• Rupture propagation and the crack tip stress field
• Cohesive zone; transition from ‘broken’ to ‘unbroken’ as a crack propagates.
• Think of this as a method to define ‘partially broken’
• Energy Balance of Dynamic Faulting
• Faulting types, stress polygons
• Wear and fault roughness
• Thermo-mechanics of faulting
• Moment, Magnitude and scaling laws for earthquake source parameters
Stress field is singular at the crack tip.
• because we assumed perfectly sharp crack (but real materials cannot support infinite stress)

\[
\begin{bmatrix}
\sigma_{22} \\
\sigma_{21} \\
\sigma_{23}
\end{bmatrix}_{\text{tip}} \approx \frac{1}{\sqrt{2\pi r}} \begin{bmatrix} K_I \\ K_{II} \\ K_{III} \end{bmatrix}
\]

\[K_I = \sqrt{\pi c} \sigma_\infty\]
Dislocation model for fracture and earthquake rupture

Dislocation model, circular crack
\[ \Delta \sigma = (\sigma_o - \sigma_f) \]

\[ \Delta u(x, y) = \frac{24}{7\pi} \frac{\Delta \sigma}{\mu} \sqrt{c^2 - (x^2 + y^2)} \]
Crack tip stress field

- Singular crack (Eshelby)

\[
\sigma_{ij}(\theta, r) = \frac{K_{\Pi}}{\sqrt{2\pi r}} \Sigma_{ij}(\theta, C_f), \quad (1)
\]


See also: Johnson and Scholz, JGR, 1976.
Crack tip stress field


See also: Johnson and Scholz, JGR, 1976.

\[ \sigma_{ij}(\theta, r) = \frac{K_{II}}{\sqrt{2\pi r}} \Sigma_{ij}(\theta, C_f), \] (1)
Cohesive zone, slip weakening crack model for friction

Shear Stress

$\sigma_0$

$\sigma_f$

$\sigma_y$

Cracked/Slipping zone

Breakdown (cohesive) zone

Intact, locked zone

Shear stress

Shear displacement

$\delta$

$\delta^*$

$E_f$

$\tau^*$

$\tau^p$

$d_c$

Slip, displacement
Energy Balance of Dynamic Faulting

The rate of energy change is a balance between work terms, surface energy, kinetic energy, and frictional work.

\[ \dot{U} = 0 = -\dot{W} + \dot{U}_e + \dot{U}_s + \dot{U}_k + \dot{U}_f \]

These terms operate over different regions.

\[
\begin{align*}
\dot{U}_k &= \frac{\partial}{\partial t} \frac{1}{2} \int_V \rho u_i \dot{u}_i dV \\
\dot{U}_s &= \frac{\partial}{\partial t} \int_{\Sigma_0} 2 \gamma dS \\
\dot{W} &= \int_{S_0} \sigma_{ij} \dot{u}_i n_j dS \\
\dot{U}_f &= \frac{\partial}{\partial t} \int_{\Sigma} \sigma_{ij} u_i n_j dS \\
\dot{U}_e &= \frac{\partial}{\partial t} \frac{1}{2} \int_{V-V_0} \sigma_{ij} \epsilon_{ij} dV
\end{align*}
\]

Recall that in the Linear Elastic Fracture Mechanics Approach to Dynamic Crack Propagation:

LEFM assumes that cracks are cohesionless. In this case the crack tip energy term, \( U_s \), can be determined.
Energy Balance of Dynamic Faulting

\[ \dot{U}_k = \frac{\partial}{\partial t} \left( \frac{1}{2} \int_V \rho u_i u_i dV \right) \]
\[ \dot{U}_s = \frac{\partial}{\partial t} \left( \int_{\Sigma_0} 2\gamma dS \right) \]
\[ \dot{W} = \int_{S_0} \sigma_{ij} u_i n_j dS \]
\[ \dot{U}_f = \frac{\partial}{\partial t} \left( \int_{\Sigma} \sigma_{ij} u_i n_j dS \right) \]
\[ \dot{U}_e = \frac{\partial}{\partial t} \left( \frac{1}{2} \int_{V-V_0} \sigma_{ij} \varepsilon_{ij} dV \right) \]

