Mechanics of Earthquakes and Faulting

Lecture 21, 15 Apr. 2021

www.geosc.psu.edu/Courses/Geosc508

- Seismic Spectra, Earthquake Scaling laws, Self-Similarity of Earthquake Rupture.

Aki, Scaling law of seismic spectrum, JGR, 72, 1217-1231, 1967.

Hanks, b Values and $\omega^{-\gamma}$ seismic source models: implications for tectonic stress variations along active crustal fault zones and the estimation of high-frequency strong ground motion, JGR, 84, 2235-2242, 1979.

Scholz, BSSA 1982

Pacheco, J. F. Scholz, C. H. Sykes, L. R. (1992). Changes in frequency-size relationship from small to large earthquakes, Nature 355, 71- 73.

Scaling of Large Earthquakes: Is slip determined (limited) by W or L?

Rectangular ruptures (large)

Slip determined by W:

$$M_o = GuLW$$
$$\Delta\sigma = C\,G\frac{u}{W}$$
$$M_o = C\Delta\sigma LW^2$$

Slip determined by L

$$M_o = GuLW$$
$$\Delta\sigma = C\,G\frac{u}{L}$$
$$M_o = C\Delta\sigma W L^2$$

L model interpretation

u = α L

4 CHRISTOPHER H. SCHOLZ

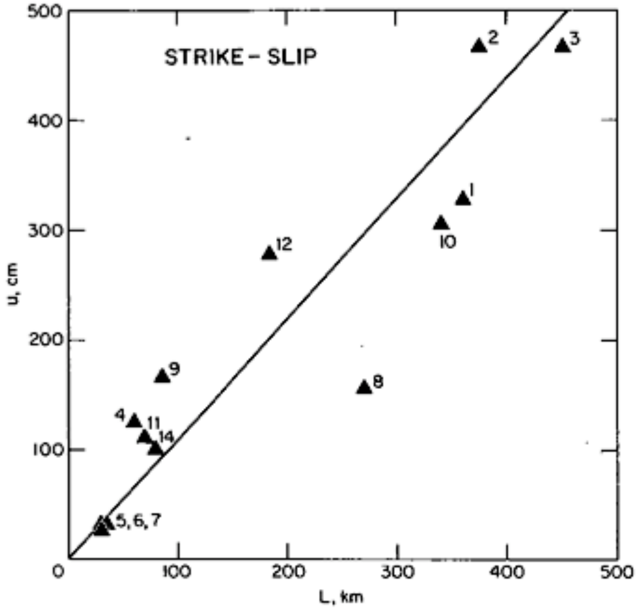

FIG. 1. A plot of mean slip, u versus fault length for the strike-slip events. The line drawn through the data has a slope of 1.25×10^{-5}. Numbers are references in Table 1.

W model interpretation: stress drop increases with L/W and seismic moment

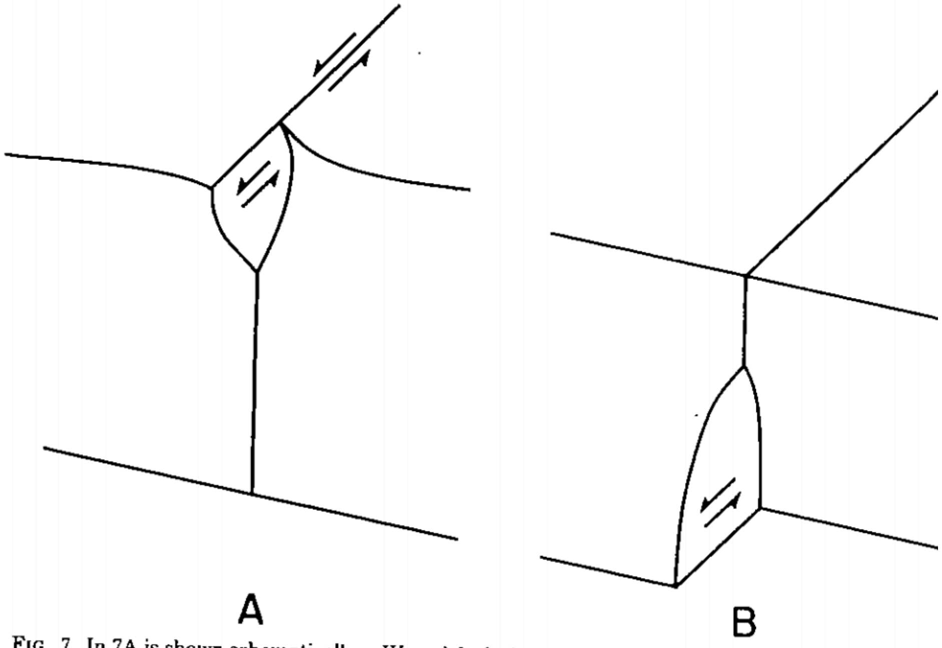

FIG. 7. In 7A is shown schematically a *W* model of a large earthquake, in which slip is constrained t be zero at the base of the fault. In 7B, we show a possible mechanism that may result in an *L* model. Th figure shows the situation just prior to a large earthquake. Aseismic slip has occurred beneath th seismogenic layer. If this preslip is larger than the slip in the earthquake, the earthquake may not b constrained at the base.

W model interpretation: stress drop increases with L/W and seismic moment

L model interpretation

$u = \alpha L$

Scaling of Large Earthquakes: Is slip determined (limited) by W or L?

Rectangular ruptures (large)

Slip determined by W:

$$M_o = C \Delta \sigma L W^2$$

Slip determined by L

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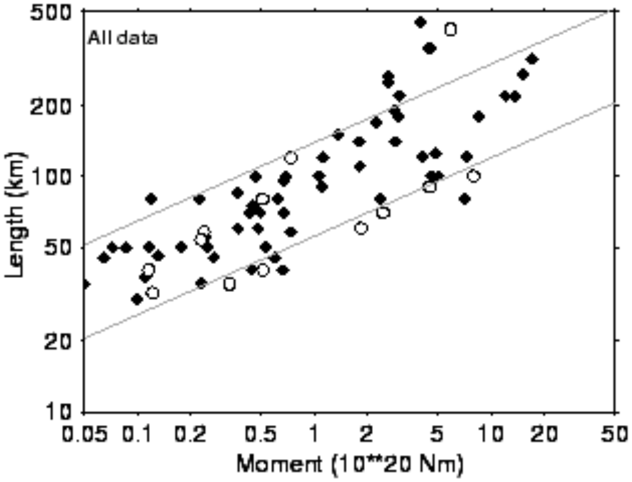


Figure 18.1: Moment-length plot for the dataset described. Lines corresponding to $n = 3$ bracketing most of the data have been drawn for reference. Circles correspond to recent data for which length was estimated from the NEIC catalog.

