• Brune Stress Drop
• Seismic Spectra & Earthquake Scaling laws.
Moment: \([N m] = [Pa m^3]\) Seismically released moment can be related to stress drop.

Stress drop \((\Delta \tau)\) in terms of shear modulus, average slip, \(u\), and fault rupture length \(L\).

\[
\Delta \tau = \mu \frac{\bar{u}}{L}
\]

replace \(\mu\) in the equation for moment. \(M_o = \Delta \tau \frac{L}{\bar{u}} A \bar{u}\)

gives: \(M_o = \Delta \tau \frac{L}{\bar{u}} A\). The right hand side is equivalent to \(F L\), where \(F\) is force.

The simple model in which crustal stiffness is given by \(k\), the seismically released elastic energy is given by \(k = \frac{F}{\bar{u}}\). Taking slip proportional to length, this gives: \(M_o \propto k u^2\)

Seismic Energy: \(\log E = 5.2 + 1.4 M_s\)

1 J = 10^7 ergs
1 kW hr = 3.6x10^6 J
Seismic Spectra & Earthquake Scaling laws.

Aki, Scaling law of seismic spectrum, JGR, 72, 1217-1231, 1967.

Hanks, $b$ Values and $\omega^{-\gamma}$ seismic source models: implications for tectonic stress variations along active crustal fault zones and the estimation of high-frequency strong ground motion, JGR, 84, 2235-2242, 1979.

Scaling and Self-Similarity of Earthquake Rupture:
Implications for Rupture Dynamics and the Mode of Rupture Propagation

0 Self-similar:
Are small earthquakes ‘the same’ as large ones?
Do small ones become large ones or are large eq’s different from the start?

1 Geometric self-similarity: aspect ratio of rupture area

2 Physical self-similarity: stress drop, seismic strain, scaling of slip with rupture dimension

3 Observation of constant $b$-value over a wide range of inferred source dimension.

4 Same physical processes operate during shear rupture of very small (lab scale, mining induced seismicity) and very large earthquakes?

5 Expectation of scaling break if rupture physics/dynamics change in at a critical size (or slip velocity, etc.). Shimazaki result. (Fig. 4.12). Length-Moment scaling and transition at $L \approx 60$km (Romanowicz, 1992; Scholz, 1994).

6 Gutenberg-Richter frequency-magnitude scaling, $b$-values.

7 G-R scaling, $b$-value data. Single-fault versus fault population. G-R versus characteristic earthquake model.

8 Crack vs. slip-pulse models
Earthquake Source Properties, Spectra, Scaling, Self-similarity

Displacement and acceleration source spectra. Spectra: zero-frequency intercept ($M_o$), corner frequency ($\omega_o$ or $f_c$), high frequency decay ($\omega^{-\gamma}$), maximum (observed, emitted) frequency $f_{\text{max}}$

\[ M_o = C \Delta \sigma r^3 \]

$\omega$-square model, $\omega^{-2}$
$\omega$-cube model, $\omega^{-3}$

Far-field body-wave spectra and relation to source slip function

Displacement waveform for P & S waves:

\[ \Omega(\chi, t) = \int_{\Sigma} \Delta \dot{u} \left( \chi, t - \frac{r}{c} \right) d\Sigma. \]

In general, very complex. $\Omega(x, t)$ and $\Omega(\omega)$ depend on slip function, azimuth to observer and relative importance of nucleation and stopping phases.
Scaling and Self-Similarity

Are small earthquakes ‘the same’ as large ones?