Recall that in the Linear Elastic Fracture Mechanics Approach to Dynamic Crack Propagation:

LEFM assumes that cracks are cohesionless. In this case the crack tip energy term, \( U_s \), can be determined

Energy Partitioning

If we choose the bounding surface sufficiently large, work at external boundaries, \( W \), is zero. Then the energy remaining to be radiated seismically is the dynamic change in kinetic energy (\( \Delta \) implies a change in the state during dynamic rupture relative to the initial state).
The change in internal strain energy can be written in terms of the drop in shear stress associated with rupture \( \Delta \sigma = \sigma_1 - \sigma_2 \)

\[
\Delta U_e = -\frac{1}{2} (\sigma_1 + \sigma_2) \bar{\Delta u} A
\]

where we assume that the initial stress, \( \sigma_1 \), is equal to the critical stress for failure.
That is, the net change in strain energy due to cracking is just equal to the work of faulting \( W_f \).
Energy Balance of Dynamic Faulting

If we assume that shear stress during slip is equal to a frictional stress, a dynamic friction term, $\sigma_f$, we can define a dynamic stress drop $\Delta \sigma_d = (\sigma_1 - \sigma_f)$, where $\sigma_1$ is the initial stress—which is not necessarily equal to the yield strength $\sigma_y$.

In this case the seismic energy is, $E_s \approx \frac{1}{2} \Delta \sigma \Delta u A$.

if we assume that $\sigma_2$ equals $\sigma_f$—e.g., the final stress is exactly the same as the dynamic frictional strength. (But what about dynamic overshoot, or healing pulse rupture models?)

The above equation for $E_s$ indicates that seismic energy depends only on the stress change, and not on the total stress. This is a big problem if you'd like to determine the complete energy balance for faulting, for example as needed for the heat flow-faulting stress problem.
Energy Balance of Dynamic Faulting

Dynamic Crack Propagation: Two Approaches

1) Energy balance, fracture mechanics, critical energy release rate.
   Problem of stress concentrations and singularities
2) Stick-slip frictional model. Critical stress needed for failure

Energy Balance: Specific fracture energy $G_c$

For a mode III crack, we can calculate a critical half length for crack extension

$$L_c = \frac{2}{\pi} \frac{G G_c}{(\sigma_1 - \sigma_2)^2}$$

Note that $G$ is shear modulus, sometimes written $\mu$.

$L_c$ is of order 1-2 m for $G_c$ of $10^2$ J/m$^2$.

Background: How does this relate to dislocation mechanics and or energy flux to/from a crack tip?

But note range of fracture energy values as a function of fracture size (Table 1.1). Earthquakes have very large values. This could imply a large process zone in those cases due to the stress concentrations associated with large ruptures.
Energy Balance of Dynamic Faulting

Seismic energy is, 

\[ E_s \approx \frac{1}{2} \Delta \sigma \Delta u A \]

if we assume that \( \sigma_2 \) equals \( \sigma_f \) - e.g., the final stress is exactly the same as the dynamic frictional strength. (But what about dynamic overshoot, or healing pulse rupture models?)

Seismic Efficiency

\[ \eta = \frac{E_s}{W_f} \approx \frac{\Delta \sigma}{\sigma_1 + \sigma_2} \]

and \( \eta \) is generally found to be 5-10%.

Static vs. Dynamic stress drop

Dynamic overshoot would make \( \Delta \sigma > \Delta \sigma_d \). Consider the role of inertia and lumped mass models.

Slip pulse rupture models (e.g., Scholz, 1980; Heaton, 1990) would make \( \Delta \sigma < \Delta \sigma_d \).
Energy Balance:  Specific fracture energy $G_c$

For a mode III crack, we can calculate a critical half length for crack extension.

$$L_c = \frac{2}{\pi} \frac{G G_c}{(\sigma_1 - \sigma_2)^2}$$

Note that $G$ is shear modulus, sometimes written $\mu$.

$L_c$ is of order 1-2 m for $G_c$ of $10^2$ J/m$^2$.