GEOPHYSICAL RESEARCH LETTERS, VOL. 29, NO. 12, 1604, 10.1029/2001GL014479, 2002

On moment-length scaling of large strike slip earthquakes and the strength of faults

B. Romanowicz[1] and L. J. Ruff [2]

Received 29 November 2001; revised 8 March 2002; accepted 11 March 2002; published 29 June 2002.

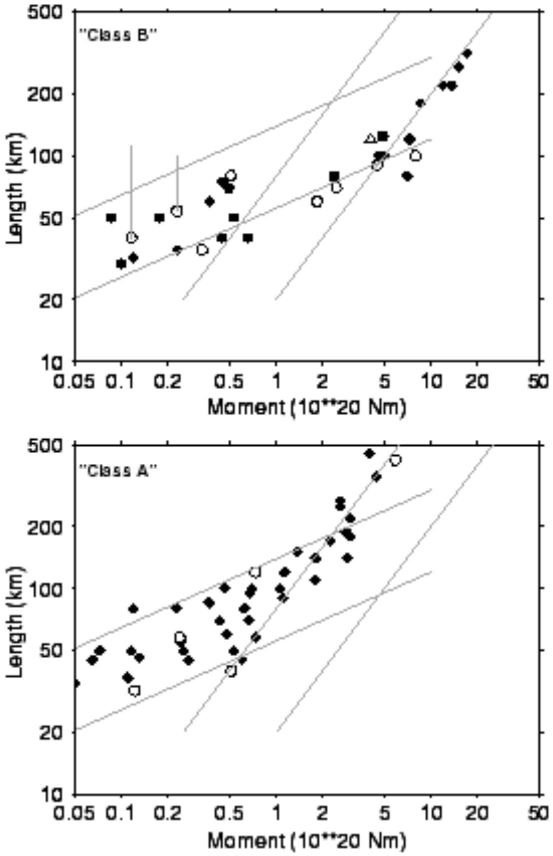
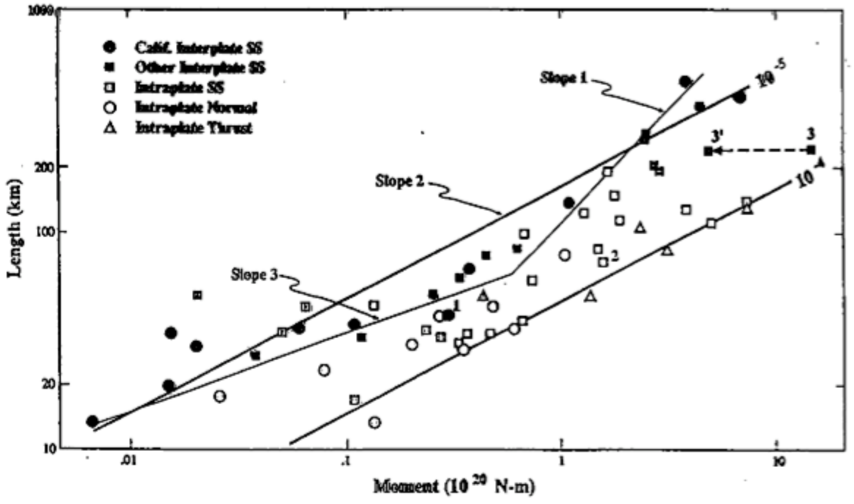


Figure 18.2: Moment-length plots for A (bottom) and B (top) events. Best fitting $n = 1$ trends are indicated for each subset of data. Circles as in Figure 1, diamonds from other sources. Triangle is Luzon'90 event. Vertical lines point to the length estimates of PD96 for Aegean Sea events

Scaling of Large Earthquakes: Is slip determined (limited) by W or L?



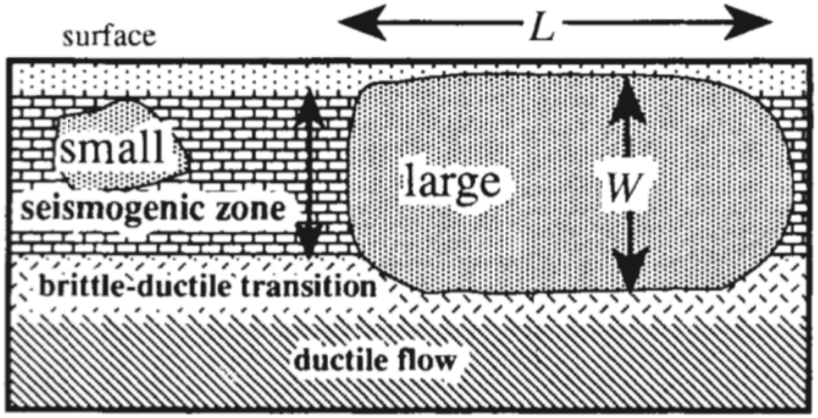


FIG. 1 Two types of earthquakes; small (unbounded) and large (bounded). *L* is rupture length, along strike of fault, *W* down-dip width of the rupture.

