

Rheology and Deformation. Definitions.

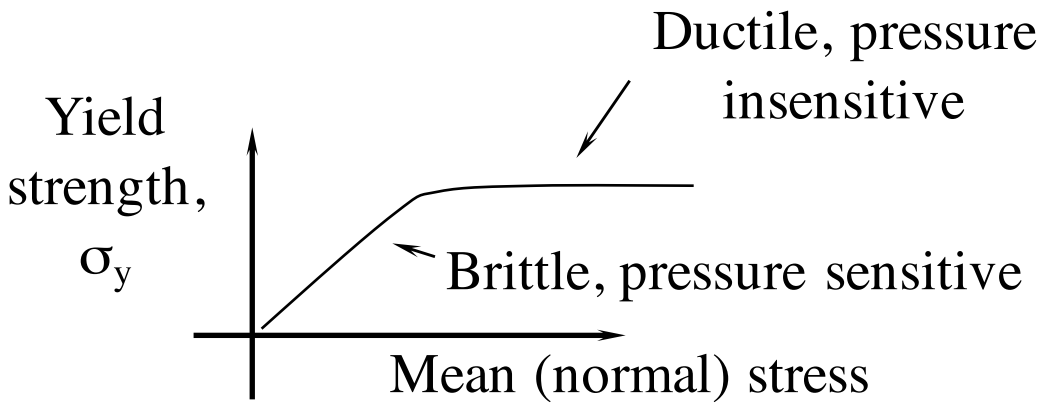
The term 'brittle' is also used to describe materials that break after very little strain. These materials have low fracture toughness.



Fracture toughness describes a materials ability to deform without breaking.

- Brittle materials (like glass or ceramics) have low toughness.
- Plastics have high toughness

What causes the pressure sensitivity of brittle deformation?

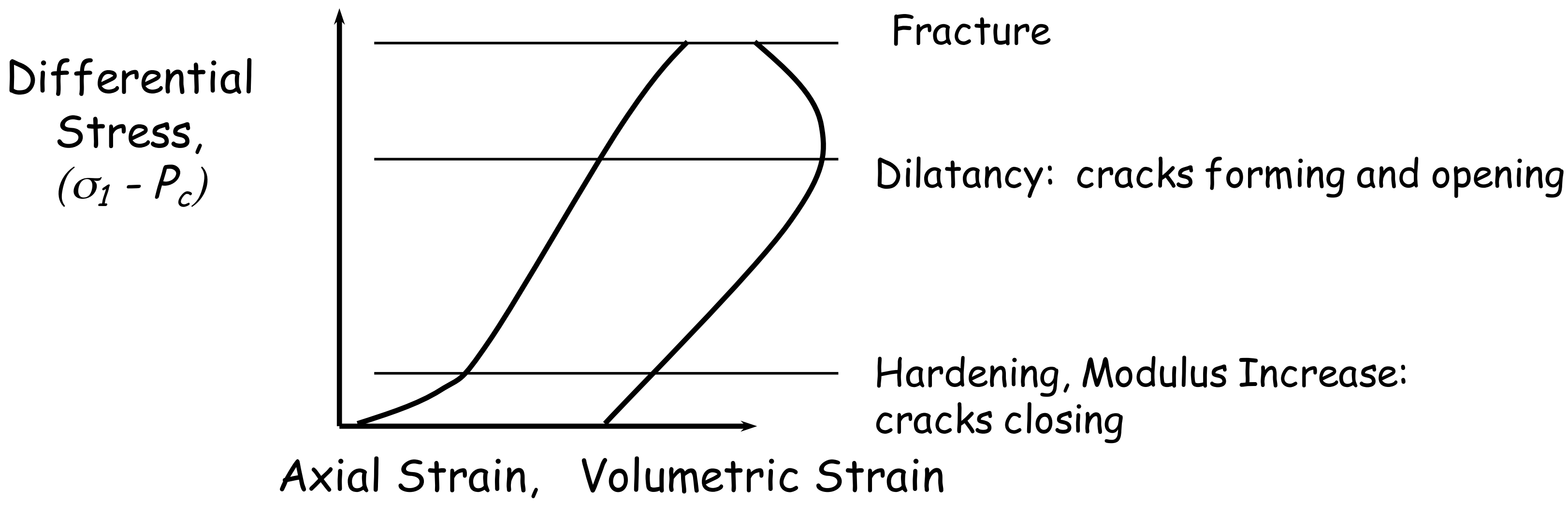


- Volume change. Brittle deformation involves volume change –dilatancy or compaction.
- 'Dilation' means volume increase. Dilatancy describes a shear induced volume increase. The term was introduced to describe deformation of granular materials – but dilation also occurs in solid brittle materials via the propagation of cracks.

- Work is done to increase volume against the mean stress during brittle deformation, thus the pressure sensitivity of brittle deformation.

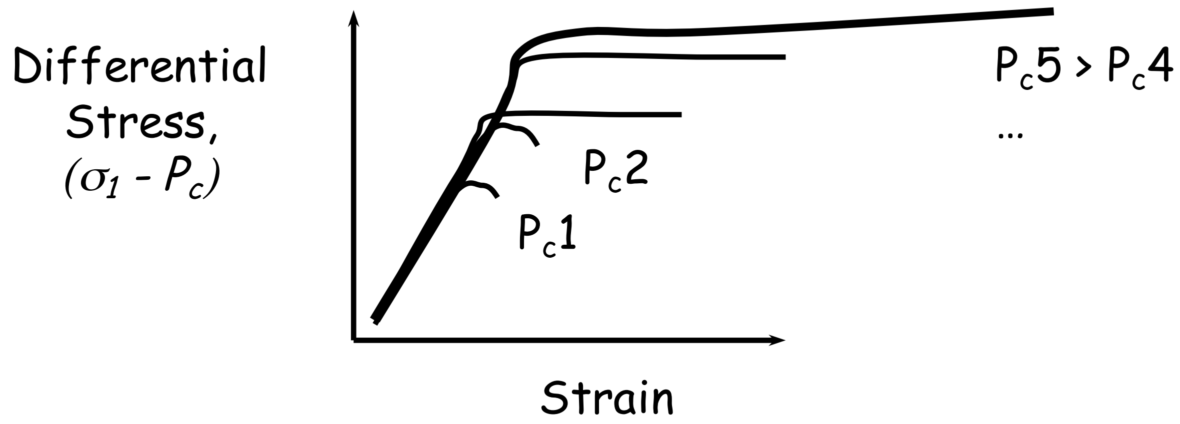
- Ductile deformation occurs without macroscopic volume change, due to the action of dislocations. Dislocation motion allows strain accommodation.

• Brittle deformation and dilatancy



Brace, Paulding & Scholz, 1966; Scholz 1968.

Stress-strain-failure curves

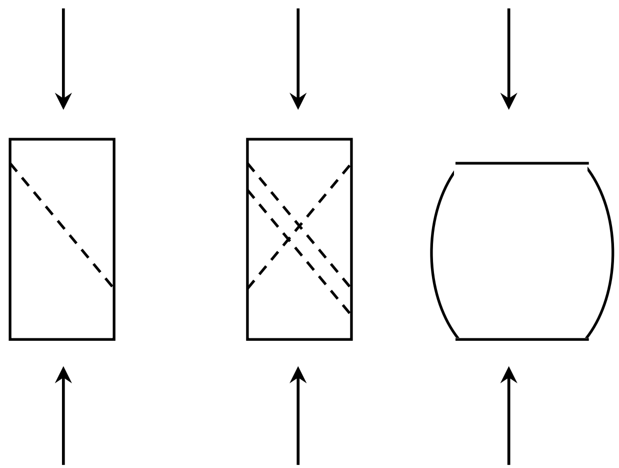


These styles can be loosely related to Brittle and Ductile deformation, respectively.

Brittle refers to pressure sensitive deformation

Brittle Failure: If we draw the stress-strain-failure curves for a range of confining pressures, we'll get a range of yield strengths, showing that σ_y is proportional to P_c.

With increasing confining pressure there is a transition from localized to more broadly distributed deformation.

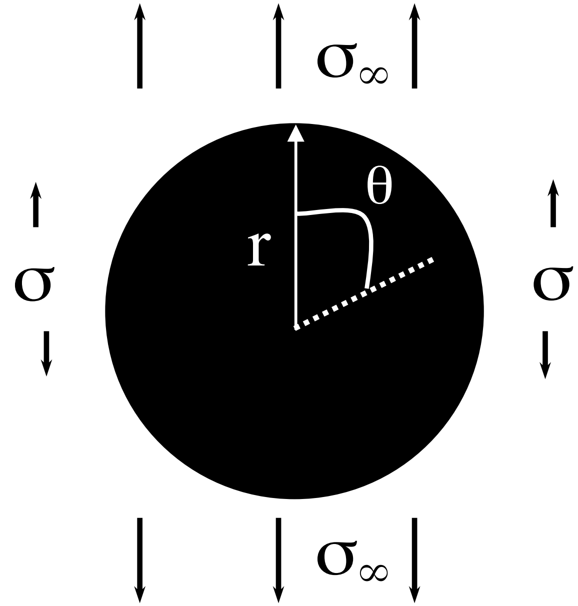


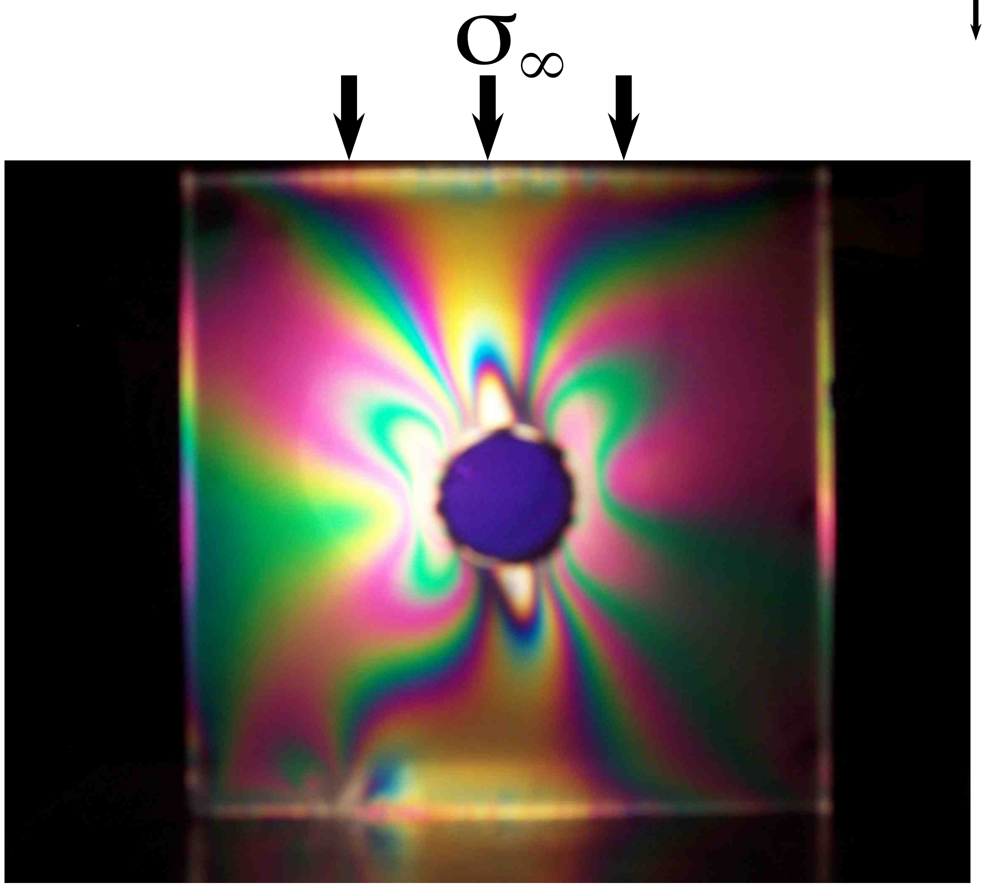
Full solution for a circular hole of radius r=a

$$\sigma_r = \frac{\sigma_\infty}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_\infty}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma_\infty}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_\infty}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

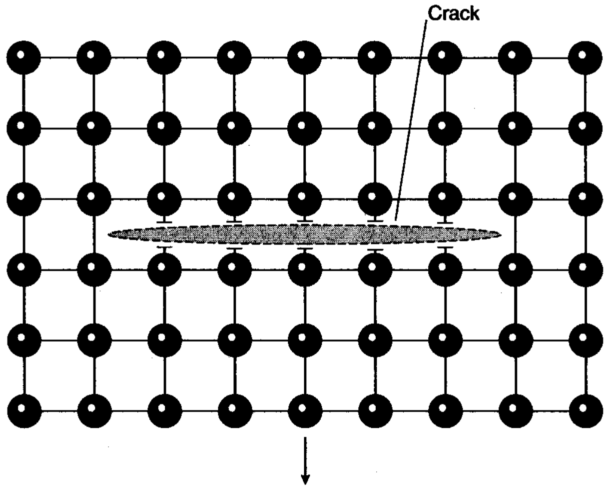
$$\tau_{r\theta} = -\frac{\sigma_\infty}{2}\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$



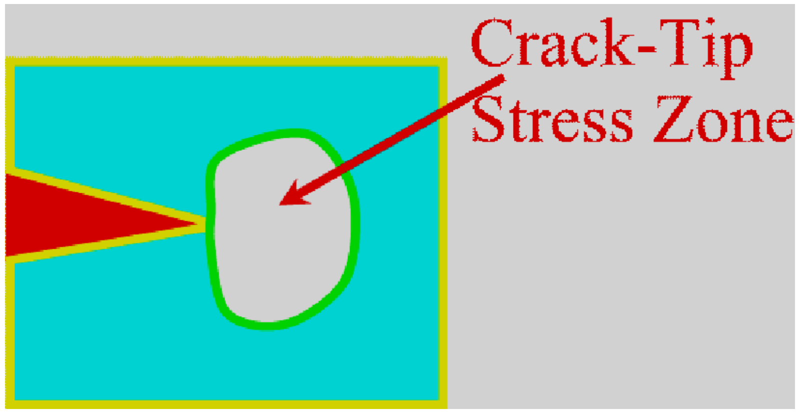


•Griffith proposed that all materials contain preexisting microcracks, and that stress will concentrate at the tips of the microcracks

•The cracks with the largest elliptical ratios will have the highest stress, and this may be locally sufficient to cause bonds to rupture



- As the bonds break, the ellipticity increases, and so does the stress concentration
- The microcrack begins to propagate, and becomes a real crack
- Today, microcracks and other flaws, such as pores or grain boundary defects, are known as Griffith defects in his honor



Bond separation and specific surface energy.