What about the stress concentration implied by such a model? How is the singularity resolved? (process zone? Irwin?)

$$\sigma_{ij} = K_n \left(2\pi r\right)^{-1/2} f_{ij}(\theta)$$

See:


See also: Johnson and Scholz, JGR, 1976.
Slip weakening model (Ida, 1972, 1973)

$G_c$ is the fracture energy

$$L_c = d_0 \frac{G}{\pi} \frac{\left(\sigma_y - \sigma_f\right)}{\left(\sigma_1 - \sigma_f\right)^2}$$

This is akin to the friction-based result, in which $G$ is Shear modulus

$$R_c = \frac{C G D_c}{\sigma_n (b-a)} \text{ or } L_c = \frac{E D_c}{2(1-\nu^2)\sigma_n \Delta \mu}$$

where $C$ is a geometric constant and $\nu$ is Poisson's ratio.

Recall the derivation of this result:
Concept of strength excess $S$

$$S = \frac{\sigma_y - \sigma_1}{\sigma_1 - \sigma_f}$$
Slip weakening model (Ida, 1972, 1973)

\[ R_c = \frac{CGD_c}{\sigma_n(b-a)} \quad \text{or} \quad L_c = \frac{ED_c}{2(1-\nu^2)\sigma_n \Delta \mu} \]

Recall the derivation of this result:

Frictional Instability

Requires \( K < K_c \)

\[ K_c = \frac{\sigma_n(b-a)}{D_c} \]

\[ \Delta \sigma = \frac{16 \pi}{7} \mu \frac{\Delta \bar{u}}{r} \]

\[ \Delta \sigma = \frac{24 \pi}{7} \mu \frac{\Delta u_{\text{max}}}{r} \]

Relation between stress drop and slip for a circular dislocation (crack) with radius \( r \)

For \( \nu = 0.25 \), Chinnery (1969)

Homework:
Determine the minimum earthquake size (magnitude and moment) assuming: \( G = \mu = 30 \) GPa, normal stress = 10 MPa, \( b-a = 0.01 \), and \( D_c = 100 \mu m \). Show all work, discuss any assumptions and empirical relations used.
Slip weakening model (Ida, 1972, 1973)

$G_c$ is the fracture energy

$$L_c = d_0 \frac{G}{\pi} \frac{(\sigma_y - \sigma_f)}{(\sigma_1 - \sigma_f)^2}$$

In this model, rupture propagation is highly dependent on the strength parameter $S$

$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$

where numerator is the strength excess and the denominator is the stress drop
cohesive zone/slip weakening crack model for friction

\[ f_{\text{max}} \text{ scales as: } f_{\text{max}} = \frac{V_r}{w} \]

\[ \Delta \sigma = \frac{7\pi}{24} G u_{\text{max}} \frac{r}{r} \]

\[ \Delta \sigma = \frac{7\pi}{24} G D_c \frac{w}{w} \]

\[ D_c = \frac{24}{7\pi} \frac{\Delta \sigma}{G} w \]
Mode II crack propagation

Cohesive zone length \( w \) scales as:

\[
N \quad \text{Propagating Rupture}
\]

\[
w \approx C \frac{D_c G}{\Delta \sigma}
\]

\[
\frac{w}{D_c} \approx C \frac{G}{\Delta \sigma}
\]
Earthquake Source Parameters and Scaling Relations

this was the original thinking. Recent work suggests there are problems with this. See Frank and Brodsky, 2019; Michel et al., 2019

\[ \Delta \sigma = \frac{7\pi}{16} G \bar{u} \]

\[ M_o = G \bar{u} A \]

\[ M_o = C \Delta \sigma r^3 \]

\[ V_r = \frac{r}{T} \]

\[ M_o = C \Delta \sigma V_r^3 T^3 \]

Ide et al., 2007; Peng and Gomberg, 2010
Rupture Patch Size for Slow Earthquakes

\[ \eta = 0.25 \]

Unstable if \( K < K_c \)

\[
K = \frac{\Delta \sigma}{\bar{u}} = \frac{7\pi \ G}{16 \ r}
\]

\[
K_c \approx \frac{\sigma_n(b - a)}{D_c}
\]

Slow earthquake nucleation when \( \frac{K}{K_c} \approx 1.0 \)

\[
h^* = r_c = \frac{GD_c}{\sigma_n(b - a)}
\]
Slow slip when effective rupture patch size is limited by heterogeneity?

\[ M_o^{\text{patch}} = G\bar{u}r^2 \]

\[ M_o = C\Delta\sigma r^3 \]

\[ M_o \approx V_r T \]
Lab data show a continuous spectrum from fast to slow slip.