Changes in frequency–size relationship from small to large earthquakes

Javier F. Pacheco, Christopher H. Scholz & Lynn R. Sykes

Lamont-Doherty Geological Observatory and Department of Geological Sciences, Columbia University, Palisades, New York 10964, USA

THE constant ‘*b* value’ observed in frequency–magnitude distributions of earthquakes has been taken as an indication of self-

Nature, 1992

Circular ruptures (small)

$$M_o = GuA$$

$$\Delta\sigma = C\,G\frac{u}{r}$$

$$M_o = C\,\Delta\sigma\,r^3$$

Transition from small to large eq's

Rectangular ruptures (large)

Slip determined by W:

$$M_o = GuLW$$

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$$M_o = C\Delta\sigma LW^2$$

Slip determined by L

$$M_o = GuLW$$

$$\Delta\sigma = C\,G\frac{u}{L}$$

$$M_o = C\Delta\sigma WL^2$$

SMALL AND LARGE EARTHQUAKES:
THE EFFECTS OF THE THICKNESS OF SEISMOGENIC LAYER AND THE FREE SURFACE

Kunihiko Shimazaki

Earthquake Research Institute, University of Tokyo, Bunkyo-ku, Tokyo, Japan 113

AGU Monograph, 1986

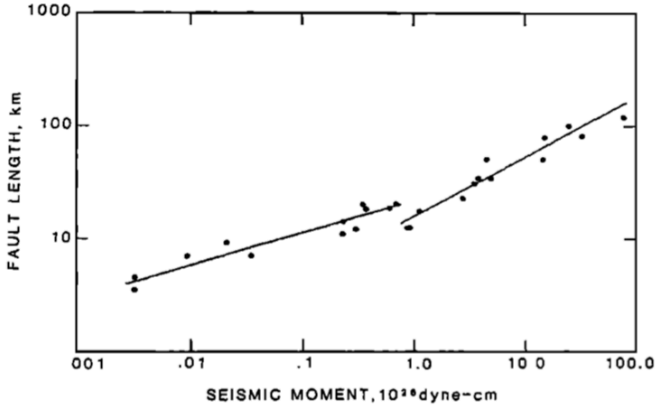
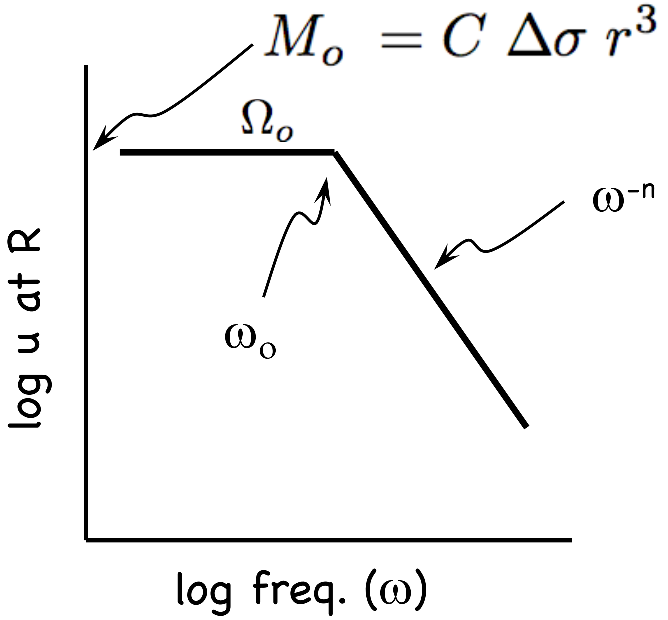


Fig. 1. A plot of logarithm of fault length against that of seismic moment. The whole dataset for Japanese intraplate earthquakes are divided into two at the seismic moment of 7.5 x 10^{25} dyne-cm and the linear regression analysis is applied to each dataset. This segmented fit is shown to be statiscally better than models shown in Figure 2.

Earthquake Source Properties, Spectra, Scaling, Self-similarity

Aki, Scaling law of seismic spectrum, JGR, 72, 1217–1231, 1967.

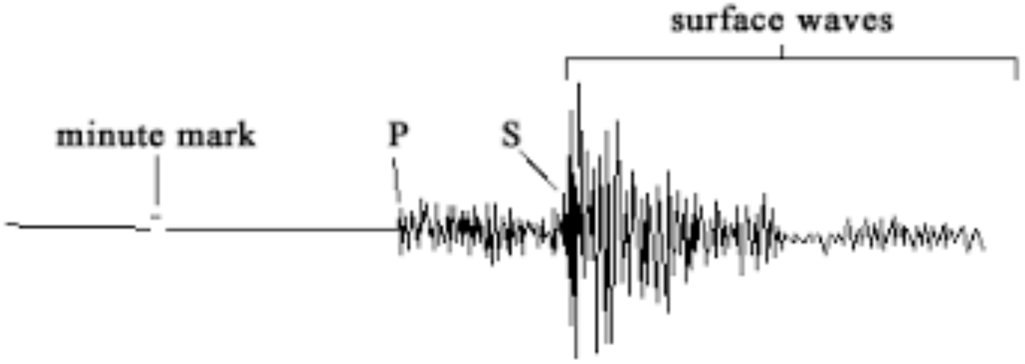
Displacement and acceleration source spectra.
Spectra: zero-frequency intercept (M_o), corner frequency (ω_o or f_c), high frequency decay ($\omega^{-\gamma}$), maximum (observed, emitted) frequency f_{max}



$$M_o = C\,\Delta\sigma\,r^3$$

ω-square model, ω^{-2}

ω-cube model, ω^{-3}



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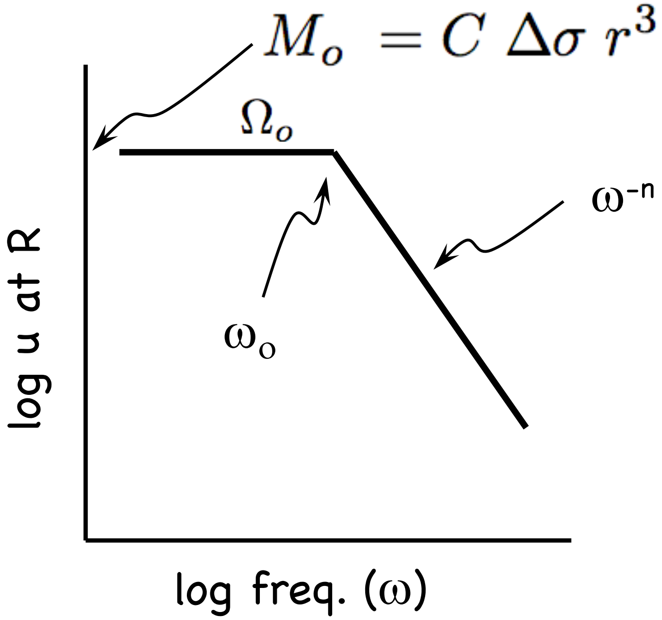