1 Geometric self-similarity: rupture aspect ratio

2 Physical self-similarity: stress drop, seismic strain, scaling of slip with rupture dimension

Circular ruptures (small)

\[ M_o = G u A \]
\[ \Delta \sigma = C \frac{G u}{r} \]
\[ M_o = C \Delta \sigma r^3 \]

Hanks, 1977
Abercrombie & Leary, 1993
Earthquake Source Properties, Spectra, Scaling, Self-similarity

\[ \Omega(\omega) = \frac{\Omega(\omega_0)}{\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]^{3/2}} \]

\(\omega\)-cube model, \(\omega^{-3}\)

Similarity condition

\[ M_o \propto L^3 \]
\[ \omega_o \propto L^{-1} \]
\[ \Omega(0) \propto \omega_o^{-3} \]

This defines a scaling law. Spectral curves differ by a constant factor at a given period (e.g., 20 s), but they have the same high-freq. asymptote.

This behavior is expected when the nucleation phase is responsible for the high-freq. asymptote -- but consider problem of time domain implication for amplitude \(M_b\) decreases with \(M_o\).
Seismic Source Spectra.

Saturation occurs for large events, particularly saturation of $M_s (T=20 \text{ s})$

\begin{align*}
M_o &= G u A \\
M_o &= C \Delta \sigma \; r^3 \\
M_o &= C \Delta \sigma \; f_c^{-3}
\end{align*}

Corner frequency, Brune Stress drop.

Aki, 1967
Earthquake Source Properties, Spectra, Scaling, Self-similarity

\[ \Omega(\omega) = \frac{\Omega(0)}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \]

\( \omega \)-square model, \( \omega^{-2} \)
Two possible explanations

1) !Similarity condition (not-similarity)
\( M_o \propto L^2 \)
\( \omega_o \propto L^{-1} \)
\( \Omega(0) \propto \omega_o^{-2} \)

2) Have similarity condition in terms of nucleation, but high-freq. asymptote is produced by “stopping phase” if rupture stops very abruptly
Earthquake Source Properties, Spectra, Scaling, Self-similarity

Displacement and acceleration source spectra.
Spectra: zero-frequency intercept ($M_o$), corner frequency ($\omega_o$ or $f_c$), high frequency decay ($\omega^{-\gamma}$), maximum (observed, emitted) frequency $f_{\text{max}}$

Aki, Scaling law of seismic spectrum, JGR, 72, 1217-1231, 1967.
Hanks, b Values and $\omega^{-\gamma}$ seismic source models: implications for tectonic stress variations along active crustal fault zones and the estimation of high-frequency strong ground motion, JGR, 84, 2235-2242, 1979.
Earthquake Source Properties, Spectra, Scaling, Self-similarity

Source spectra for two events of equal stress drop: \textit{omega cube model}

\[
\Omega(\omega) = \frac{\Omega(o)}{[1 + \left(\frac{\omega}{\omega_o}\right)^2]^{3/2}}
\]

High-freq. spectral properties: produced by rupture growth, represent nucleation and enlargement
Source spectra for two events of equal stress drop: \textit{omega square model}

\[ \Omega(\omega) = \frac{\Omega(\omega_0)}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \]

High-freq. spectral properties: produced by rupture growth, represent nucleation and enlargement

\( \omega^L_0 \quad \omega^S_0 \quad \omega^{-2} \)

\( f_{\text{max}}^L \quad f_{\text{max}}^S \)

Large and Small Eq

log u at R

log freq. (\(\omega\))

log a at R

log freq. (\(\omega\))
Earthquake Source Properties, Spectra, Scaling, Self-similarity

Relation between source
(a) displacement
(b) velocity
(c) acceleration
history and asymptotic behavior of spectrum
Earthquake Source Properties, Spectra, Scaling, Self-similarity

Hanks (1979)

0. Consider two events that differ in size by 10x and assume self-similarity of rupture so that their moments differ by a factor of $10^3$ and durations differ by a factor of 10 (rupture velocity is the same and constant for each event.) Take the events to be large enough so that their corner frequency is well below 1 sec.