- Fracture involves creation of new surface area.
- The specific surface energy is the energy per unit area required to break bonds.

Two surfaces are created by separating the material by a distance $\lambda/2$ and the work per area is given by stress times displacement.

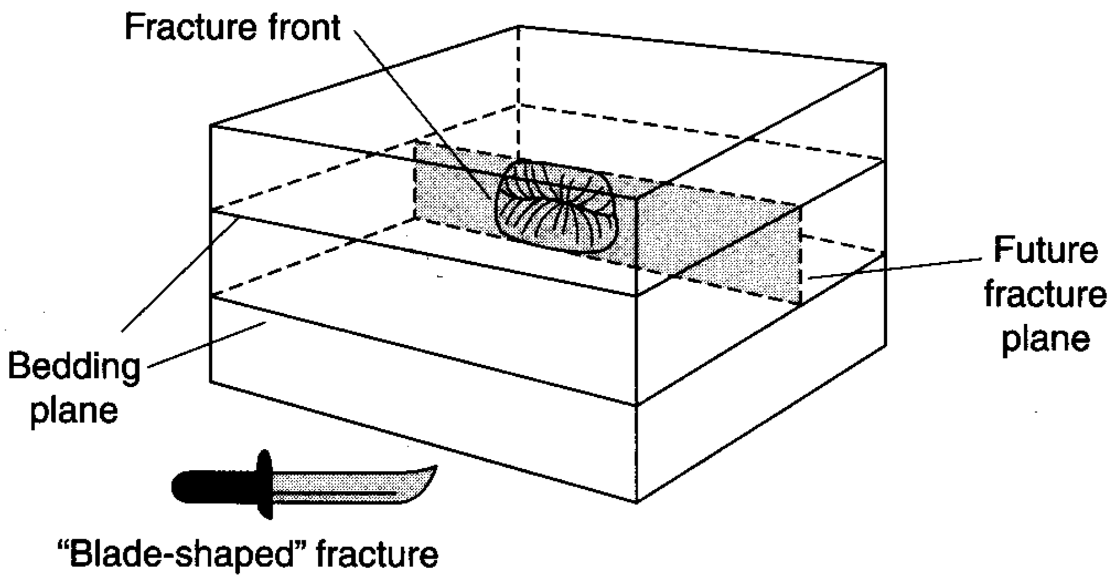
$$2\gamma = \int_0^{\frac{\lambda}{2}} \sigma_t \sin\left(\frac{2\pi(r-a)}{\lambda}\right) \; d(r-a)$$

$$= \frac{\lambda\sigma_t}{\pi}$$

This yields the estimate: $$\gamma = \frac{Ea}{4\pi^2}$$

The surface energy is a fundamental physical quantity and we will return to it when we talk about the energy balance for crack propagation and the comparison of laboratory and seismic estimates of G, the fracture energy.

Crack mechanics and crack propagation



(b)

Griffith posed the problem of crack propagation at a fundamental level, on the basis of thermodynamics.

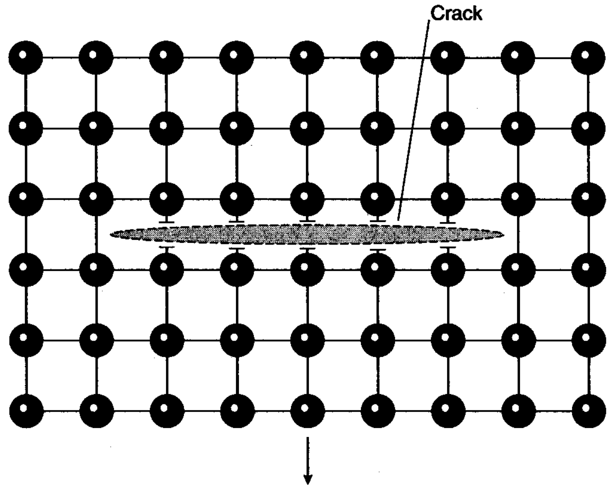
He considered the total energy of the system, including the region at the crack tip and just in front of a propagating crack.

Total energy of the system is U and the crack length is 2c, then the (cracked) solid is at equilibrium when $dU/dc = 0$

- Work to extend the crack is W
- Change in internal strain energy is U_e
- Energy to create surface area is U_s

Then: $U = (-W + U_e) + U_s$

Crack mechanics and crack propagation, Griffith theory



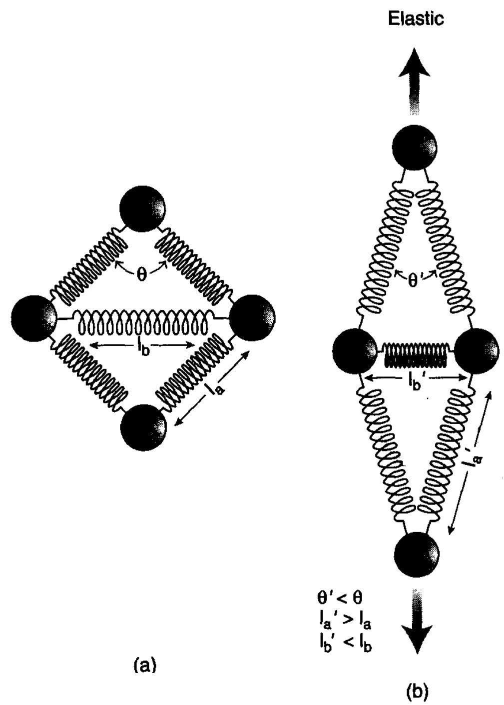
- Work to extend the crack is W
- Change in internal strain energy is U_e
- Energy to creation surface area is U_s

Then: $U = (-W + U_e) + U_s$

- Mechanical energy $(-W + U_{\underline{e}})$ decreases w/ crack extension. This is the *energy supply* during crack extension.
- $(-W + U_{\underline{e}})$ may come from the boundary or from local strain energy.

- The decrease in mechanical energy is balanced by an increase in surface energy (Us is related to specific surface energy, γ, discussed above.

- The crack will extend if $dU/dc < 0$



Energy balance for crack propagation, Griffith theory

$U = (-W + U_e) + U_s$

- Crack will extend if $dU/dc < 0$
- System is at equilibrium if $dU/dc = 0$

Consider a rod of length y, modulus E and unit cross section loaded in tension:

- Internal energy is: $U_e = y\sigma^2/2E$, for uniform tensile stress σ
- For a crack of length 2c, internal strain energy will increase by $\pi c^2\sigma^2/E$

- Introduction of the crack means that the rod becomes more compliant:
  - The effective modulus is then: $E' = yE/(y+2\pi c^2)$

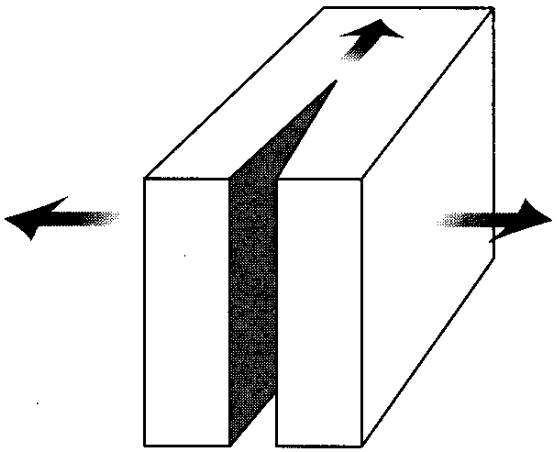
- The work to introduce the crack is: $W = \sigma y(\sigma/E' - \sigma/E) = 2\pi c^2\sigma^2/E$
- Change in surface energy is $U_s = 4c\gamma$

- Thus: $U = -\pi c^2\sigma^2/E + 4c\gamma$,
- At equilibrium: the critical stress for crack propagation (failure stress) is: $\sigma_f = (2E\gamma/\pi c)^{1/2}$

$U = (-W + U_e) + U_s$

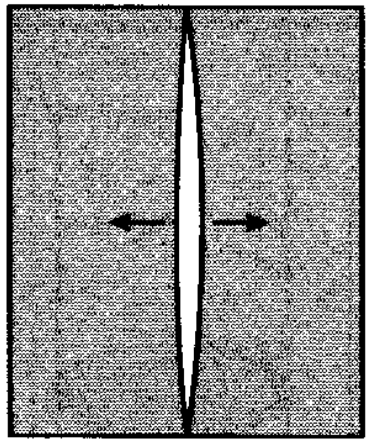
- Crack will extend if $dU/dc < 0$
- System is at equilibrium if $dU/dc = 0$

- The critical stress for crack propagation (failure stress): $\sigma_f = (4E\gamma/\pi c)^{1/2}$



(a)

$$\sigma_\infty = \sqrt{\frac{E\gamma}{4c}}$$

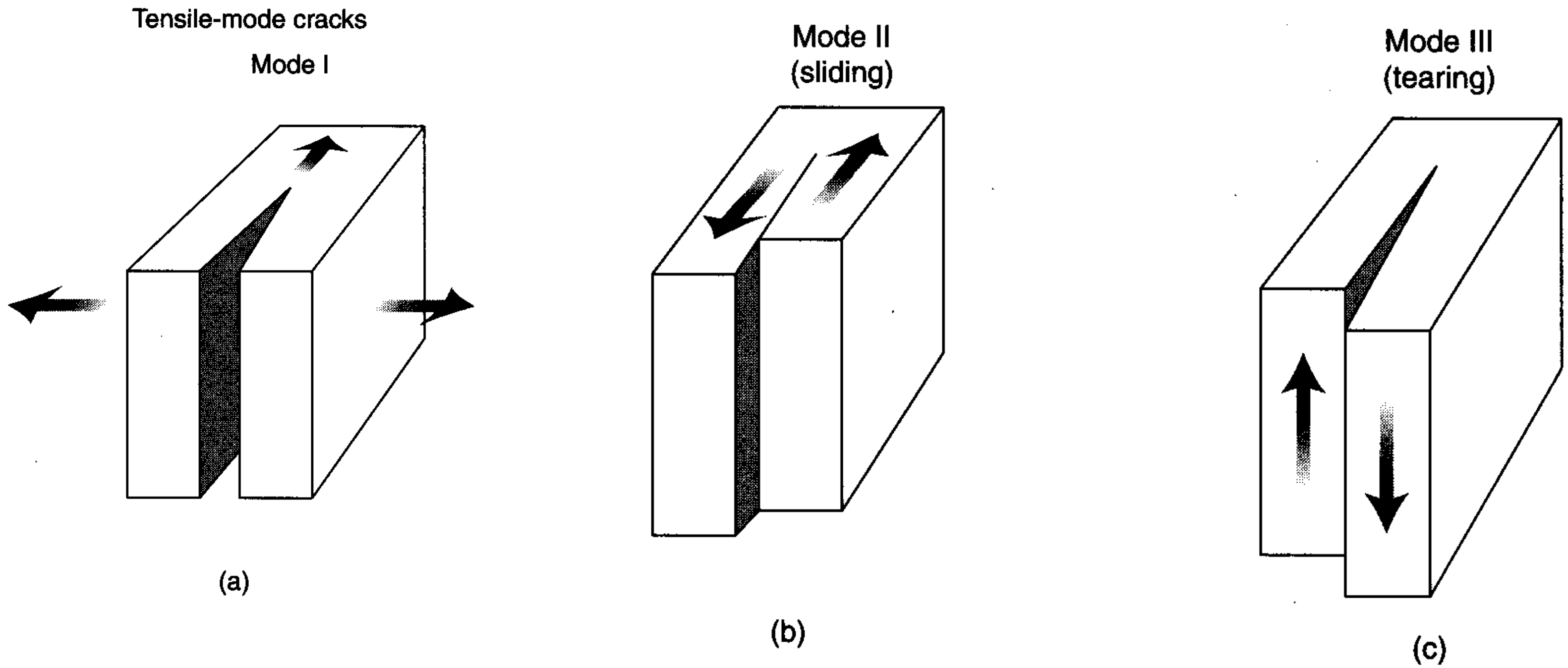


(b)

Taking σ_∞ of 10 MPa, E= 10 GPa and γ of 4×10^{-2} J/m^2, gives a crack half length c of 1 micron.

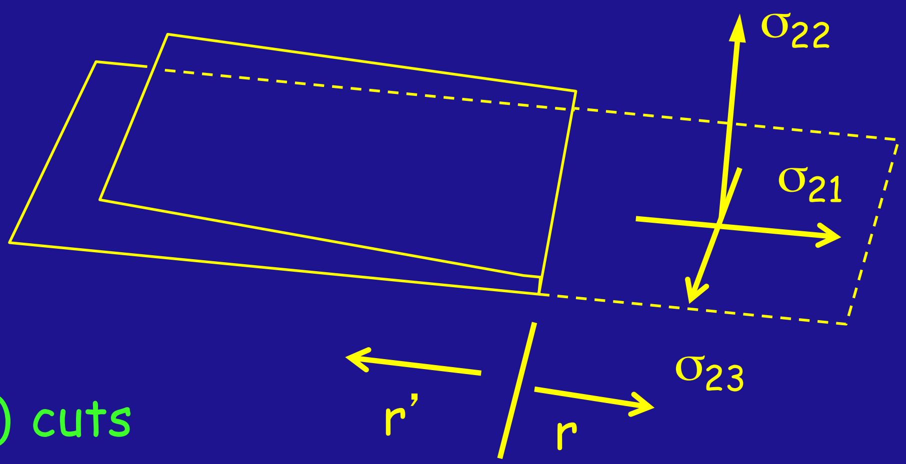
$U = (-W + U_e) + U_s$

- Crack will extend if $dU/dc < 0$
- System is at equilibrium if $dU/dc = 0$



Fracture Mechanics and Stress intensity factors for each mode

K_I, K_{II}, K_{III}



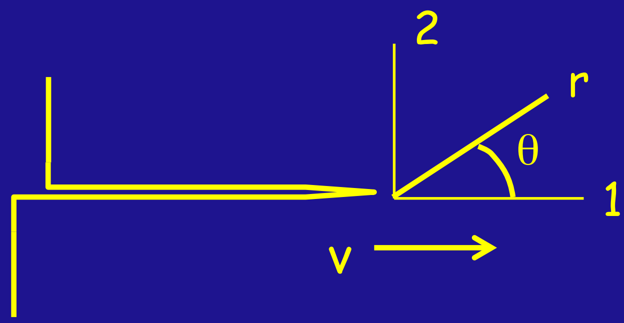
Linear Elastic Fracture Mechanics

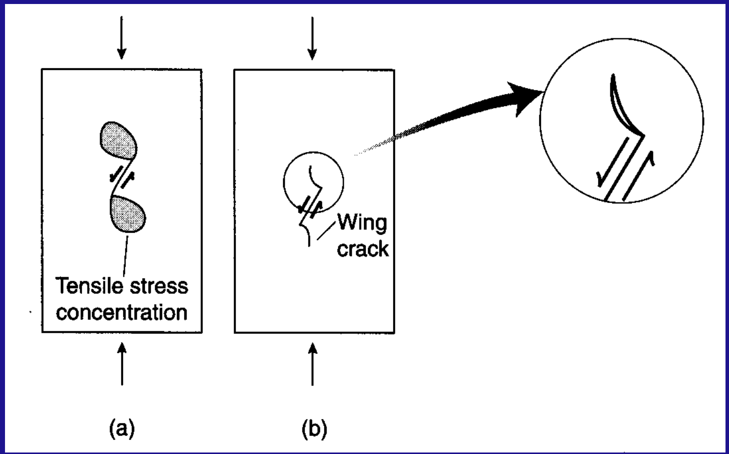
- Frictionless cracks
- Planar, perfectly sharp (mathematical) cuts