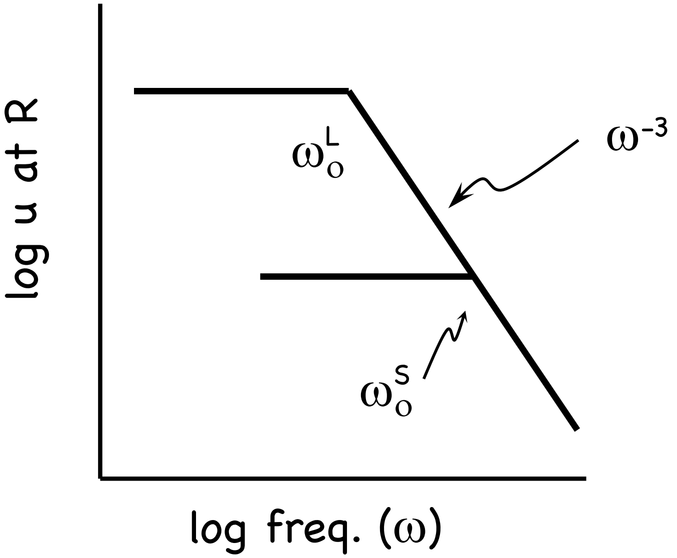
ω-square model, ω^{-2}

ω-cube model, ω^{-3}

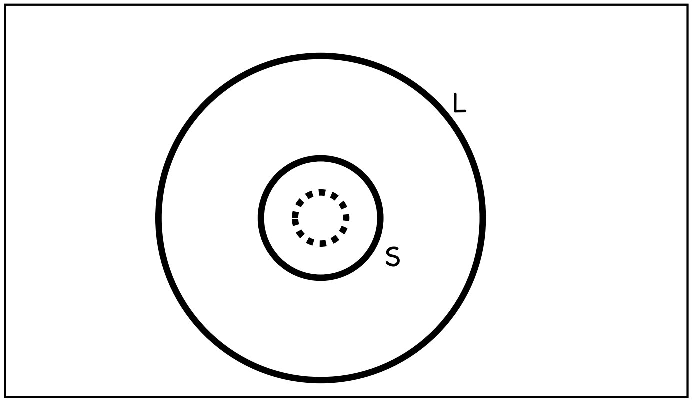
Earthquake Source Properties, Spectra, Scaling, Self-similarity

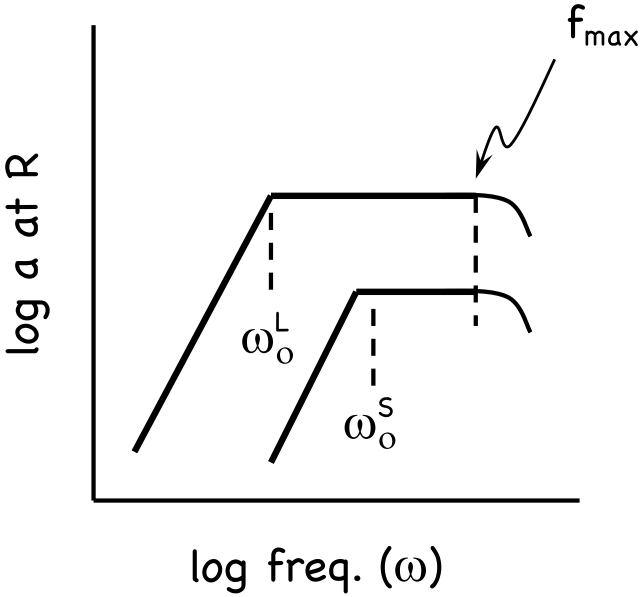
Source spectra for two events of equal stress drop: **omega cube model**

$$\Omega(\omega) = \frac{\Omega(o)}{[1 + (\frac{\omega}{\omega_o})^2]^{\frac{3}{2}}}$$



Large and Small Eq





High-freq. spectral properties:
produced by rupture growth,
represent nucleation and enlargement

Earthquake Source Properties, Spectra, Scaling, Self-similarity

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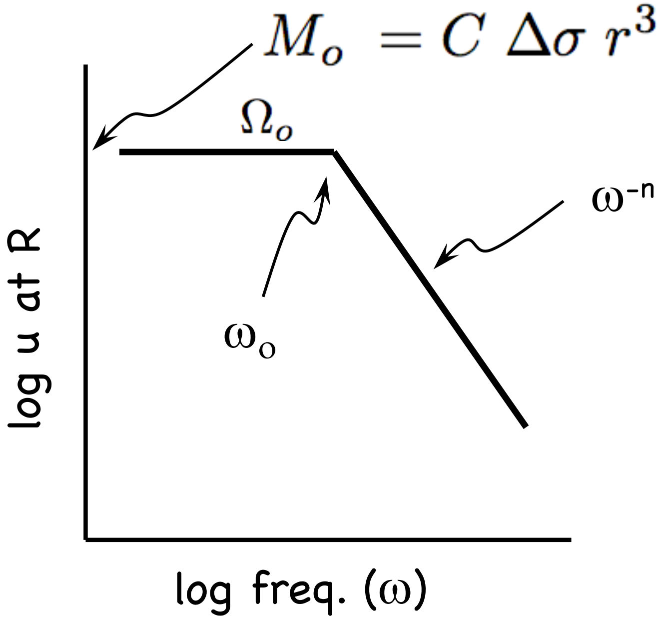


ω-square model, ω^{-2}

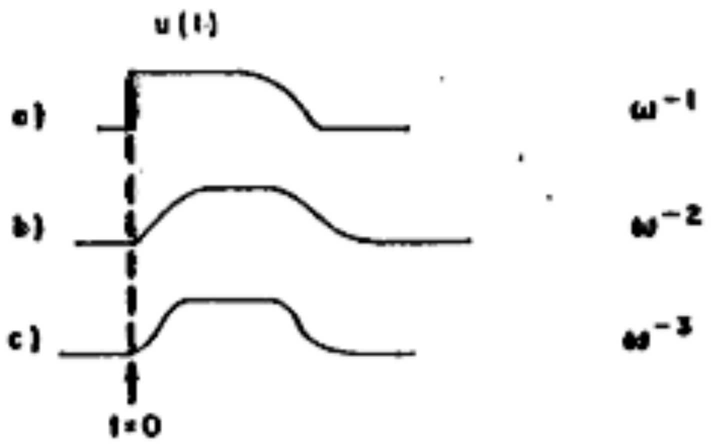
ω-cube model, ω^{-3}

Far-field body-wave spectra and relation to source slip function

Displacement waveform for P & S waves:

$$\Omega(x,t) = \int_{\Sigma} \dot{\Delta u}\left(\chi, t - \frac{r}{c}\right) d\Sigma.$$