Gomberg et al., 2016
Similar scaling laws for earthquakes and Cascadia slow-slip events

https://doi.org/10.1038/s41586-019-1673-6

Sylvain Michel\textsuperscript{1,2,4*}, Adriano Gualandi\textsuperscript{1,3} & Jean-Philippe Avouac\textsuperscript{1,5}

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Accepted: 1 August 2019
Published online: 23 October 2019

Faults can slip not only episodically during earthquakes but also during transient aseismic slip events\textsuperscript{1-5}, often called slow-slip events. Previous studies based on
Source Parameters and Scaling Relations for Slow Earthquakes

GEOPHYSICS

Daily measurement of slow slip from low-frequency earthquakes is consistent with ordinary earthquake scaling

William B. Frank and Emily E. Brodsky

Slow slip transients on faults can last from seconds to months and stitch together the earthquake cycle. However, no single geophysical instrument is able to observe the full range of slow slip because of bandwidth limitations. Here, we connect seismic and geodetic data from the Mexican subduction zone to explore an instrumental blind spot. We establish a calibration of the daily median amplitude of the seismically recorded low-frequency earthquakes to the daily geodetically recorded moment rate of previously established slow slip events. This calibration allows us to use the precise evolution of low-frequency earthquake activity to quantitatively measure the moment of smaller, subdaily slip events that are unresolvable by geodesy alone. The resulting inferred slow slip moments scale with duration and inter-event time like ordinary earthquakes. These new quantifications help connect slow and fast events in a broad spectrum of transient slip and suggest that slow slip events behave much like ordinary earthquakes.
Michel et al., Nature, 2019

Frank and Brodsky, 2019
Anderson’s Theory of Faulting:
- Free Surface and Principal Stresses
Adhesive and Abrasive Wear: Fault gouge is wear material

Chester et al., 2005

![Diagram of fault gouge and wear mechanism](image)

where $T$ is gouge zone thickness, $\kappa$ is a wear coefficient, $D$ is slip, and $h$ is material hardness

$$T = \frac{\kappa \sigma D}{3h}$$

This describes steady-state wear. But wear rate is generally higher during a ‘run-in’ period.

And what happens when the gouge zone thickness exceeds the surface roughness?

We’ll come back to this when we talk about fault growth and evolution.
Fault Growth and Development

Fault gouge is wear material

\[ T = \frac{k \sigma D}{3h} \]

‘run in’ and steady-state wear rate

This describes steady-state wear. But wear rate is generally higher during a ‘run-in’ period.
Fault Growth and Development

Fault gouge is wear material

‘run in’ and steady-state wear rate

\[ T = \frac{\kappa \sigma D}{3h} \]

This describes steady-state wear. But wear rate is generally higher during a ‘run-in’ period.
Fault Growth and Development

Fault gouge is wear material

\[ T = \frac{k\sigma D}{3h} \]

- \( T \) is the gouge zone thickness
- \( k \) is a constant
- \( \sigma \) is the shear stress
- \( D \) is the fault offset
- \( h \) is the surface roughness

‘run in’ and steady-state wear rate

This describes steady-state wear. But wear rate is generally higher during a ‘run-in’ period.

And what happens when the gouge zone thickness exceeds the surface roughness?
Fault Growth and Development

Fault gouge is wear material

‘run in’ and steady-state wear rate

Fault offset, D

Gouge Zone Thickness, T

Scholz, 1987
- Fault Growth and Development
- Fault Roughness

Fig. 3.28 Schematic diagram illustrating the definition of jogs and steps.