Crack tip stress field written in a generalized form

$$\sigma_{ij} = K_n \frac{1}{\sqrt{2\pi r}} f_{ij}^n(\theta)$$

$$\begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{tip} \approx \frac{1}{\sqrt{2\pi r}} \begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix}$$





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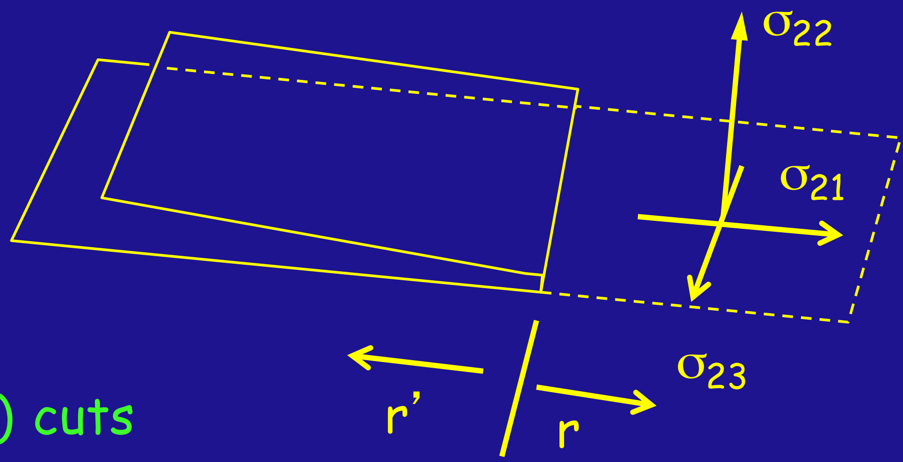

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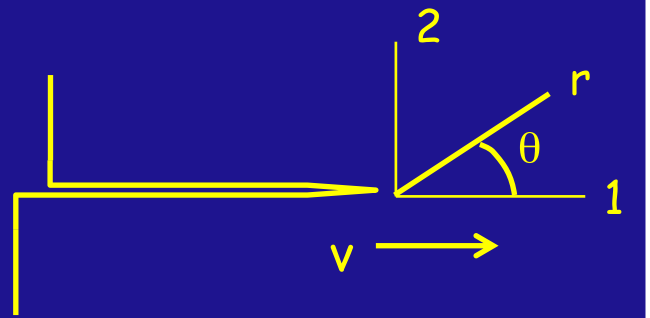

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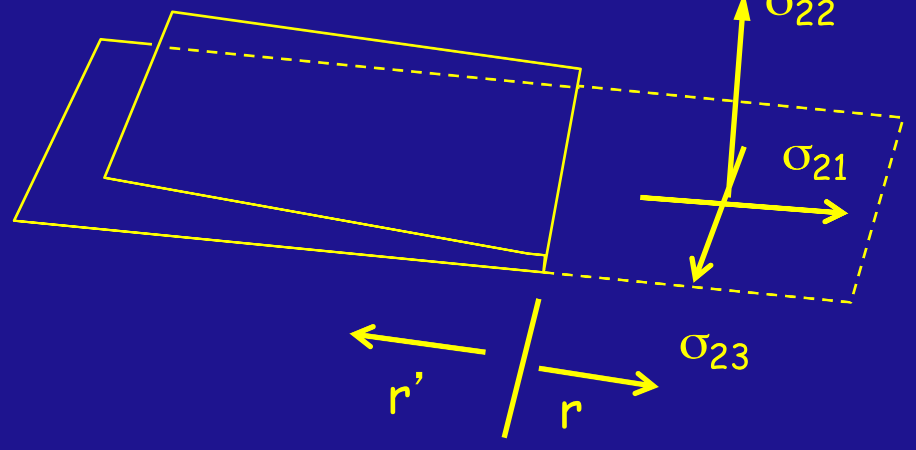
Note that the functions $f_{ij}^n(\theta)$ vary from ±2, so are not major factors

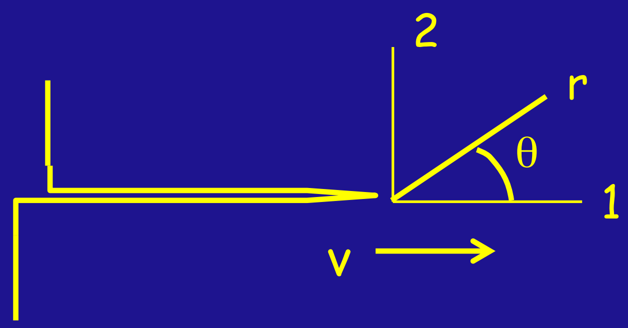
For uniform remote loading of a crack of length 2c:

$$\begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix} = \sqrt{\pi c} \begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{applied}$$



$$\begin{Bmatrix} \sigma_{22} \\ \sigma_{21} \\ \sigma_{23} \end{Bmatrix}_{tip} \approx \frac{1}{\sqrt{2\pi r}} \begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix}$$

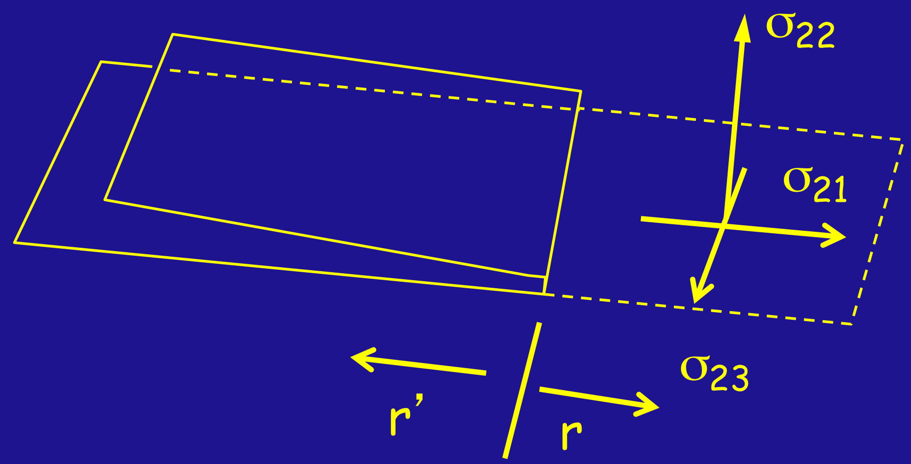
$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}} K_n$$



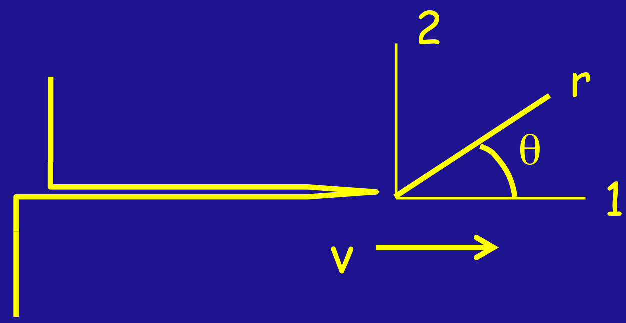
$$\sigma_{22}^{tip} = \frac{\sqrt{c}}{\sqrt{2r}} \sigma_{22}$$

$$\begin{Bmatrix} \Delta u_2 \\ \Delta u_1 \\ \Delta u_3 \end{Bmatrix} \approx \frac{4(1-\eta)}{\mu} \sqrt{\frac{r'}{2\pi}} \begin{Bmatrix} K_I \\ K_{II} \\ \dfrac{K_{III}}{(1-\eta)} \end{Bmatrix}$$

Static vs. dynamic fracture mechanics, relativistic effects



$$\begin{Bmatrix} \Delta u_2 \\ \Delta u_1 \\ \Delta u_3 \end{Bmatrix} \approx \frac{4(1-\eta)}{\mu} \sqrt{\frac{r'}{2\pi}} \begin{Bmatrix} g_I(v)\, K_I \\ g_{II}(v)\, K_{II} \\ g_{III}(v)\, \dfrac{K_{III}}{(1-\eta)} \end{Bmatrix}$$



$$g_I(0) = g_{II}(0) = g_{III}(0) = 1$$

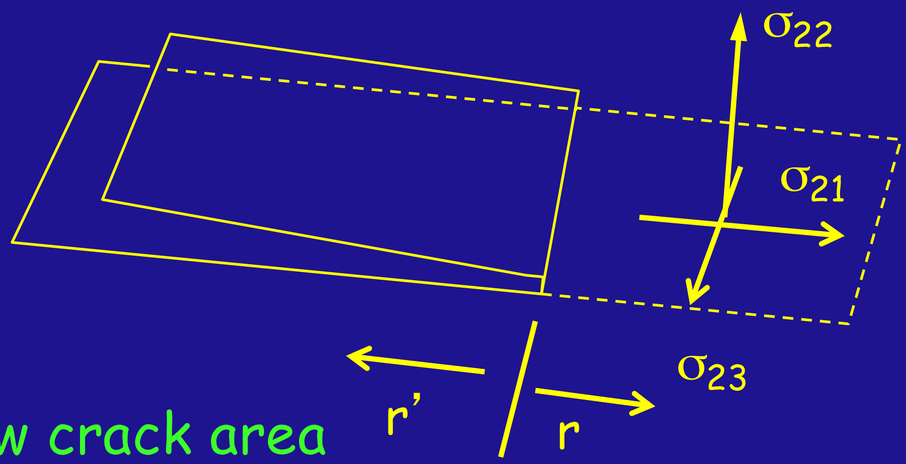
Static

Dynamic crack propagation

$$g_I(v) \to \infty \;\; and = g_{II}(v) \to \infty, \;\; as \;\; v \to C_R$$

$$g_{III}(v) = \frac{1}{\sqrt{1-\eta^2 / C_s^2}} \to \infty, \;\; as \;\; v \to C_s$$

$$\begin{Bmatrix} \Delta u_2 \\ \Delta u_1 \\ \Delta u_3 \end{Bmatrix} \approx \frac{4(1-\eta)}{\mu}\sqrt{\frac{r'}{2\pi}} \begin{Bmatrix} g_I(v)\ K_I \\ g_{II}(v)\ K_{II} \\ g_{III}(v)\ \dfrac{K_{III}}{(1-\eta)} \end{Bmatrix}$$



G is Energy flow to crack tip per unit new crack area

$$G = \frac{(1-\eta)}{2\mu}\left[g_I(v)K_I^2 + g_{II}(v)K_{II}^2\right] + \frac{1}{2\mu} g_{III}(v)K_{III}^2$$

$$G = G_{critical} = 2\gamma$$

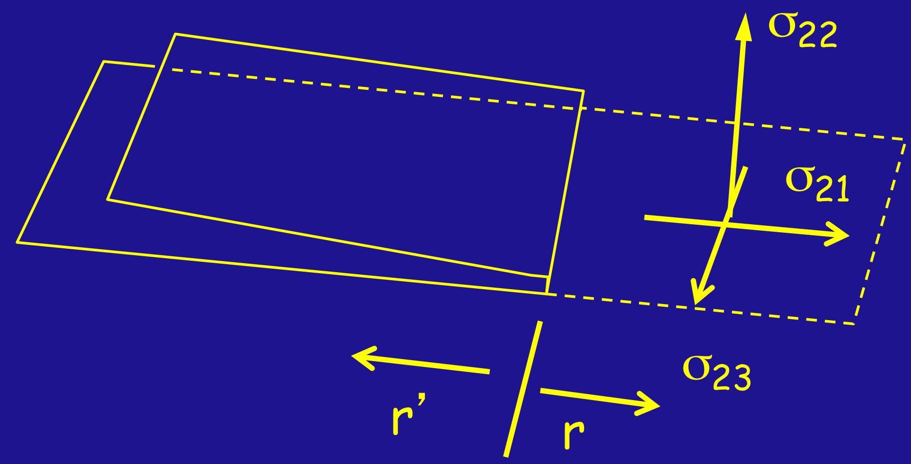
Critical energy release rate

G_{crit} is a material property --the “fracture energy”

$G_{crit} = K_c^2 / E = 2\gamma$, where K_c is the critical stress intensity factor (also known as the fracture toughness).

$$\begin{Bmatrix}\sigma_{22}\\ \sigma_{21}\\ \sigma_{23}\end{Bmatrix}_{tip} \approx \frac{1}{\sqrt{2\pi r}}\begin{Bmatrix}K_I\\ K_{II}\\ K_{III}\end{Bmatrix}$$

$$K_I = \sqrt{\pi c}\;\sigma_\infty$$

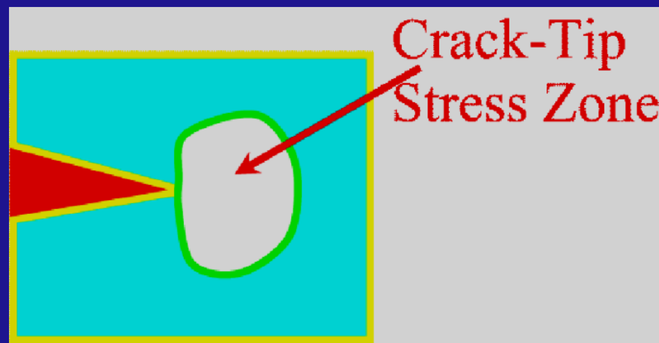


Stress field is singular at the crack tip.