In general, very complex. $\Omega(x, t)$ and $\Omega(\omega)$ depend on slip function, azimuth to observer and relative importance of nucleation and stopping phases



Earthquake Source Properties, Spectra, Scaling, Self-similarity

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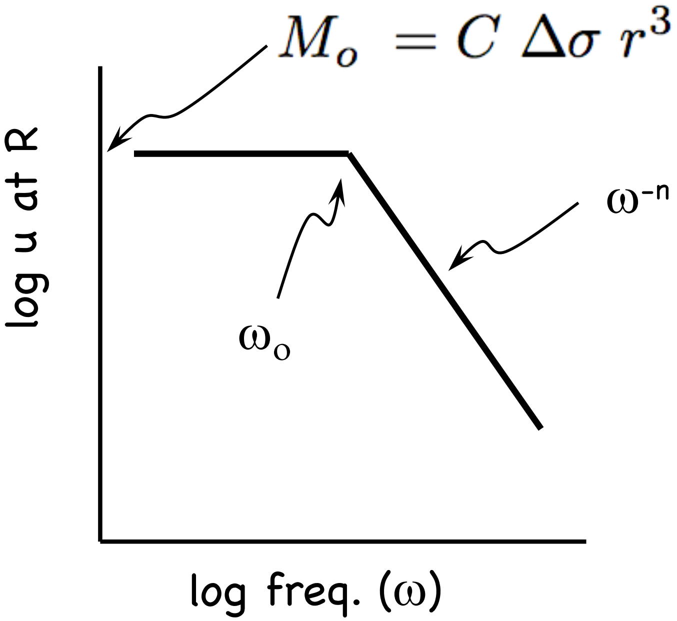

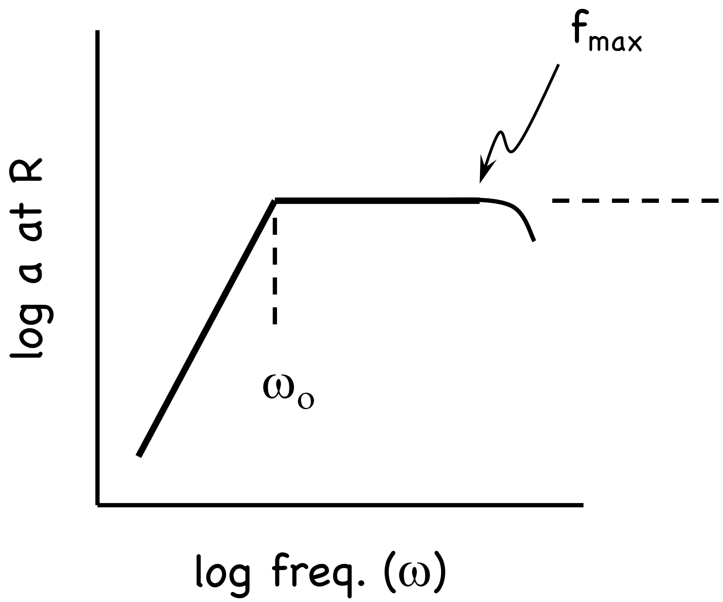

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Hanks, b Values and $\omega^{-\gamma}$ seismic source models: implications for tectonic stress variations along active crustal fault zones and the estimation of high-frequency strong ground motion, JGR, 84, 2235-2242, 1979.

Seismic Source Spectra.

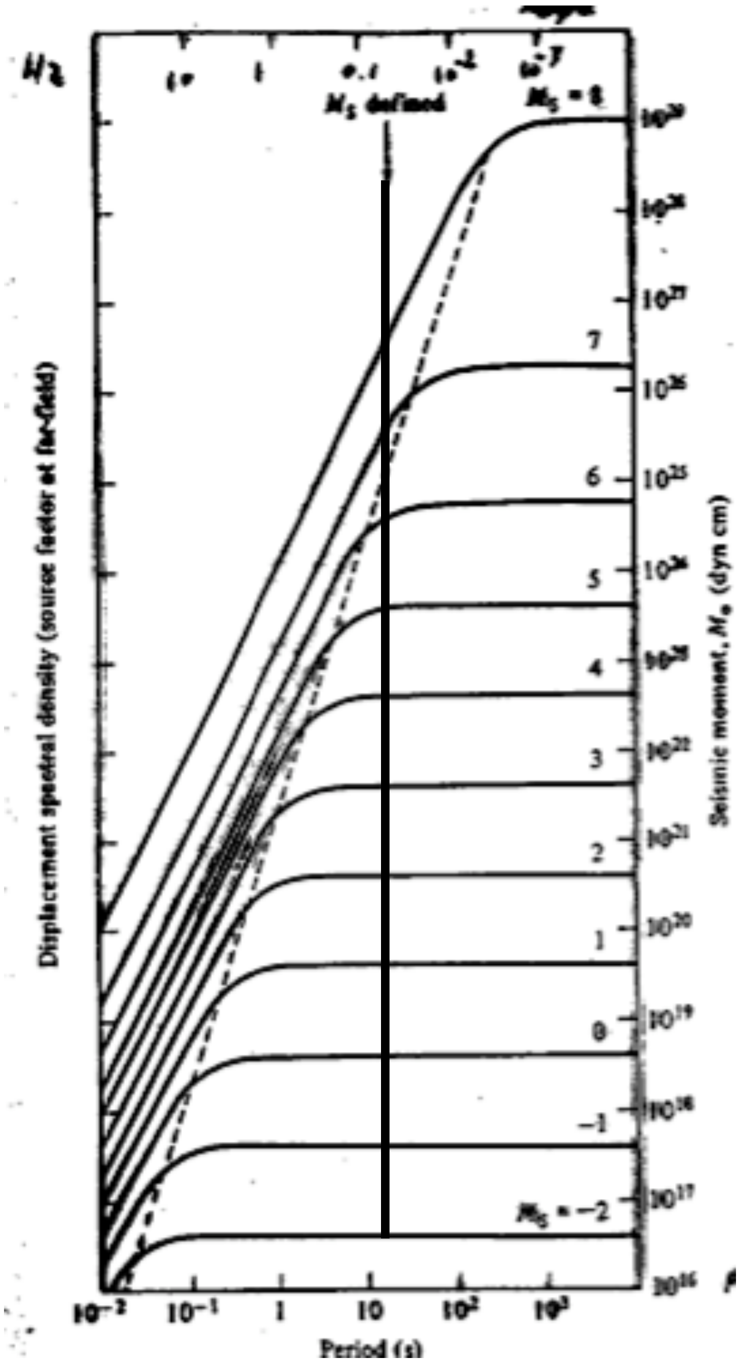
Saturation occurs for large events, particularly saturation of M_s (T=20 s)