Scholz, 1990
Fault Growth and Development

Scholz, 1990
Fault Growth and Development

Fig. 17. Examples of complex fault zones. (a) Dasht-e-Bayaz, Iran, (b) Ales, France, and (c) Taranaki graben, New Zealand—(a)–(c) all map views adapted from Naylor et al. (1986). (d) Cross-section through a fault array in the Coeur d’Alene mine, Idaho, compiled from observations in an extensive series of mine galleries (shown as thin dotted lines, after Wallace & Morris 1986). (e) Fresh fracture in a deep mine, Republic of South Africa (after McGarr et al. 1979). J—pre-existing joint, P—primary and S—secondary fractures as described by McGarr et al. (1979).

Cox and Scholz, JSG, 1988
Fault Growth and Development

Fault zone width

Tchalenko, GSA Bull., 1970
Fig. 3.27 Power spectra for topographic profiles of fault surfaces. Para and perp mean parallel and perpendicular to the slip vector. (From Power et al., 1987.)

Scholz, 1990
Thermo-mechanics of faulting II

• San Andreas fault strength, heat flow.
• Consider: \( W_f = \tau v \geq q \)

• If \( \tau \sim 100 \text{ MPa} \) and \( v \) is \( \sim 30 \text{ mm/year} \), then \( q \) is:
  • \( 1e8 \text{ (N/m}^2\) \( 3e-2 \text{ (m/3e7s)} = 1e-2 \text{ (J/s m}^2\) \( \sim 100 \text{ mW/m2}. \)

• Problem of finding very low strength materials.

• Relates to the very broad question of the state of stress in the lithosphere?
  • Byerlee’s Law, Rangley experiments, Bore hole stress measurements, bore hole breakouts, earthquake focal mechanisms.

  • Seismic stress drop vs. fault strength.
Fault Strength, State of Stress in the Lithosphere, and Earthquake Physics
• Thermo-mechanics of faulting...
• Fault strength, heat flow.

• Consider shear heating:

\[ \dot{W} = \int_{S_0} \sigma_{ij} \dot{u}_i n_j \, dS \]

• If \( \tau \sim 100 \text{ MPa} \) and \( v \) is \( \sim 30 \text{ mm/year} \), then \( q \) is:

\[ 1e8 \text{ (N/m}^2) \times 3e-2 \text{ (m/3e7s)} = 1e-1 \text{ (J/s m}^2) \approx 100 \text{ mW/m}^2 \]
Fault Strength and State of Stress

- Heat flow
- Stress orientations

Data from Lachenbruch and Sass, 1980

Heat flow (mW/m²)

Distance from fault (km)

Strong fault

- \( \bar{\tau}_F = 100 \text{ MPa} \)

Weak fault

- \( \bar{\tau}_F = 20 \text{ MPa} \)
Fault Strength and State of Stress

- Heat flow
- Stress orientations

Have been used to imply that the SAF is weak, $\mu \approx 0.1$.

- Inferred stress directions

e.g. Townend & Zoback, 2004; Hickman & Zoback, 2004
SAFOD The San Andreas Fault Observatory at Depth

• Is the San Andreas anomalously weak?
SAF - Geology

Based on Zoback et al., EOS, 2010
Frictional Strength, SAFOD Phase III Core


Carpenter, Saffer and Marone, *Geology*, 2012
Weak Fault in a Strong Crust

Carpenter, Saffer, and Marone. 
*Geology*, 2012
Magnitude and Seismic Moment. Moment is a most robust measure of earthquake size because magnitude is a measure of size at only one frequency.

\[ M_o = \mu A u, \]

where \( \mu \) is shear modulus, \( A \) is fault Area and \( u \) is mean slip.

Relation to magnitude:
\[ M_w = \frac{2}{3} \log M_o - 6 \]
\[ M_o = \frac{3}{2} M_w + 9 \] (for \( M_o \) in N-m)
Earthquakes represent failure on geologic faults. The rupture occurs on a pre-existing surface.

Faults are finite features—*the Earth does not break in half every time there is an earthquake*.

Earthquakes represent failure of a limited part of a fault. Most earthquakes within the crust are shallow.