1. Four cases for time-domain interpretation of spectral source models, 2 for each model.

- $\omega$-square (spectral amplitude at 1 sec period is 10x greater for larger event)
  a) If: 1-s energy arrives continuously over the complete faulting duration. Then: 1-s time domain amplitude is the same and $M_b$ is the same for both events. $M_b$ is independent of $M_o$.
  b) If: all 1-s energy radiated at the same time (arrives at the same time) Then: 1-s time domain amplitude is 10x larger for the larger event. $M_b$ scales directly with $M_o$.

- $\omega$-cube (spectral amplitude at T=1 sec is the same for each event)
  c) If: 1-s energy arrives continuously over the complete faulting duration. Then: 1-s time domain amplitude is smaller for the larger event. $M_o$ scales inversely with $M_b$.
  d) If: all 1-s energy radiated at the same time (arrives at the same time) Then: 1-s time domain amplitude is the same for both events. $M_b$ is independent of $M_o$. 

Fig. 1. Spectral representation of the $\omega^{-2}$ and $\omega^{-1}$ source models for two constant stress drop earthquakes observed at the same distance $R$ in a uniform, elastic, isotropic full space.
Earthquake Source Properties, Spectra, Scaling, Self-similarity

Hanks (1979)

0. Consider two events that differ in size by 10x and assume self-similarity of rupture so that their moments differ by a factor of $10^3$ and durations differ by a factor of 10 (rupture velocity is the same and constant for each event.) Take the events to be large enough so that their corner frequency is well below 1 sec.

1. Four cases for time-domain interpretation of spectral source models, 2 for each model.

$\omega$-square (spectral amplitude at 1 sec period is 10x greater for larger event)

a) If: 1-s. energy arrives continuously over the complete faulting duration.

Then: 1-s time domain amplitude is the same and $M_b$ is the same for both events

$M_b$ is independent of $M_o$.

b) If: all 1-s energy radiated at the same time (arrives at the same time)

Then: 1-s time domain amplitude is 10x larger for the larger event

$M_b$ scales directly with $M_o$.

$\omega$-cube (spectral amplitude at T=1 sec is the same for each event)

c) If: 1-s energy arrives continuously over the complete faulting duration.

Then: 1-s time domain amplitude is smaller for the larger event

$M_b$ scales inversely with $M_o$.

d) If: all 1-s energy radiated at the same time (arrives at the same time)

Then: 1-s time domain amplitude is the same for both events

$M_b$ is independent of $M_o$.

From data: cases (c) and (b) are clearly wrong: $M_b$ does not decrease with $M_o$ and $M_b$ does not increase beyond about 6.5. Either (a) or (d) could be right, but very simplified approach. Data tend to support $\omega$-square model (See Boatwright and Choy 1989) but also see $\omega^{-5/2}$ and lots of scatter. Propagation effects very hard to remove in practice.
Earthquake Source Properties, Spectra, Scaling, Self-similarity

Hanks (1979)

0. Consider two events that differ in size by 10x and assume self-similarity of rupture so that their moments differ by a factor of $10^3$ and durations differ by a factor of 10 (rupture velocity is the same and constant for each event.)

Take the events to be large enough so that their corner frequency is well below 1 sec.

1. Four cases for time-domain interpretation of spectral source models, 2 for each model.

ω-square (spectral amplitude at 1 sec period is 10x greater for larger event)

a) If: 1-s. energy arrives continuously over the complete faulting duration.

Then: 1-s time domain amplitude is the same and $M_b$ is the same for both events $M_b$ is independent of $M_o$.

b) If: all 1-s energy radiated at the same time (arrives at the same time)

Then: 1-s time domain amplitude is 10x larger for the larger event $M_b$ scales directly with $M_o$.
Consider two events that differ in size by 10x and assume self-similarity of rupture so that their moments differ by a factor of $10^3$ and durations differ by a factor of 10 (rupture velocity is the same and constant for each event.) Take the events to be large enough so that their corner frequency is well below 1 sec.