$$M_o = GuA$$

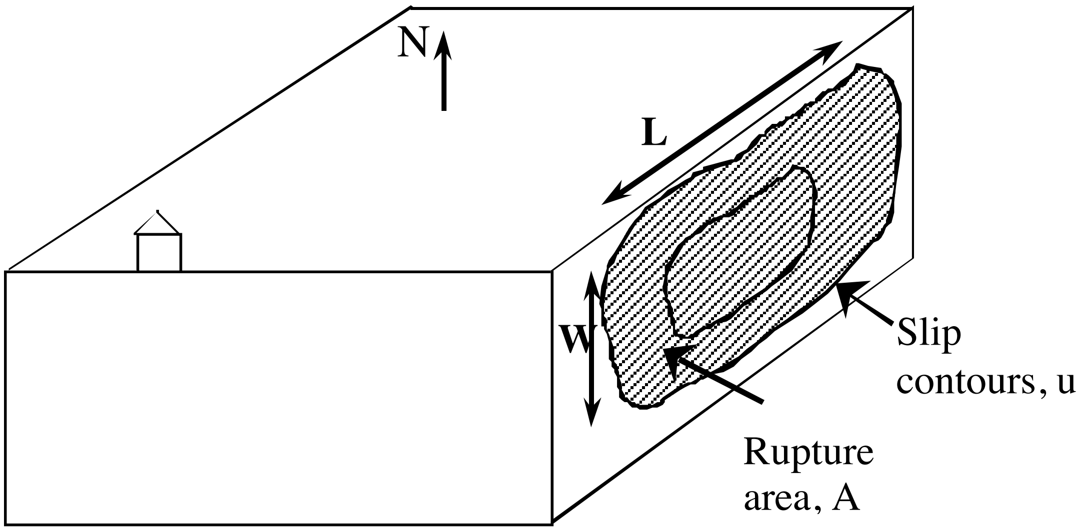
$$M_o = C\,\Delta\sigma\, r^3$$

$$M_o = C\,\Delta\sigma\, f_c^{-3}$$

Corner frequency, Brune Stress drop.



Aki, 1967

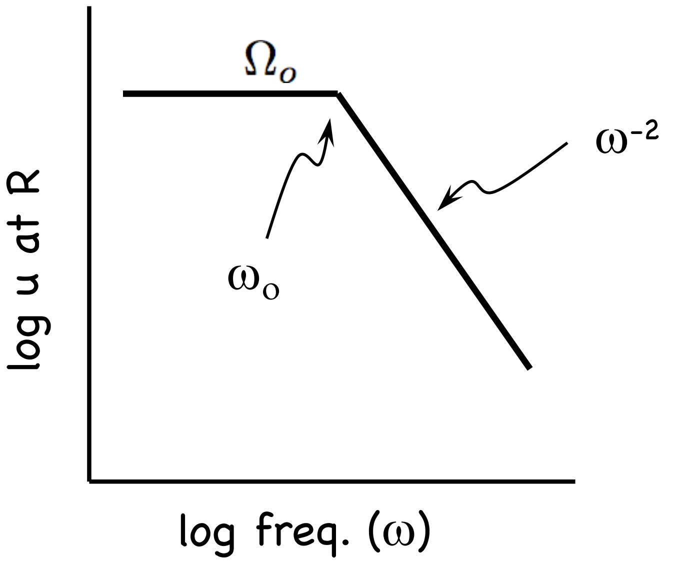


Magnitude and Seismic Moment. Moment is a most robust measure of earthquake size because magnitude is a measure of size at only one frequency.

$M_o = \mu A u$, where μ is shear modulus, A is fault Area and u is mean slip.

Moment and Moment Magnitude (Hanks and Kanamori, JGR, 1979):
$M_w = 2/3 \log M_o - 6$ or
$M_o = 3/2 M_w + 9$ (for M_o in N-m)

Earthquake Source Properties, Spectra, Scaling, Self-similarity



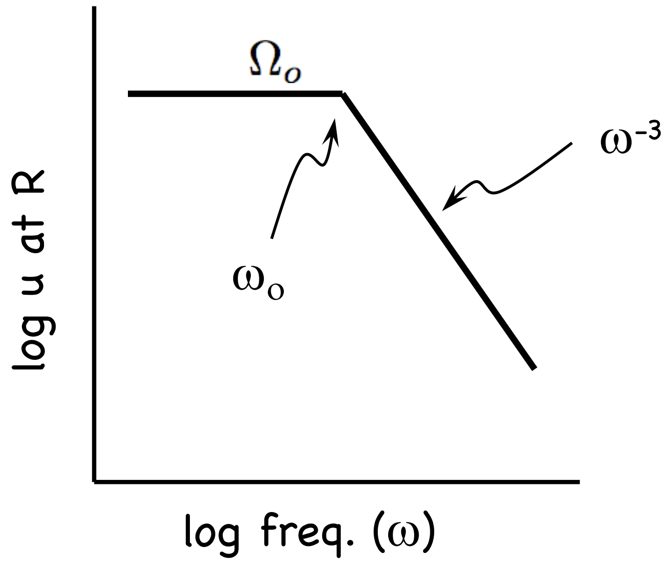
$$\Omega(\omega) = \frac{\Omega(o)}{1 + (\frac{\omega}{\omega_o})^2}$$

ω-square model, ω^{-2}
Two possible explanations

1) !Similarity condition (not-similarity)
$M_o \alpha L^2$
$\omega_o \alpha L^{-1}$
$\Omega(0) \alpha \omega_o^{-2}$

2) Have similarity condition in terms of nucleation, but high-freq. asymptote is produced by “stopping phase” if rupture stops very abruptly