Definitions of **Focus, Epicenter**

NOTE: *Epicenter* is also the Rancho Cucamonga Quakes’ stadium—they are single-A team of the Anaheim (LA) Angels: http://www.rcquakes.com/

---

**Earthquake Size (Source Properties)**

Measures of earthquake size: Fault Area, Ground Shaking, Radiated Energy

<table>
<thead>
<tr>
<th>Fault dimensions for some large earthquakes:</th>
<th>L (km)</th>
<th>W (km)</th>
<th>U (m)</th>
<th>Mw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile 1960</td>
<td>1000</td>
<td>100</td>
<td>&gt;10</td>
<td>9.7</td>
</tr>
<tr>
<td>Landers, CA 1992</td>
<td>70</td>
<td>15</td>
<td>5</td>
<td>7.3</td>
</tr>
<tr>
<td>San Fran 1906</td>
<td>500</td>
<td>15</td>
<td>10</td>
<td>8.5</td>
</tr>
<tr>
<td>Alaska 1964</td>
<td>750</td>
<td>180</td>
<td>~12</td>
<td>9.3</td>
</tr>
</tbody>
</table>
Wave resulting from the interaction of P and S waves with the free surface.

Their wave motion is confined to and propagating along the surface of the body.
Magnitude is a measure of earthquake size base on:

- Ground shaking
- Seismic wave amplitude at a given frequency
Magnitude is a measure of earthquake size based on:

- Ground shaking
- Seismic wave amplitude at a given frequency

Magnitude accounts for three key aspects:

- Huge range of ground observed displacements due to very large range of earthquake sizes
- Distance correction to account for attenuation of elastic disturbance during propagation
- Site, station correction – small empirical correction to account for local effects at source or receiver

\[ M_L = \log_{10} \left( \frac{u}{T} \right) + q(\Delta, h) + a \]
**Magnitude** is a measure of earthquake size based on:

- Ground shaking
- Seismic wave amplitude at a given frequency

\[ M_L = \log_{10}\left(\frac{u}{T}\right) + q(\Delta, h) + a \]

\( M_L \) (Richter --*local*-- Magnitude) & \( M_s \), based on 20-s surface wave

\( M_B \), Body-wave mag. Is based on 1-s wave p-wave

\( M_w \), Moment mag. (see Hanks and Kanamori, JGR, 1979)

Systematic differences between \( M_s \) and \( M_b \) --due to use of different periods.

Source Spectra isn’t flat.
Saturation occurs for large events, particularly saturation of \( M_s \).

e.g: http://neic.usgs.gov/neis/nrg/bb_processing.html
Magnitude and Seismic Moment. Moment is a most robust measure of earthquake size because magnitude is a measure of size at only one frequency.

\[ M_o = \mu A u, \] where \( \mu \) is shear modulus, \( A \) is fault area and \( u \) is mean slip.

Moment and Moment Magnitude (Hanks and Kanamori, JGR, 1979):

\[ M_w = \frac{2}{3} \log M_o - 6 \quad \text{or} \]
\[ M_o = \frac{3}{2} M_w + 9 \quad (\text{for } M_o \text{ in N-m}) \]
Moment: [N m] = [Pa m³]  

Seismically released moment can be related to stress drop.

Stress drop ($\Delta \tau$) in terms of shear modulus, average slip, $u$, and fault rupture length $L$.

$$\Delta \tau = \mu \frac{u}{L}$$

replace $u$ in the equation for moment.  

$$M_o = \Delta \tau \frac{L}{u} A \bar{u}$$

gives:  

$$M_o = \Delta \tau \ L \ A \ . \ The \ right \ hand \ side \ is \ equivalent \ to \ F \ L, \ where \ F \ is \ force.$$

The simple model in which crustal stiffness is given by $k$, the seismically released elastic energy is given by  

$$k = \frac{F}{u} \ . \ Taking \ slip \ proportional \ to \ length, \ this \ gives: \ M_o \ proportional \ to \ k \ u^2$$

Seismic Energy:  

$$\log E = 5.2 + 1.4 \ M_s$$

1J = $10^7$ ergs  

1 kW hr = $3.6 \times 10^6$ J