1. Four cases for time-domain interpretation of spectral source models, 2 for each model.

   - $\omega$-cube (spectral amplitude at $T=1$ sec is the same for each event)
   - c) If: 1-s energy arrives continuously over the complete faulting duration.
     Then: 1-s time domain amplitude is smaller for the larger event $M_b$ scales inversely with $M_o$.
   - d) If: all 1-s energy radiated at the same time (arrives at the same time)
     Then: 1-s time domain amplitude is the same for both events $M_b$ is independent of $M_o$. 

\[ \text{log } u \text{ at } R \quad \omega^3 \quad \omega_o \]

\[ \text{log freq. (}\omega\text{)} \]
Earthquake Scaling: Size-frequency of occurrence

Gutenberg-Richter frequency-magnitude scaling.

\[
N(M_o) \geq M_o \\
N(M_s) = aM_s^{-b} \\
N(M_o) = aM_o^{-B}
\]

GR scaling, with constant \( b \) implies self-similarity of earthquakes (rupture physics, fracture process, fault roughness, etc.)

Observed for the world-wide eq catalog

\[ b \sim 1 \]
\[ B \sim 2/3 \]
Earthquake Scaling: Size-frequency of occurrence

Gutenberg-Richter frequency-magnitude scaling.

\[ N(M_o) \geq M_o \]
\[ N(M_s) = aM_s^{-b} \]
\[ N(M_o) = aM_o^{-B} \]

\[ b \sim 1 \]
\[ B \sim 2/3 \]

Observed for the world-wide eq catalog

Scholz, 1990
Earthquake Scaling: Size–frequency of occurrence

\[ N(M_o) = aM_o^{-B} \]

Earthquake Size Distributions

- A: single fault
- B: fault population

Note dimensions

Cumulative number

Scholz, 1990
Gutenberg-Richter frequency-magnitude scaling.

What about scaling breaks?

\[ N(M_s) = aM_s^{-b} \]

\[ M_o = C \Delta \sigma r^3 \]

It could imply that stress drop is not independent of size or it could imply a preferred or characteristic size.

Earthquake Size Distributions

Note dimensions.
Gutenberg-Richter frequency-magnitude scaling.

What about scaling breaks?

It could imply that stress drop is not independent of size or it could imply a preferred or characteristic size.
Gutenberg-Richter frequency-magnitude scaling.

What about scaling breaks?

It could imply that stress drop is not independent of size or
It could imply a preferred or characteristic size

Ms = 7.3

Characteristic Earthquake model

Scholz, 1990
Scaling and Self-Similarity
Are small earthquakes ‘the same’ as large ones?
1 Geometric self-similarity: rupture aspect ratio
2 Physical self-similarity: stress drop, seismic strain, scaling of slip with rupture dimension

Circular ruptures (small)

\[ M_o = G u A \]
\[ \Delta \sigma = C \frac{G u}{r} \]
\[ M_o = C \Delta \sigma r^3 \]

Read Scholz, BSSA 1982
and
Scaling and Self-Similarity
Are small earthquakes ‘the same’ as large ones?
1 Geometric self-similarity: rupture aspect ratio
2 Physical self-similarity: stress drop, seismic strain, scaling of slip with rupture dimension

Circular ruptures (small)

\[
M_o = G_u A \\
\Delta \sigma = C \frac{G u}{r} \\
M_o = C \Delta \sigma r^3
\]

Hanks, 1977
Abercrombie & Leary, 1993
Circular ruptures (small)

\[ M_o = G u A \]
\[ \Delta \sigma = C \frac{u}{r} \]
\[ M_o = C \Delta \sigma r^3 \]

Rectangular ruptures (large)

Slip determined by \( W \):

\[ M_o = G u L W \]
\[ \Delta \sigma = C \frac{u}{W} \]
\[ M_o = C \Delta \sigma L W^2 \]