Earthquake Source Properties, Spectra, Scaling, Self-similarity



$$\Omega(\omega) = \frac{\Omega(o)}{[1+(\frac{\omega}{\omega_o})^2]^{\frac{3}{2}}}$$

ω-cube model, ω^{-3}

Similarity condition

$M_o \propto L^3$

$\omega_o \propto L^{-1}$

$\Omega(0) \propto \omega_o^{-3}$

This defines a scaling law. Spectral curves differ by a constant factor at a given period (e.g., 20 s), but they have the same high-freq. asymptote

This behavior is expected when the nucleation phase is responsible for the high-freq. asymptote --but consider problem of time domain implication for amplitude (M_b decreases with M_o)

Earthquake Source Properties, Spectra, Scaling, Self-similarity

Source spectra for two events of equal stress drop: **omega cube model**

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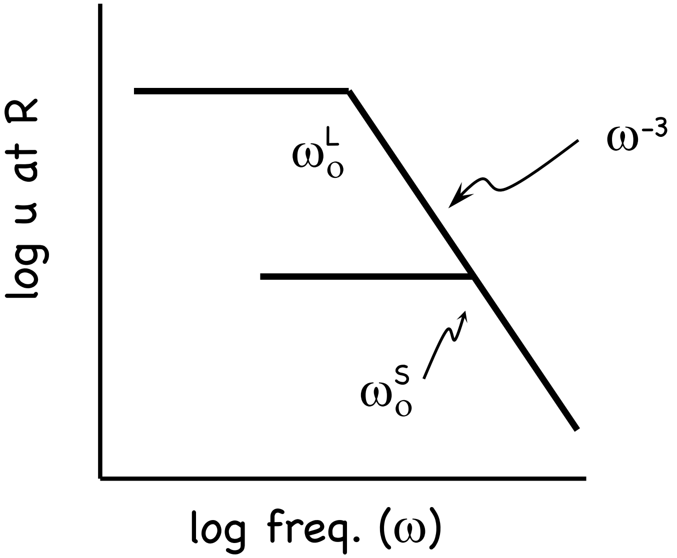
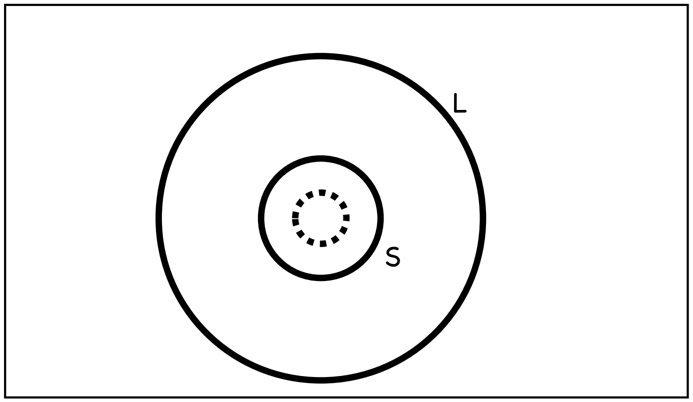
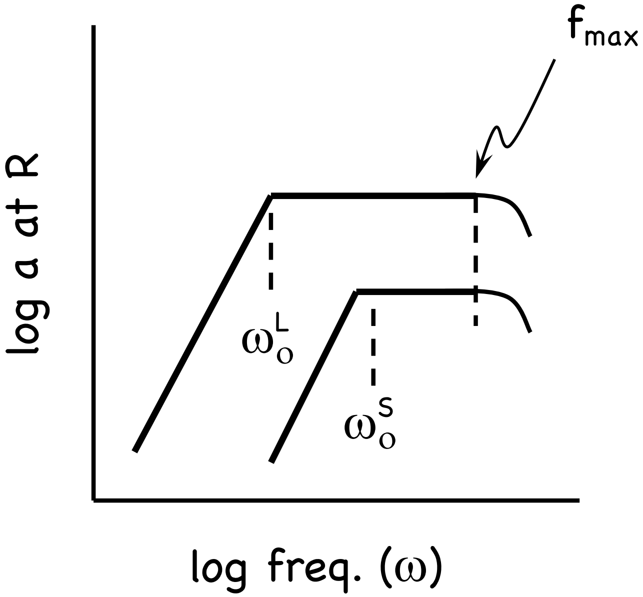


Large and Small Eq

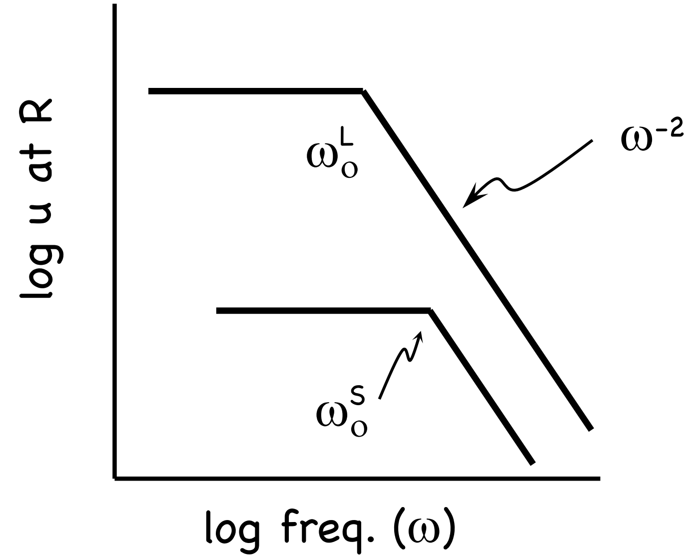


High-freq. spectral properties:
produced by rupture growth,
represent nucleation and enlargement



Source spectra for two events of equal stress drop: omega square model

$$\Omega(\omega) = \frac{\Omega(o)}{1 + (\frac{\omega}{\omega_o})^2}$$



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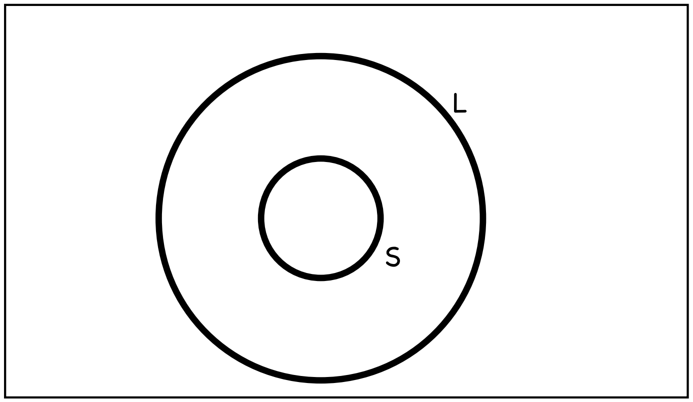