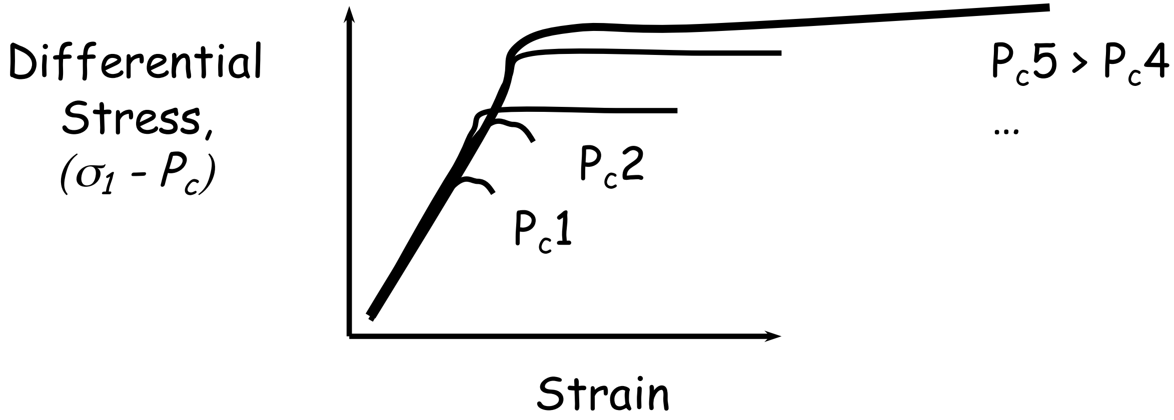
- because we assumed perfectly sharp crack
- but real materials cannot support infinite stress

Process zone (Irwin) to account for non-linear zone of plastic flow and cracking

- Size of this zone will depend upon crack velocity, material properties and crack geometry
- Energy dissipation in the crack tip region helps to limit the stresses there (why?)



Stress-strain-failure curves

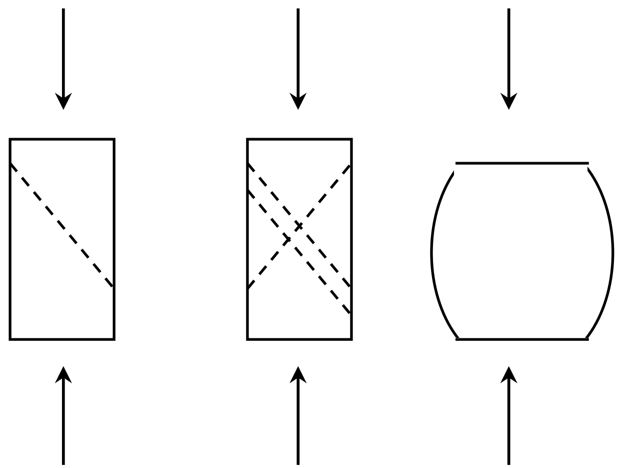


These styles can be loosely related to Brittle and Ductile deformation, respectively.

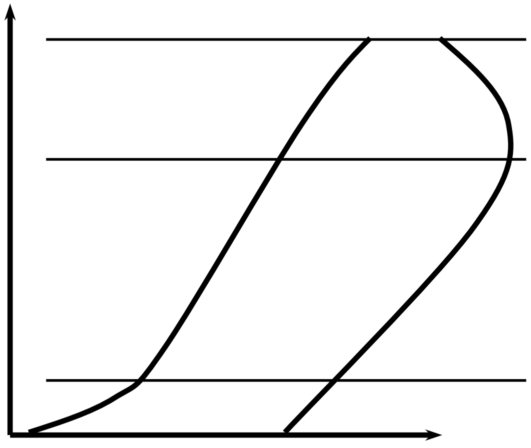
Brittle refers to pressure sensitive deformation

Brittle Failure: If we draw the stress-strain-failure curves for a range of confining pressures, we'll get a range of yield strengths, showing that σ_y is proportional to P_c.

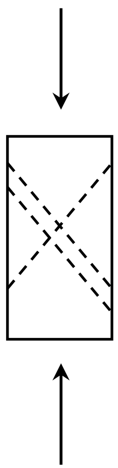
With increasing confining pressure there is a transition from localized to more broadly distributed deformation.



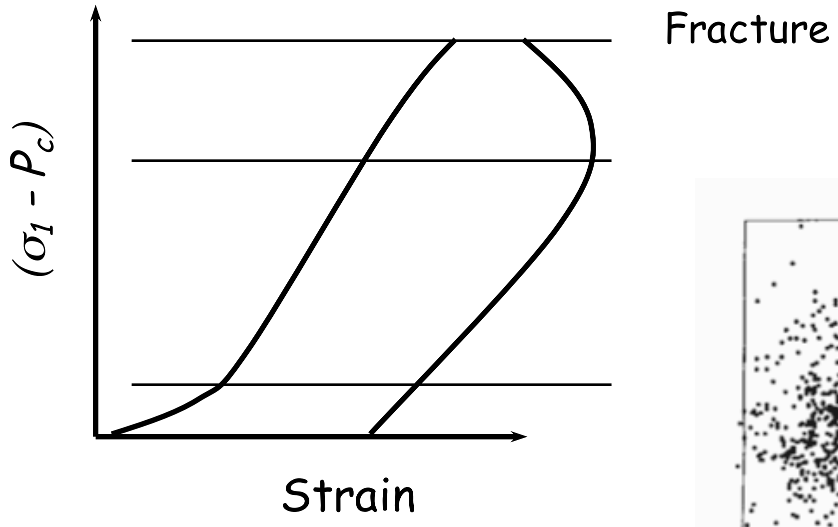
Shear Fracture Energy from Postfailure Behavior

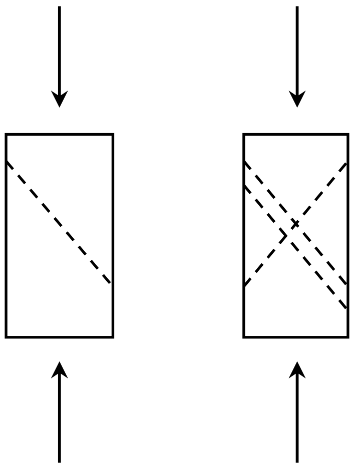


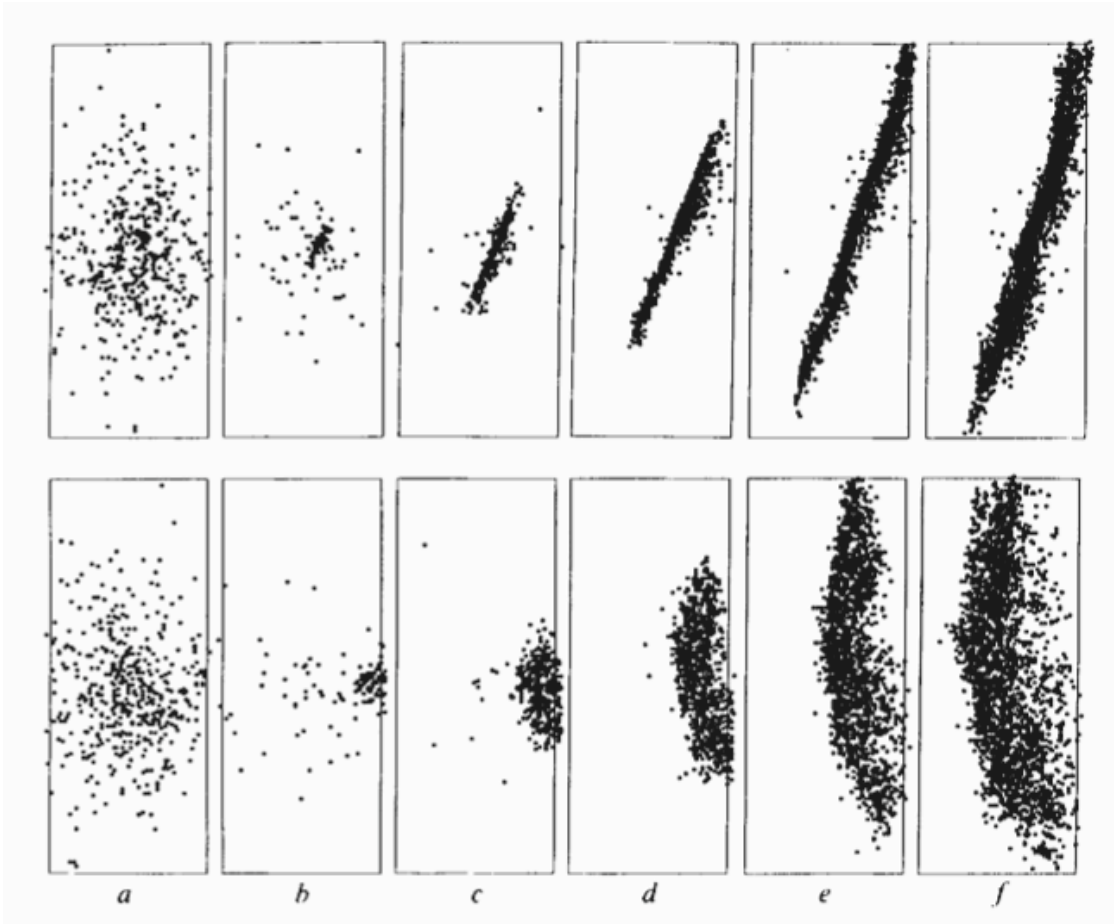
Brace, Paulding & Scholz, 1966; Scholz 1968.



Shear Fracture Energy from Postfailure Behavior

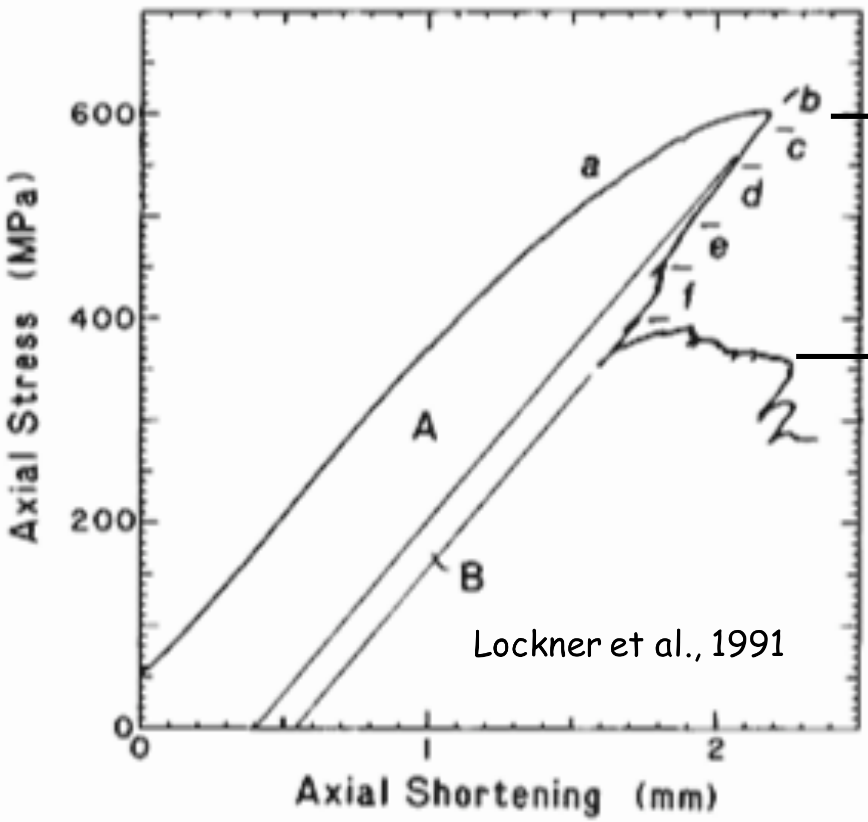


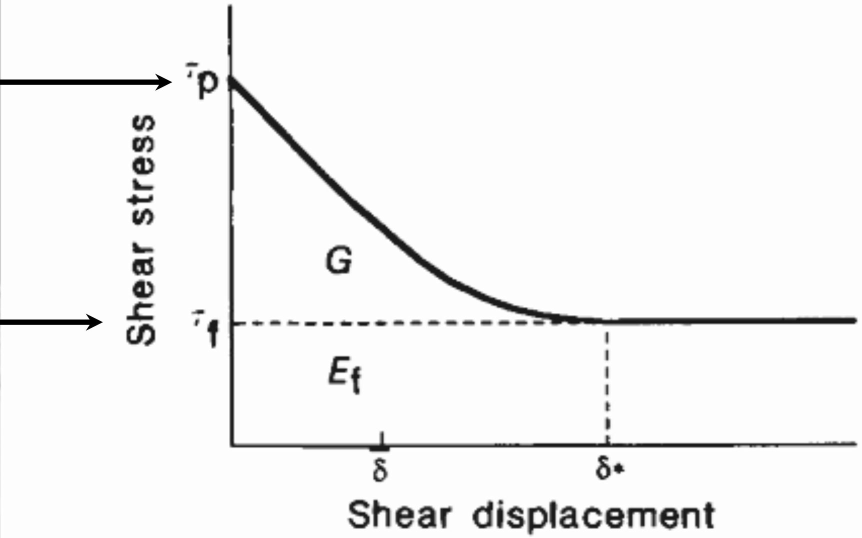




Lockner et al., 1991

Shear Fracture Energy from Postfailure Behavior





Inferred shear stress vs. slip relation for slip-weakening model. (based on Wong, 1982)

Wong, 1982, found that shear stress dropped ~ 0.2 GPa over a slip distance of ~50 microns.

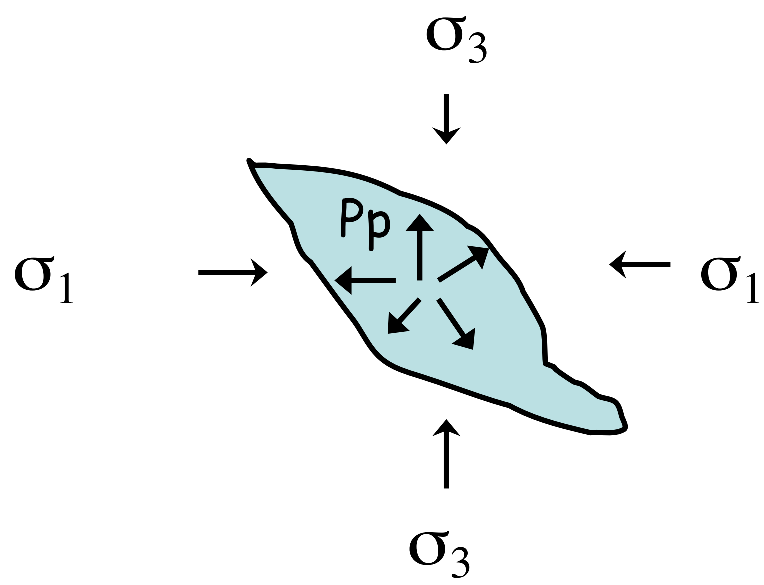
Exercise: Estimate G from these data and compare it to the values reported in Scholz (Table 1.1) and Wong, 1982.