Slip determined by \( L \)

\[ M_o = G u L W \]
\[ \Delta \sigma = C \frac{u}{L} \]
\[ M_o = C \Delta \sigma W L^2 \]

note dimensions
Circular ruptures (small)

\[ M_o = G u A \]
\[ \Delta \sigma = C \ G \frac{u}{r} \]
\[ M_o = C \Delta \sigma \ r^3 \]

Transition from small to large eq's

Rectangular ruptures (large)

Slip determined by \( W \):

\[ M_o = G u L W \]
\[ \Delta \sigma = C \ G \frac{u}{W} \]
\[ M_o = C \Delta \sigma L W^2 \]

Slip determined by \( L \):

\[ M_o = G u L W \]
\[ \Delta \sigma = C \ G \frac{u}{L} \]
\[ M_o = C \Delta \sigma W L^2 \]

Shimazaki, 1986

FIG. 1. Two types of earthquakes: small (unbounded) and large (bounded). \( L \) is rupture length along strike of fault, \( W \) down-dip width of the rupture.
Rectangular ruptures (large)

Slip determined by W:

\[ M_o = GuLW \]
\[ \Delta \sigma = C \frac{u}{W} \]
\[ M_o = C\Delta \sigma LW^2 \]

Slip determined by L

\[ M_o = GuLW \]
\[ \Delta \sigma = C \frac{u}{L} \]
\[ M_o = C\Delta \sigma WL^2 \]

http://seismo.berkeley.edu/annual_report/ar01_02/node22.html

Fig. 1. A plot of mean slip, \( u \) versus fault length for the strike-slip events. The line drawn through the data has a slope of \( 1.25 \times 10^{-11} \). Numbers are references in Table 1.
Scaling of Large Earthquakes: Is slip determined (limited) by $W$ or $L$?

Rectangular ruptures (large)

- Slip determined by $W$:
  \[ M_o = C \Delta \sigma LW^2 \]

- Slip determined by $L$:
  \[ M_o = C \Delta \sigma WL^2 \]

http://seismo.berkeley.edu/annual_report/ar01_02/node22.html

**Figure 18.1:** Moment-length plot for the dataset described. Lines corresponding to $n = 3$ bracketing most of the data have been drawn for reference. Circles correspond to recent data for which length was estimated from the NEIC catalog.

**Figure 18.2:** Moment-length plots for $A$ (bottom) and $B$ (top) events. Best fitting $n = 1$ trends are indicated for each subset of data. Circles as in Figure 1, diamonds from other sources. Triangle is Luzon'90 event. Vertical lines point to the length estimates of PD96 for Aegean Sea events.
Scaling of Large Earthquakes: Is slip determined (limited) by $W$ or $L$?

Rectangular ruptures (large)

Slip determined by $W$:

$$M_o = C \Delta \sigma LW^2$$

Slip determined by $L$

$$M_o = C \Delta \sigma WL^2$$

---

Figure 18.2: Moment-length plots for $A$ (bottom) and $B$ (top) events. Best fitting $n = 1$ trends are indicated for each subset of data. Circles as in Figure 1, diamonds from other sources. Triangle is Luzon'90 event. Vertical lines point to the length estimates of PD96 for Aegean Sea events
Some Topics in the Mechanics of Earthquakes and Faulting

• What determines the size of an earthquake?
• What physical features and factors of faulting control the extent of dynamic earthquake rupture?  --Fault Area, Seismic Moment
• What is the role of fault geometry (offsets, roughness, thickness) versus rupture dynamics?
• What controls the amount of slip in an earthquake?  Average Slip, Slip at a point
• What controls whether fault slip occurs dynamically or quasi-statically?
• Nucleation: How does the earthquake process get going?
• What is the size of a nucleation patch at the time that slip becomes dynamic?  How do we define dynamic versus quasi-dynamic and quasi-static?  Nucleation patch: physical size, seismic signature
• What controls dynamic rupture velocity?
• How do faults grow and evolve with time?