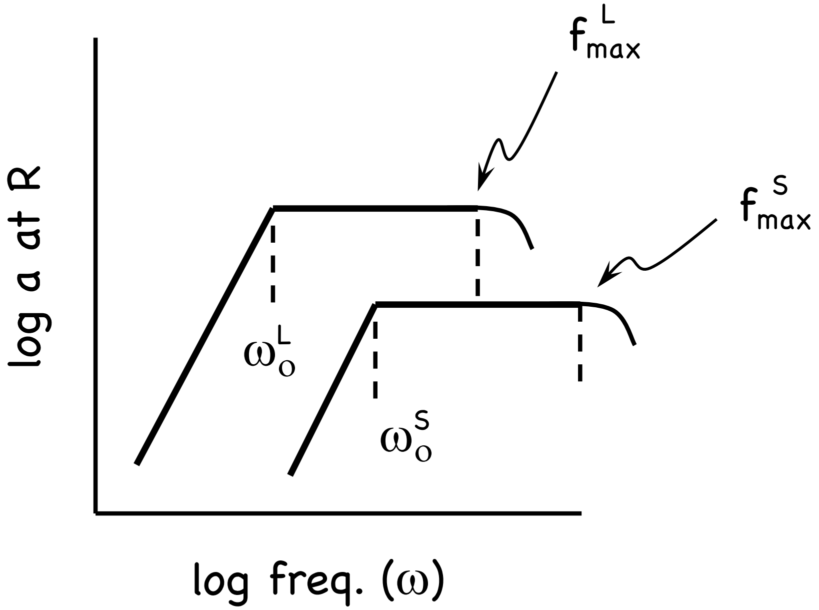

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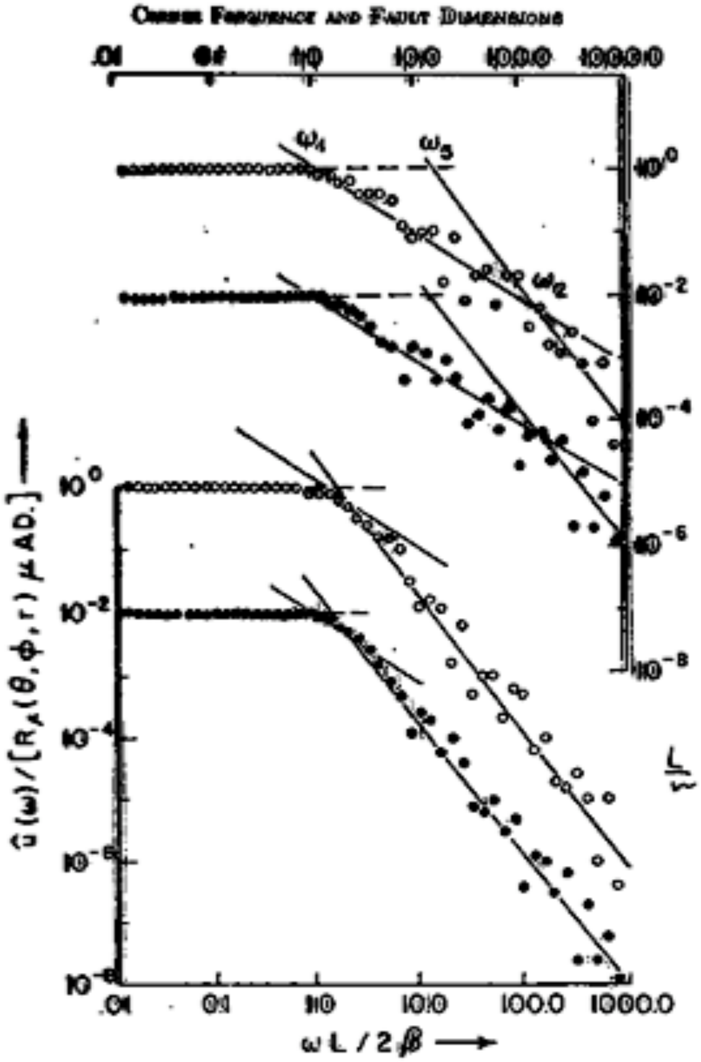
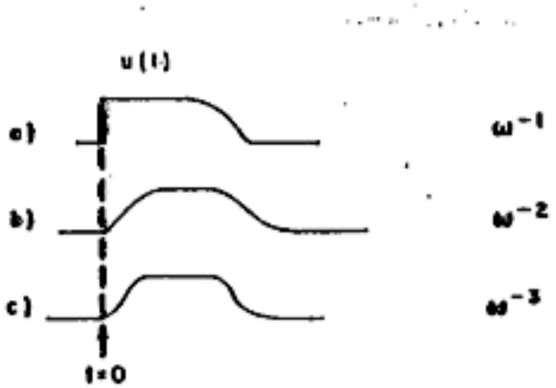
Earthquake Source Properties, Spectra, Scaling, Self-similarity



$\frac{L}{w} \sim 100$

$\frac{L}{w} \sim 1.0$



Relation between source
(a) displacement
(b) velocity
(c) acceleration
history and asymptotic behavior of spectrum

Some Topics in the Mechanics of Earthquakes and Faulting

- What determines the size of an earthquake?
- What physical features and factors of faulting control the extent of dynamic earthquake rupture? --Fault Area, Seismic Moment
- What is the role of fault geometry (offsets, roughness, thickness) versus rupture dynamics ?
- What controls the amount of slip in an earthquake? Average Slip, Slip at a point
- What controls whether fault slip occurs dynamically or quasi-statically?
- Nucleation: How does the earthquake process get going?
- What is the size of a nucleation patch at the time that slip becomes dynamic? How do we define dynamic versus quasi-dynamic and quasi-static? Nucleation patch: physical size, seismic signature
- What controls dynamic rupture velocity?
- How do faults grow and evolve with time?