Fluids: Consider the affects on shear strength

- Mechanical Effects
- Chemical Effects

Mechanical Effects: Effective Stress Law

$$\sigma_{effective} = \sigma_n - Pp$$



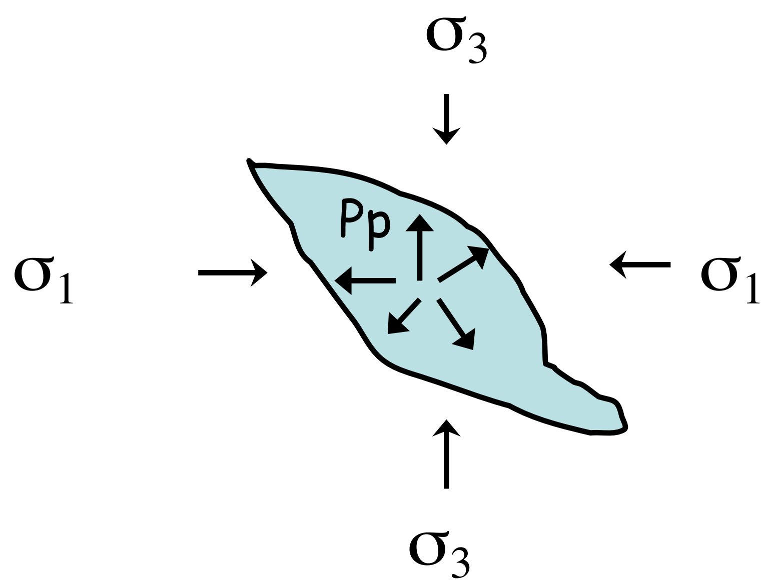
Fluids: Consider the affects on shear strength

- Mechanical Effects
- Chemical Effects

Mechanical Effects: Effective Stress Law

$\sigma_{effective} = \sigma_n - Pp$

$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$



$$\delta_{ij} = 1; i = j$$
$$\delta_{ij} = 0; i \neq j$$

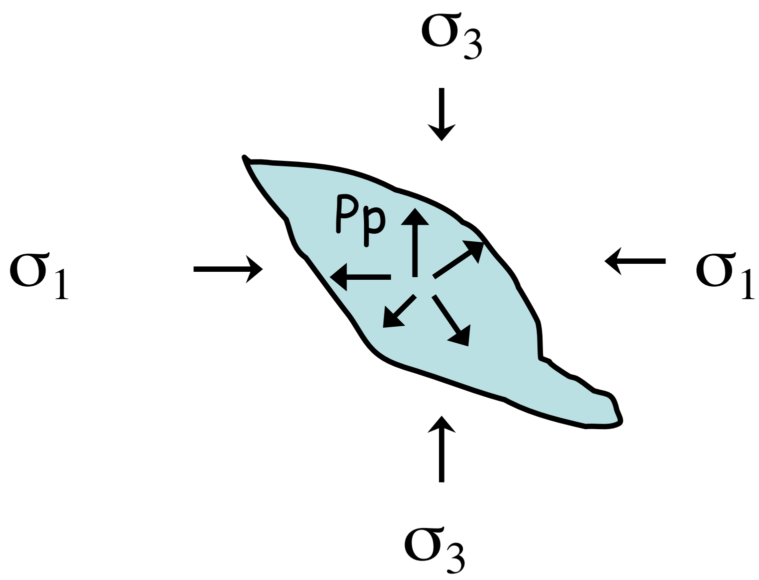
Leopold Kronecker (1823–1891)

Rock properties depend on effective stress: Strength, porosity, permeability, Vp, Vs, etc.

$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$

$$\sigma_{effective} = \sigma_n - Pp$$

$$\delta_{ij} = 1; i = j$$
$$\delta_{ij} = 0; i \neq j$$



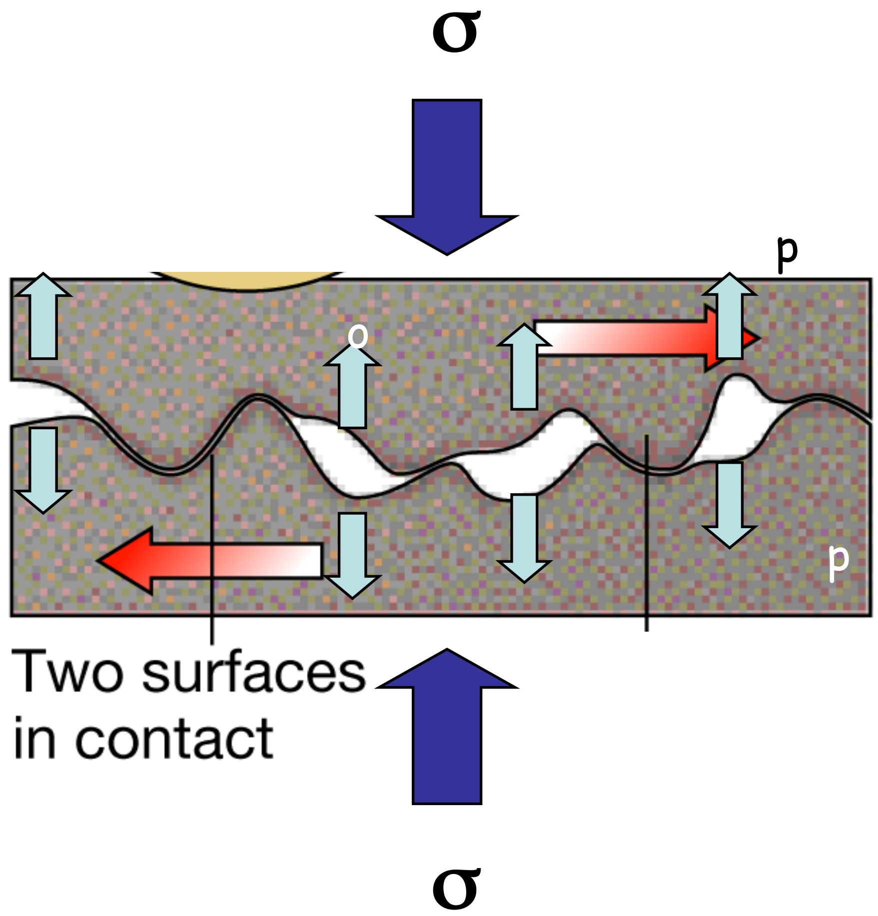
Exercise: Follow through the implications of Kronecker's delta to see that pore pressure only influences normal stresses and not shear stresses. Hint: see the equations for stress transformation that led to Mohr's circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

Void space filled with a fluid at pressure Pp

But what if $Ar \neq A$?



Fluids play a role by opposing the normal stress

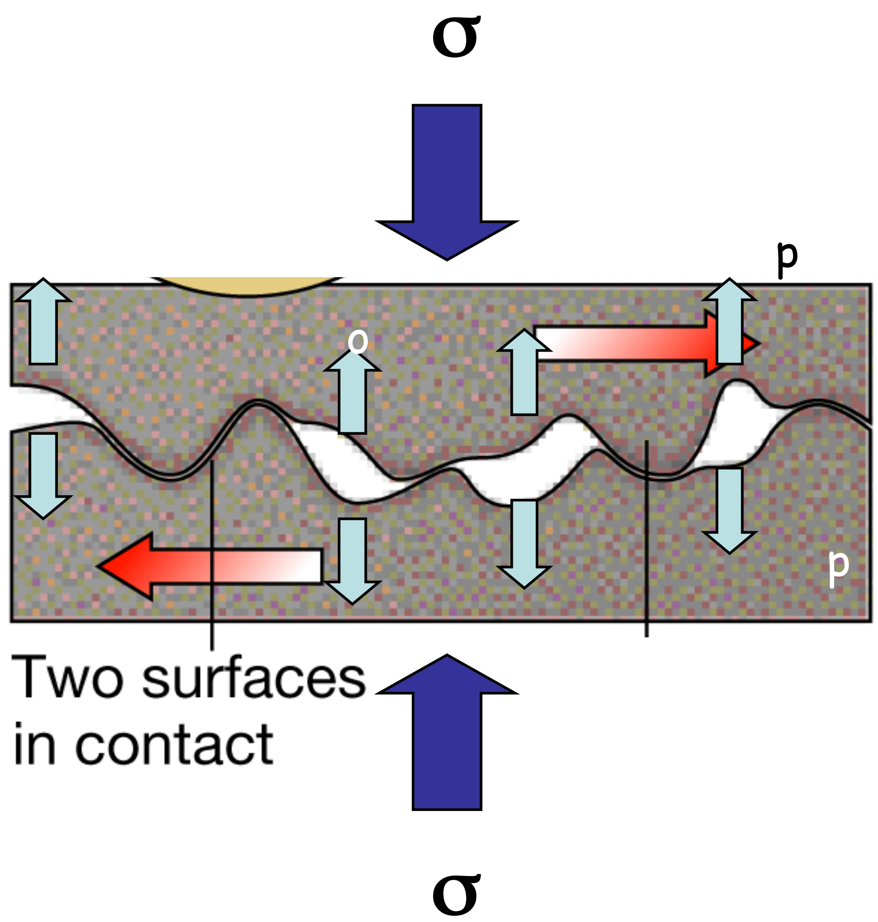
Void space filled with a fluid at pressure Pp

But what if $Ar \neq A$?

For example, we expect that shear strength depends on effective stress, but perhaps not in the way envisioned by:

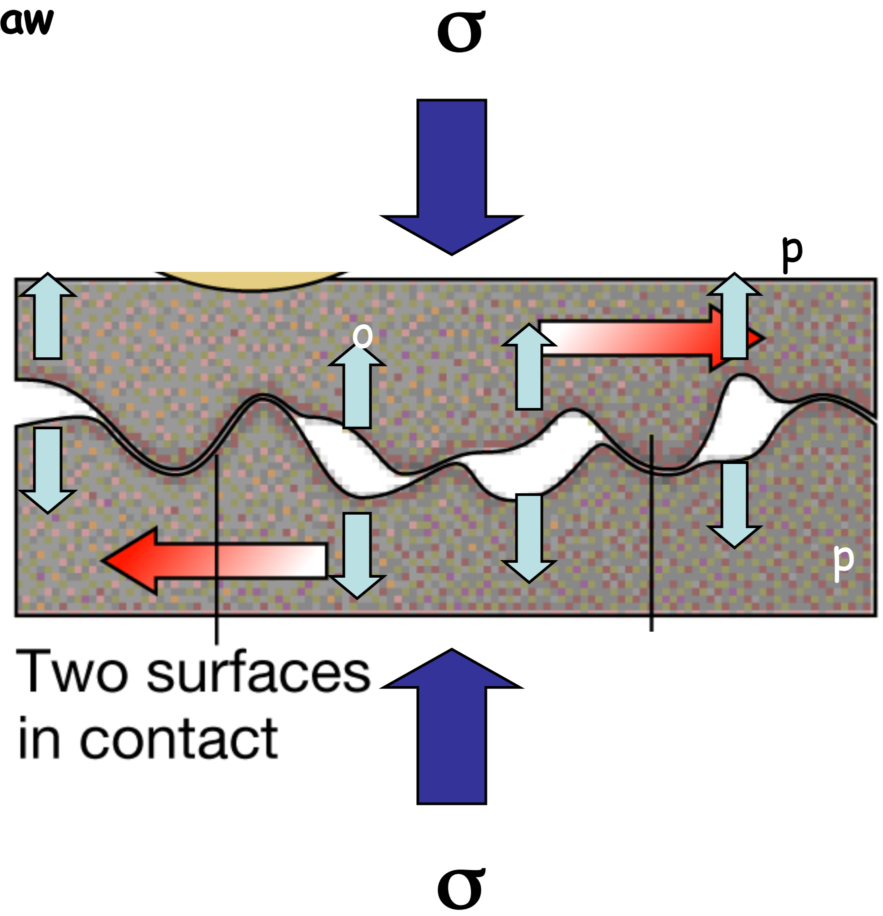
$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$

$$\tau = C + \mu_i \sigma'$$



Mechanical Effects: Effective Stress Law

$$\sigma'_{ij} = \sigma_{ij} - P_p\delta_{ij}$$



For brittle conditions,
Ar / A ~ 0.1

$$\sigma'_{ij} = \sigma_{ij} - \alpha\ P_p\delta_{ij}$$

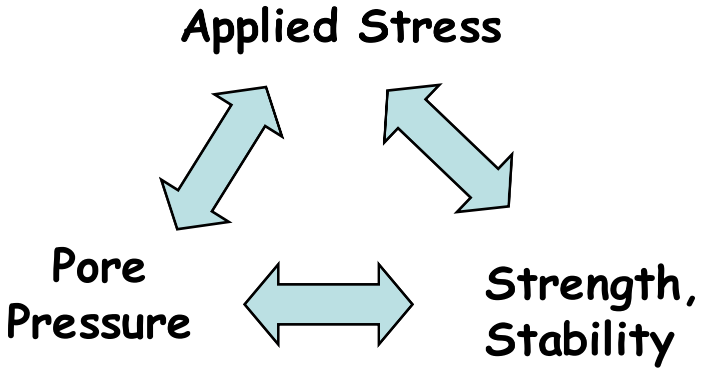
$$\alpha = (1 - \frac{A_r}{A})$$

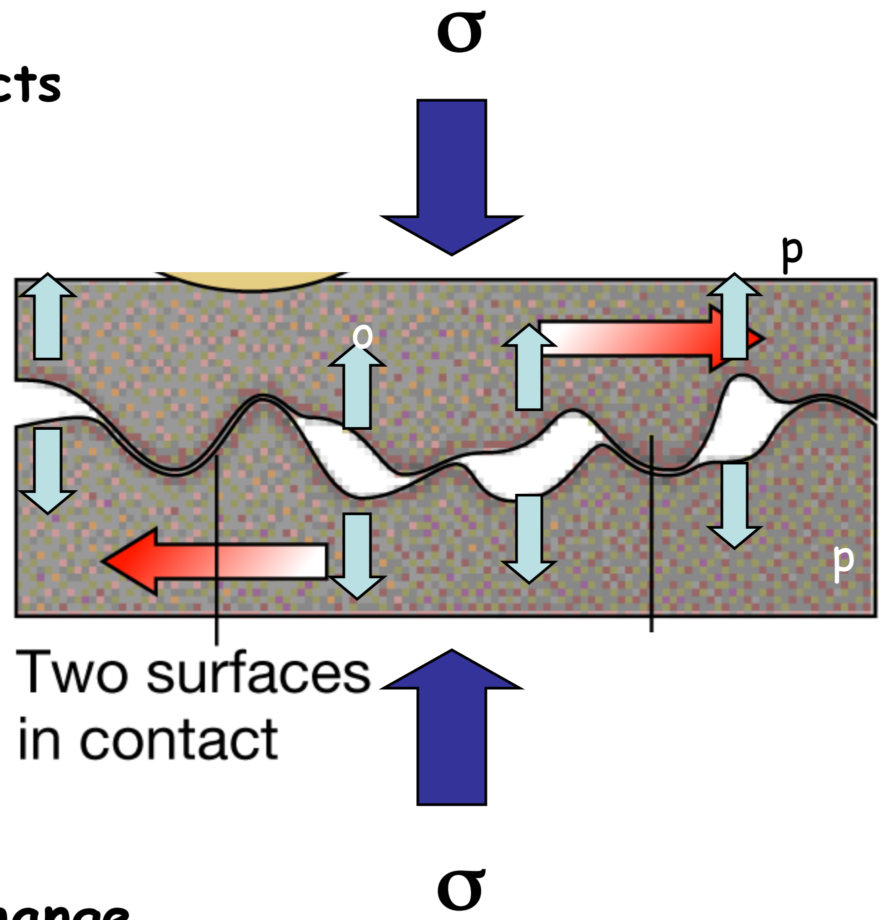
$$\tau = C + \mu_i\sigma'$$

Exercise: Consider how a change in applied stress would differ from a change in Pp in terms of its effect on Coulomb shear strength. Take α = 0.9

Effective Stress Law

Coupled Effects



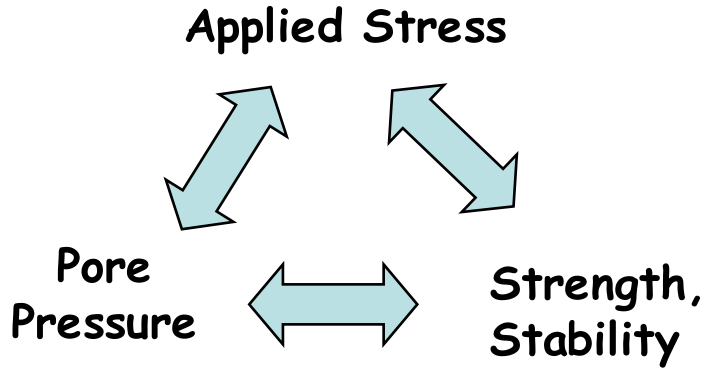


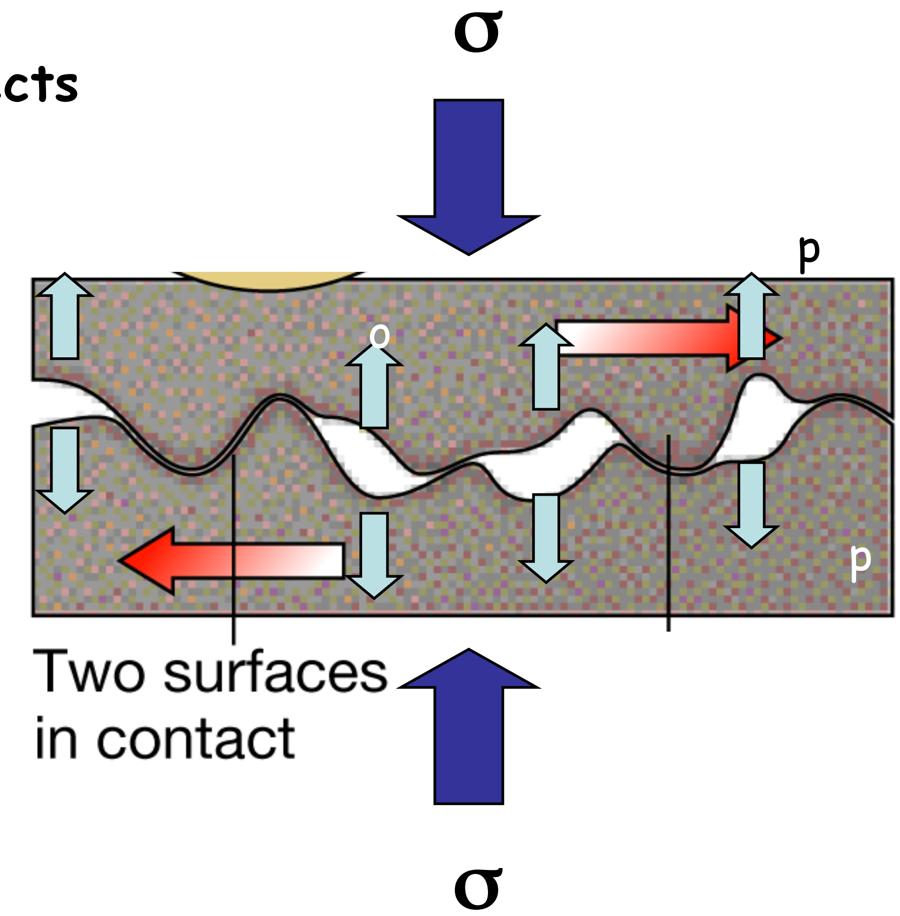
Dilatancy: Shear driven volume change

Exercise: Make the dilatancy demo described by Mead (1925) on pages 687-688. You can use a balllon, but a plastic bottle with a tube works better. Bring to class to show us. Feel free to work in groups of two.

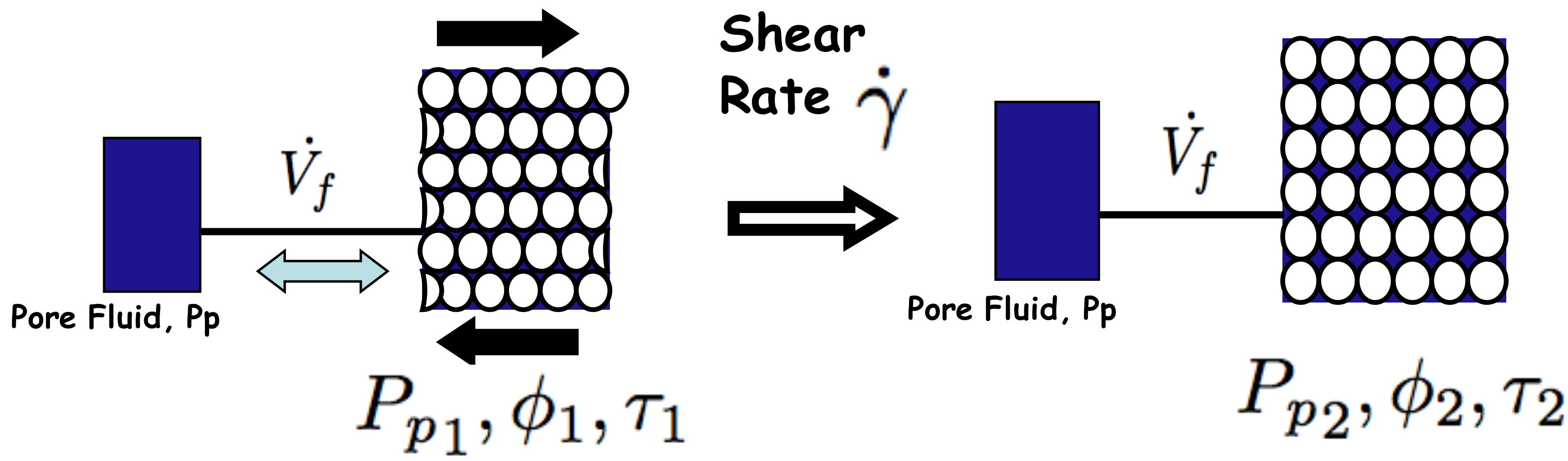
Effective Stress Law

Coupled Effects





Dilatancy

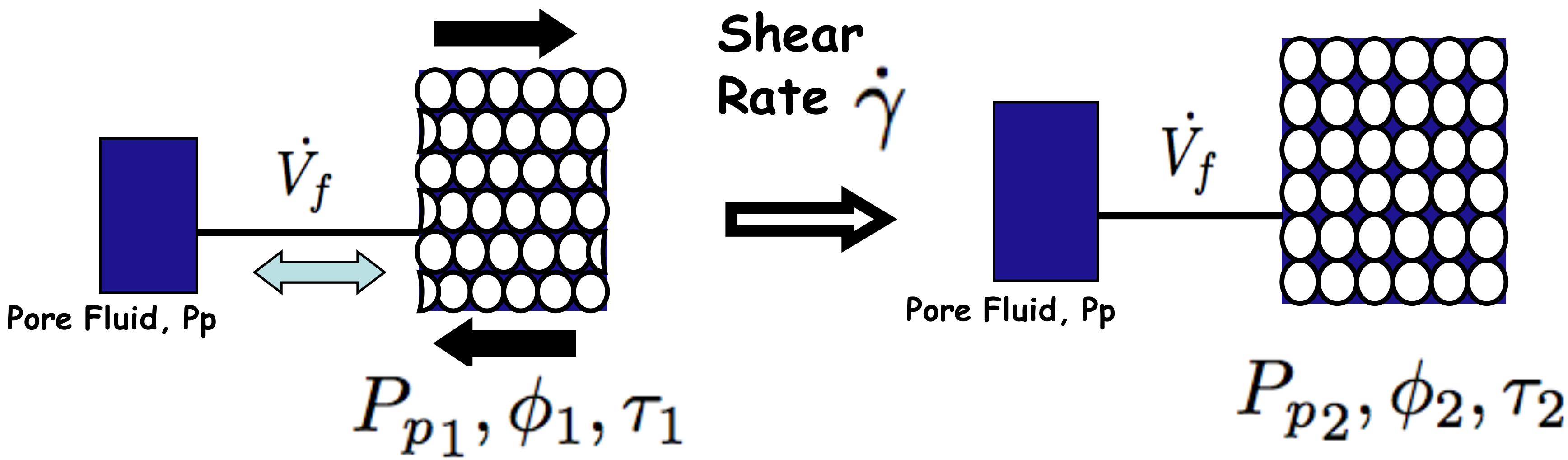


Dilatancy: $$\phi_1 \neq \phi_2$$

Volumetric Strain: $$d\theta = \frac{(V_2 - V_1)}{V}$$

Assume no change in solid volume

Dilatancy Rate: $$\beta = \frac{d\theta}{d\gamma}$$



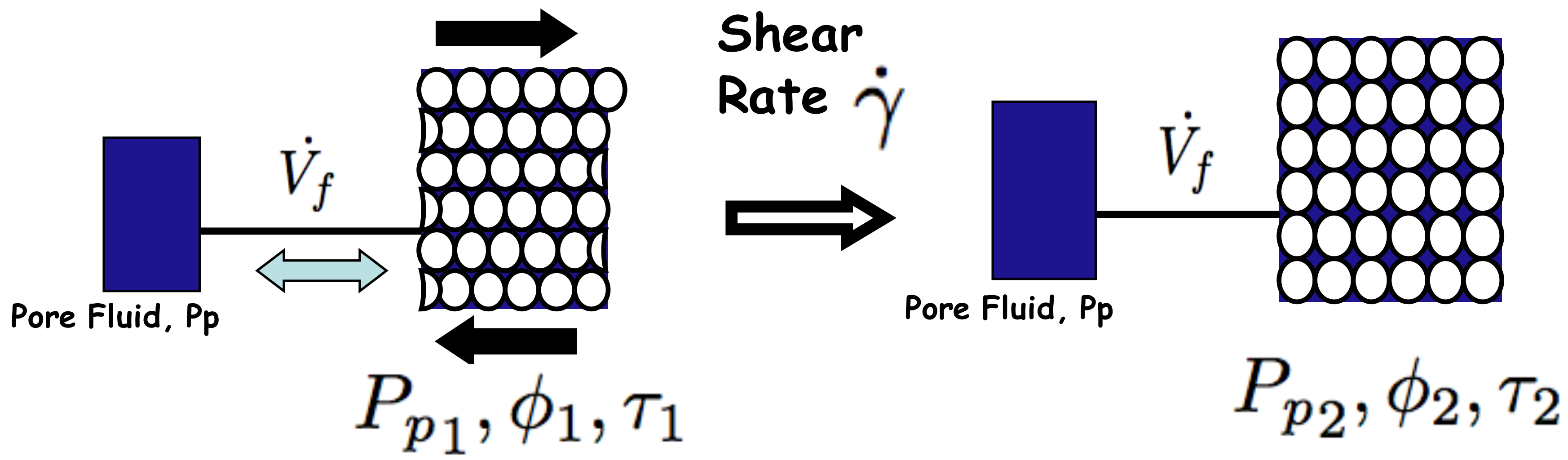
Dilatancy: $\phi_1 \neq \phi_2;$ $P_{p1} \neq P_{p2}$ Undrained loading

Volumetric Strain: $$d\theta = \frac{(V_2 - V_1)}{V}$$

Assume no change in solid volume

Dilatancy Rate: $$\beta = \frac{d\theta}{d\gamma}$$

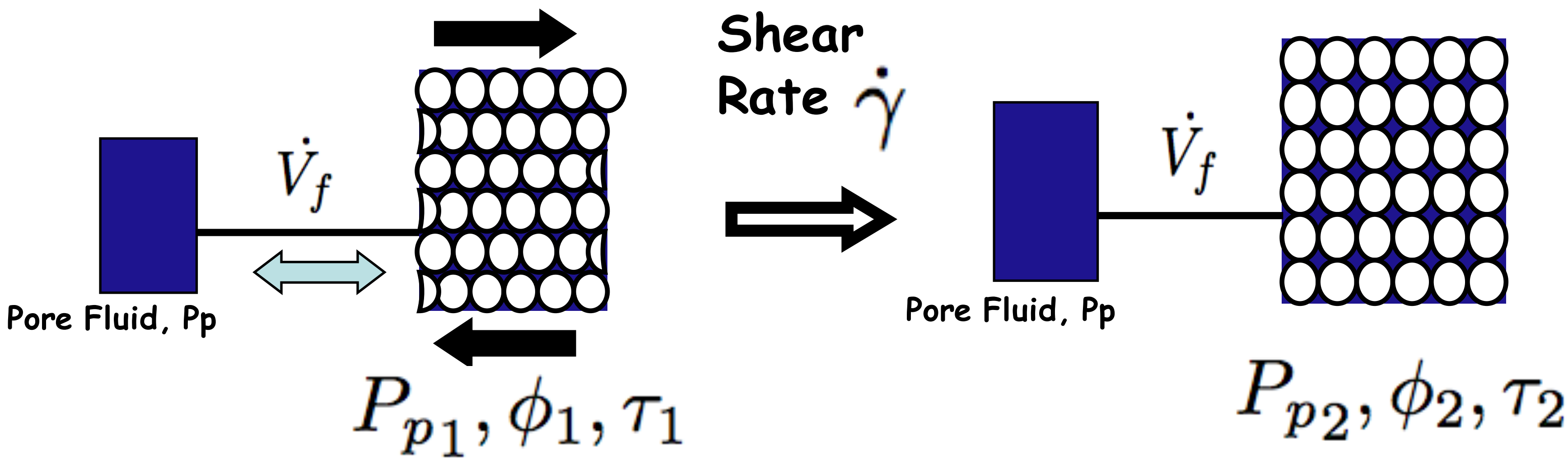
Dilatancy Hardening if $\dot{d\theta} > \dot{V}_f$ **or** $\dot{V}_f < \dot{\beta}$



Dilatancy Hardening if : $\dot{V}_f < \dot{\beta}$

$$\sigma'_{ij} = \sigma_{ij} - P_p \delta_{ij}$$

$$\tau = C + \mu_i \sigma'$$

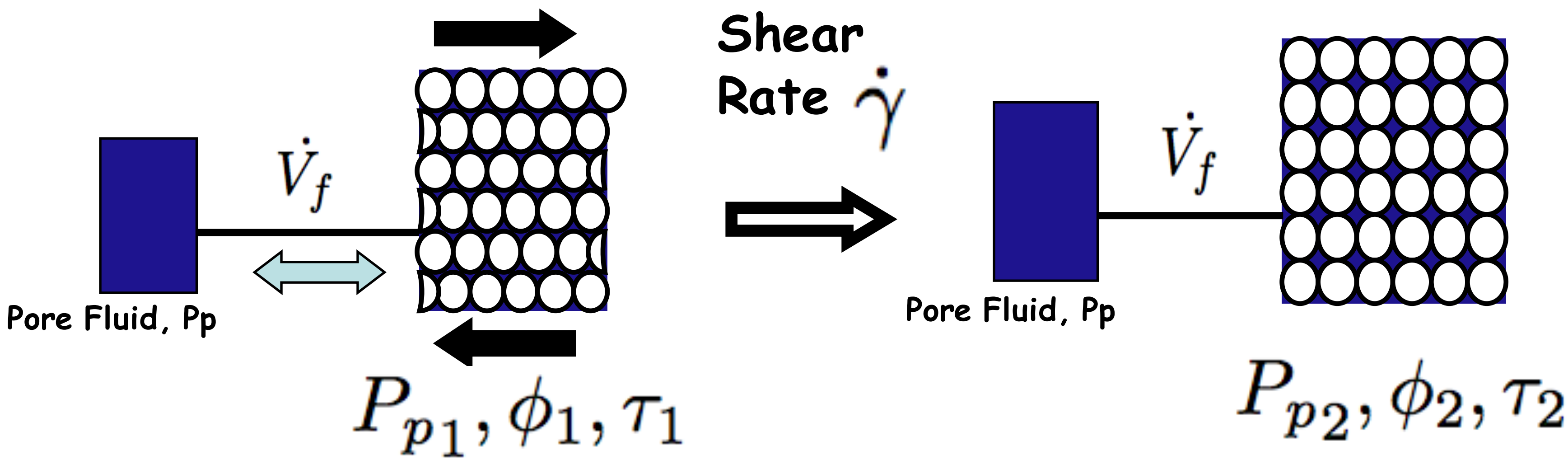


Dilatancy Weakening can occur if: $d\theta < 0 \; and \; |d\dot{\theta}| > \dot{V}_f$

This is shear driven compaction

$$\sigma'_{ij} = \sigma_{ij} - P_p\delta_{ij}$$

$$\tau = C + \mu_i \sigma'$$

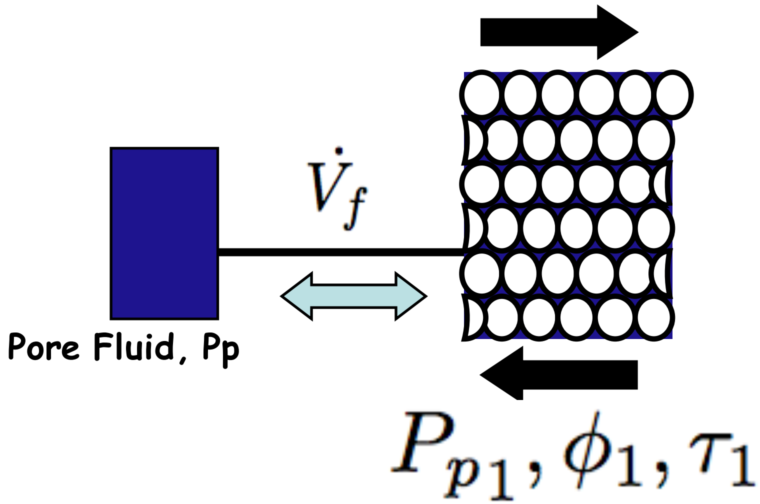


Consider the implications of dilatancy and volume change for the total work of shearing, W

$$W = \tau_p \, d\gamma + \sigma \, d\theta$$

W is total work of shearing

$W = \tau \, d\gamma = \sigma \mu \, d\gamma$



Consider the implications of dilatancy and volume change for the total work of shearing, W

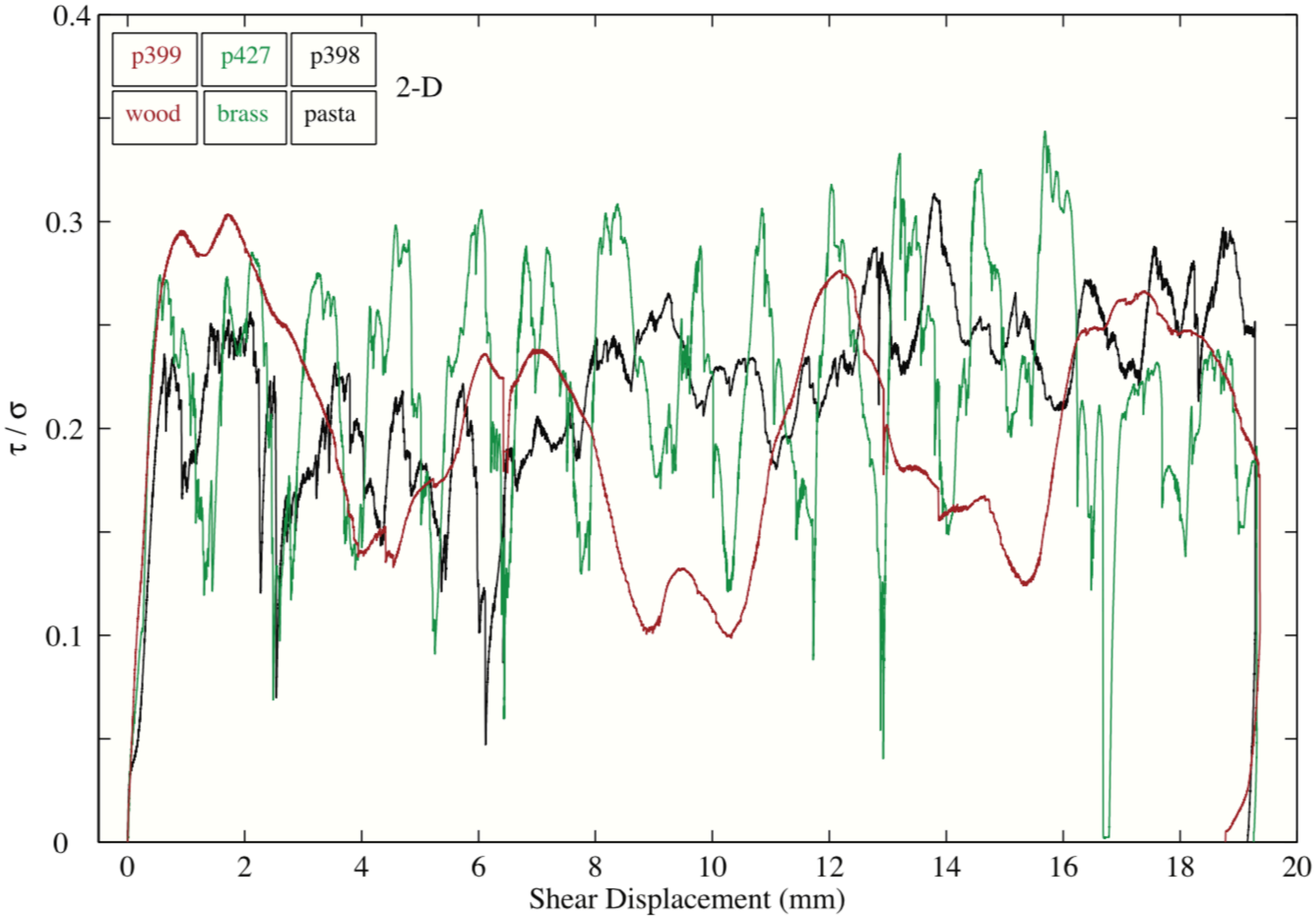
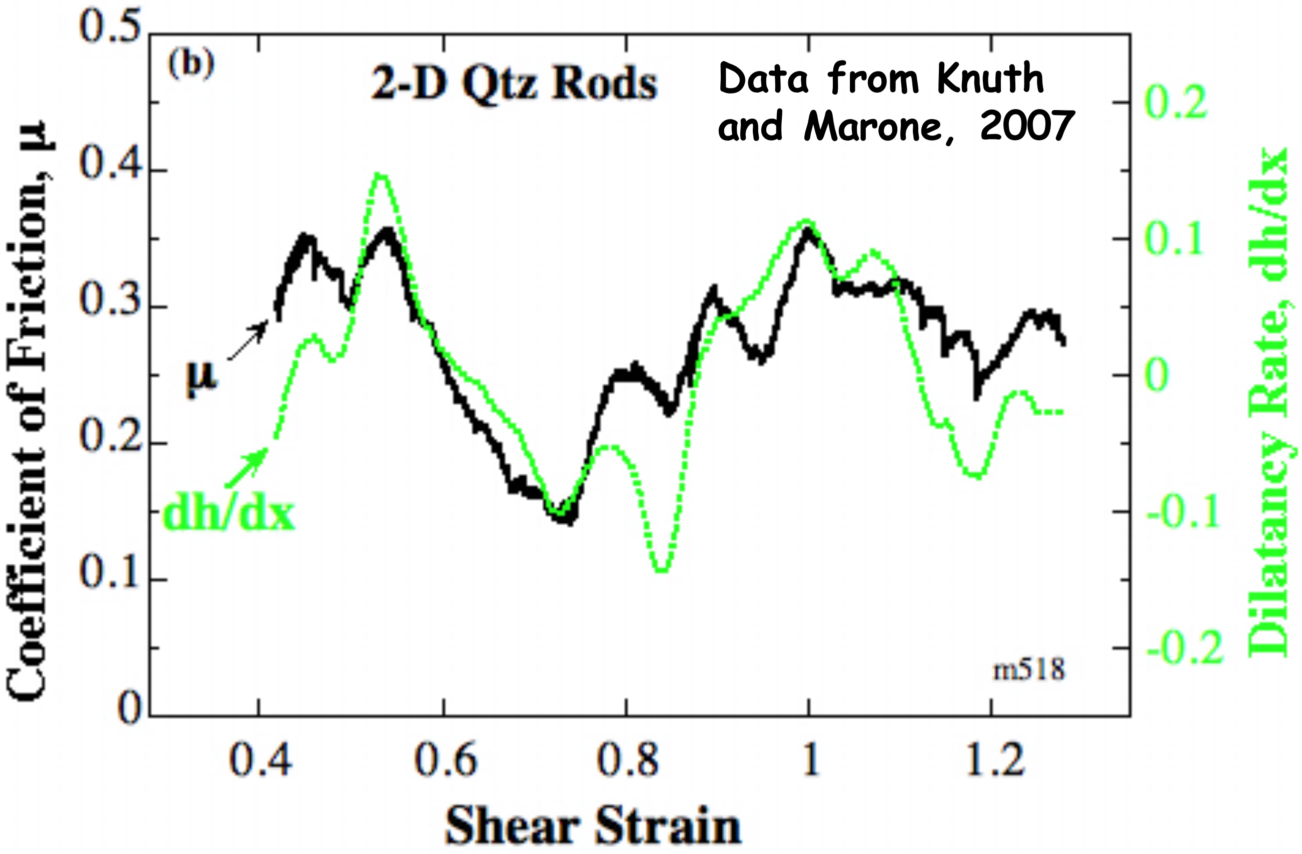
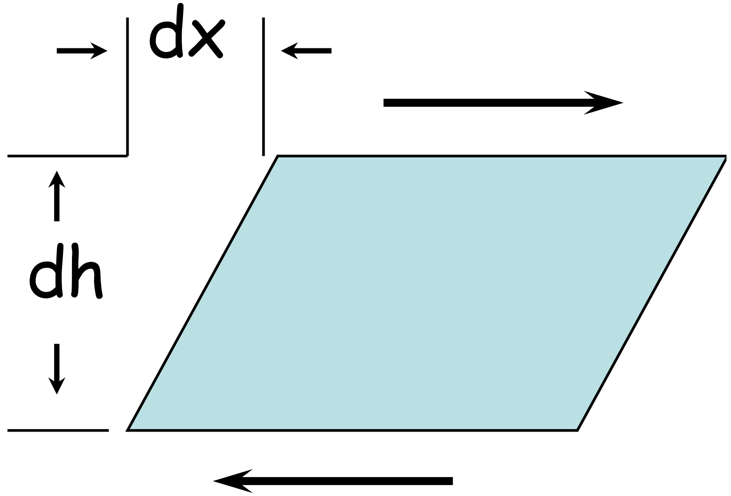


Figure 8. Comparison of granular materials in the 2-D configuration. Wood layers exhibit long wavelength, smooth variations. Pasta and brass exhibit higher-frequency variation. In all cases, sliding is stable and without abrupt, audible stick-slips.

Friction mechanics of 2-D particles



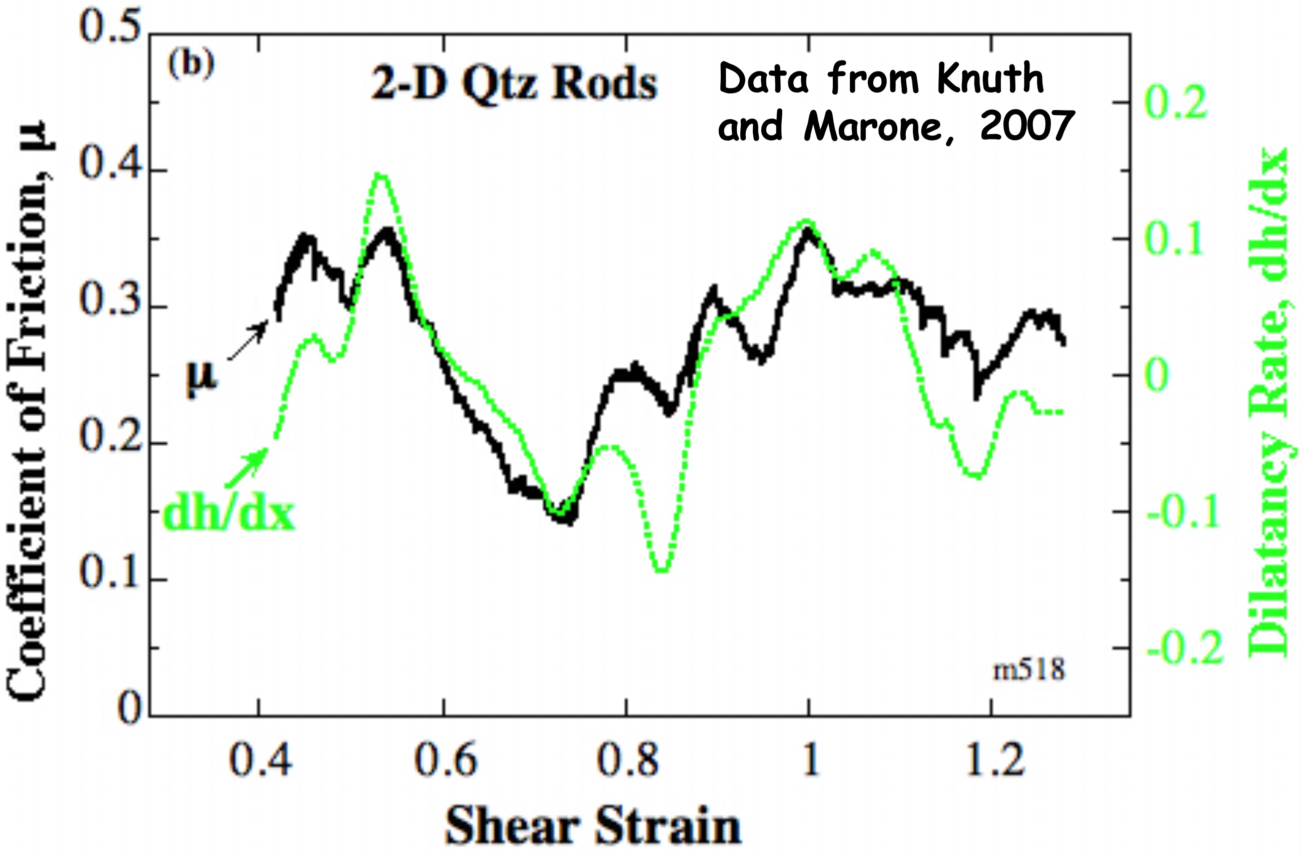


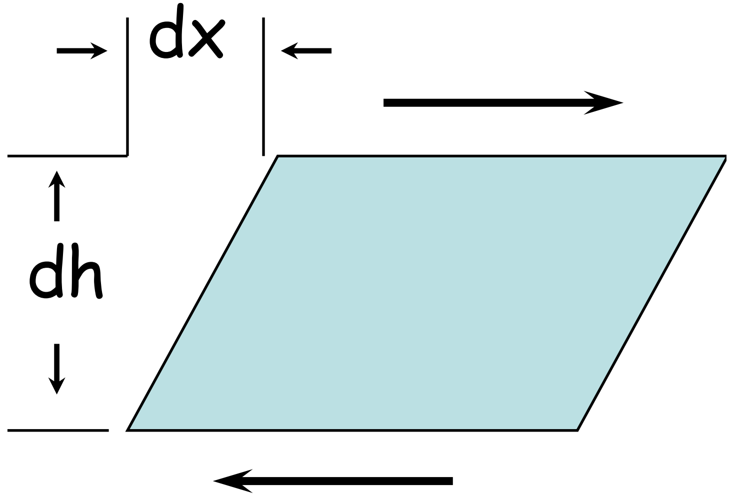
$$W = \tau_p \, d\gamma + \sigma \, d\theta$$

W is total work of shearing

$W = \tau \, d\gamma = \sigma \mu \, d\gamma$

Friction mechanics of 2-D particles



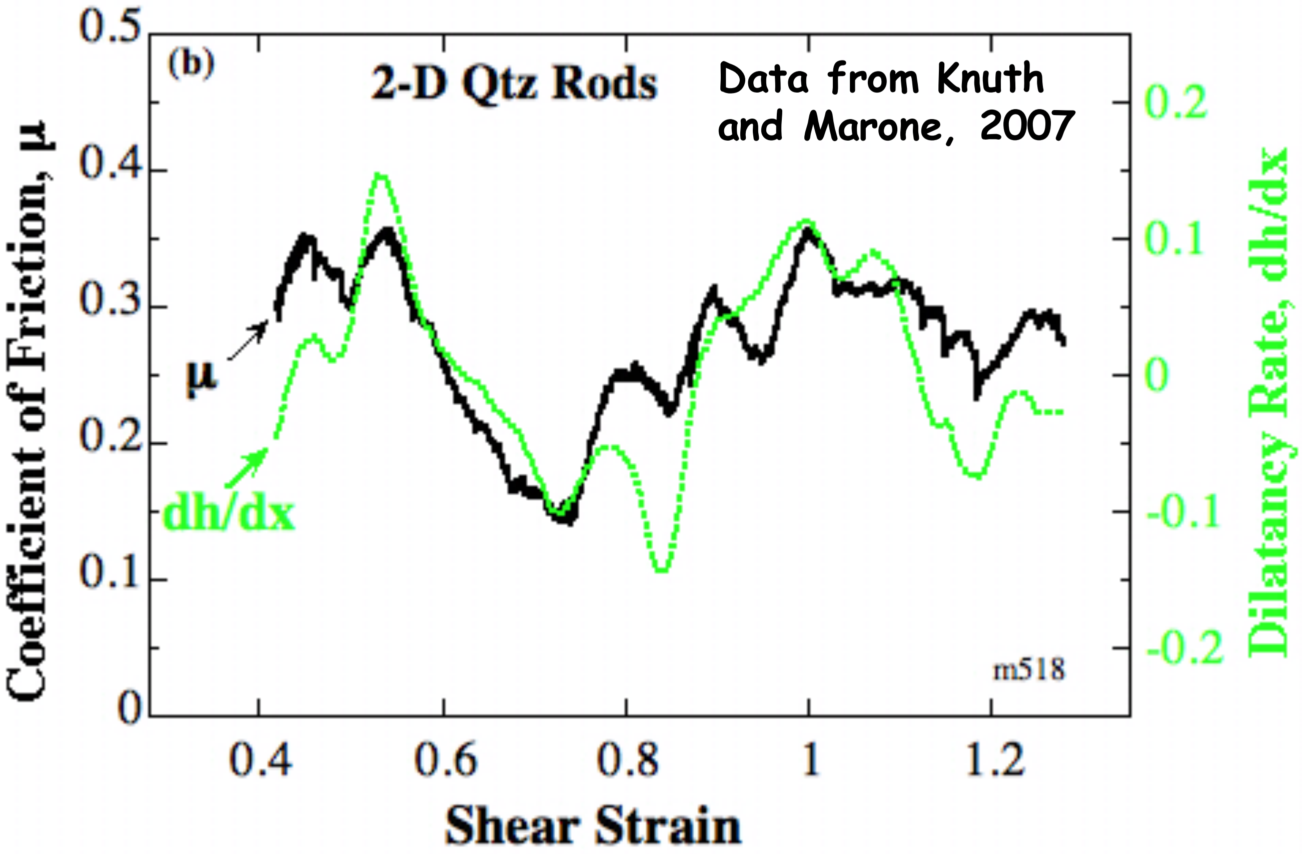


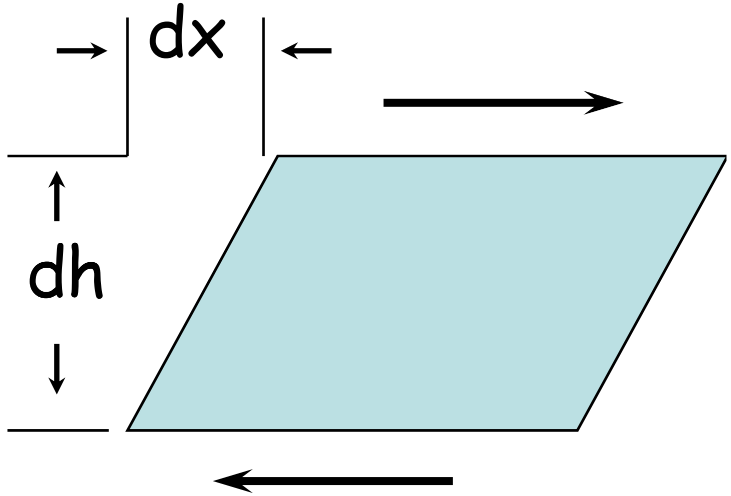
$$W = \tau_p \, d\gamma + \sigma \, d\theta$$

$$\tau = \sigma\left(\mu_p + d\theta / d\gamma\right)$$

$$d\theta = dV / V \; ; \; d\gamma = dx / h$$

Friction mechanics of 2-D particles



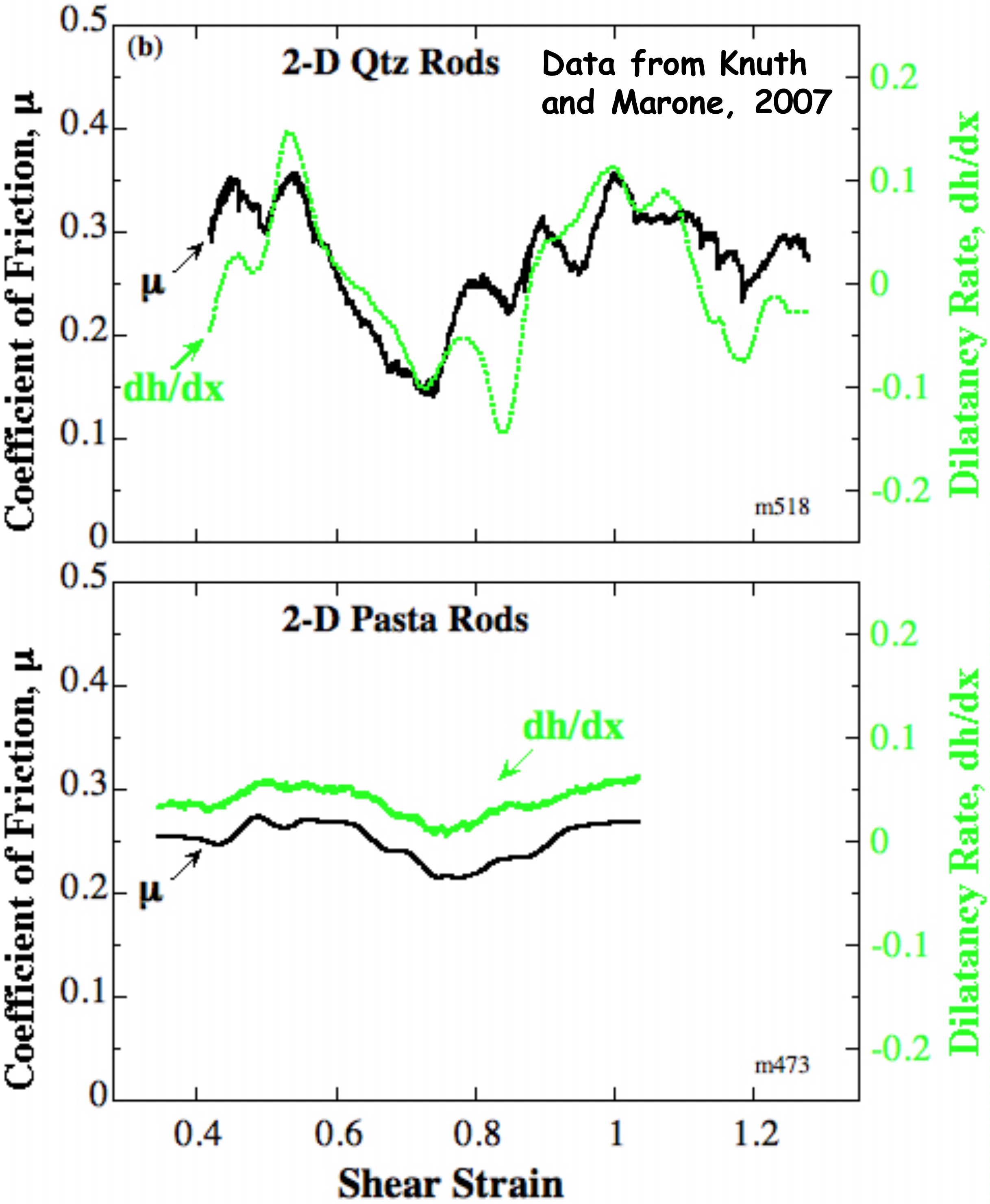


$$W = \tau_p \; d\gamma + \sigma \; d\theta$$

$$\tau = \sigma\left(\mu_p + d\theta / d\gamma\right)$$

$$\tau = \sigma\left(\mu_p + dh / dx\right)$$

- Dilatancy rate plays an important role in setting the frictional strength



- Macroscopic variations in friction are due to variations in dilatancy rate.
- Smaller amplitude fluctuations in dilatancy rate produce smaller amplitude friction fluctuations.