Mechanics of Earthquakes and Faulting

Lecture 13, 8 Oct. 2015

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- Stability of frictional slip, Stick-slip dynamics
- Static and dynamic friction and the nature of the transition between them
- Slip weakening distance
- Time and velocity dependent friction

- Rabinowicz, 1951 & 1956
- Read Chapter 2 of Scholz
Rate \((v)\) and State \((\theta)\) Friction Constitutive Laws

\[
\mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right)
\]

- state variable, characterizes physical state of surface or shearing region
- critical slip distance
- reference velocity
- reference value of base friction

\[
\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}
\]

Dieterich, aging law

\[
\frac{d\theta}{dt} = -\frac{V \theta}{D_c} \ln \left( \frac{V \theta}{D_c} \right)
\]

Ruina, slip law

![Graph showing coefficient of friction versus load point displacement and loading rate](image)
Rate (v) and State (θ) Friction Constitutive Laws

1) $\mu(\theta, V) = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \theta}{D_c} \right)$

2) $\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}$

Adhesive Theory of Friction (Bowden and Tabor)
- Real contact area << nominal area
- Contact junctions at inelastic (plastic) yield strength
- Contacts grow with "age"
- Add: Rabinowicz’s observations of static/dynamic friction
- "Static" friction is higher than "Dynamic" friction because contacts are older (larger)
- $\Rightarrow$ implies that contact size decreases as velocity increases

$\mu \propto \exp \left( - \frac{u}{D_c} \right)$

Direct Effect

Evolution Effect

Fading memory of past state
Rate \((v)\) and State \((\theta)\) Friction Constitutive Laws

1) \[\mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right)\]

2) \[\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}\]

Adhesive Theory of Friction (Bowden and Tabor)
- Real contact area \(<\) nominal area
- Contact junctions at inelastic (plastic) yield strength
- Contacts grow with “age”
- Add: Rabinowicz’s observations of static/dynamic friction
- “Static” friction is higher than “Dynamic” friction because contacts are older (larger)
- \(\rightarrow\) implies that contact size decreases as velocity increases

The “explains” healing and velocity dependence, but what about \(D_c\)?
Rate (v) and State (θ) Friction Constitutive Laws

1) \( \mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right) \)

2) \( \frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c} \)

Convention is to use a, b for friction and A, B for Stress

\( \tau(\theta, v) = \tau_o + A \ln \left( \frac{V}{V_o} \right) + B \ln \left( \frac{V_o \theta}{D_c} \right) \)

\( A - B = \frac{\Delta \tau}{\Delta \ln V} \)

Steady-state velocity strengthening if a-b > 0, velocity weakening if a-b < 0
Rate ($v$) and State ($\theta$) Friction Constitutive Laws

1) $\mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right)$

2) $\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}$  

3) $\frac{d\mu}{dt} = k(V_{lp} - V)$  

Elastic Coupling

$V = V_o \exp \left[ \frac{\mu - \mu_o - b \ln \left( \frac{V_o \theta}{D_c} \right)}{a} \right]$ 

Solve:

$\frac{d\mu}{dt} = k \left( V_{lp} - V_o \exp \left[ \frac{\mu - \mu_o - b \ln \left( \frac{V_o \theta}{D_c} \right)}{a} \right] \right)$

$\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}$
Rate and State Friction

\[
(a-b) = \frac{\Delta \mu_{ss}}{\Delta \ln \nu}
\]

Marone, 1998

\[
\mu(\theta, \nu, \sigma) = \mu_0 + a \ln\left(\frac{\nu}{\nu_0}\right) + b \ln\left(\frac{\nu_0 \theta}{D_c}\right)
\]

\[
\frac{d\theta}{dt} = 1 - \frac{\nu \theta}{D_c}
\]

Dieterich State Evolution

\[
\theta = \theta_0 \left(\frac{\sigma_{initial}}{\sigma_{final}}\right)^\frac{a}{b}
\]
Rate/State Friction

Measuring the friction constitutive parameters

Empirical laws, based on laboratory friction data

Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

\[ \mu(\theta,v) = \mu_0 + a\ln\left(\frac{v}{v_0}\right) + b\ln\left(\frac{v_0\theta}{D_c}\right) \]

\[ \frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c} \]

\[ \theta_{ss} = \frac{D_c}{v} \]

\[ \Delta\mu_{ss} = (a-b)\ln\left(\frac{v}{v_0}\right) \]

\[ \frac{d\mu}{dt} = k'\left(v_{lp} - v\right) \]
Rate/State Friction
Measuring the friction constitutive parameters

Constitutive Modelling
Rate and State Friction Law
Elastic Interaction, Testing Apparatus

\[ \mu(\theta,v) = \mu_0 + a\ln\left(\frac{v}{v_0}\right) + b\ln\left(\frac{v_0\theta}{D_c}\right) \]

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Rate/State Friction
Measuring the friction constitutive parameters
Stressed Aging

Aging rate depends on the rate of shearing

(Marone, 1998, Nature)
The rate of frictional healing depends on the rate of shearing (Marone, 1998, *Nature*).

Rate State Friction Laws predict this behavior.
The rate of frictional healing depends on the rate of shearing (Marone & Saffer, 2015).

**Friction Law**

\[ \mu = \mu_o + a \ln(V/V_o) + b \ln(V) \]

**State Evolution**

\[ \frac{d\theta}{dt} = 1 - V \frac{\theta}{D_c} \]
\[ \frac{d\theta}{dt} = - V \frac{\theta}{D_c} \ln(V \theta/D) \]

**Elastic Coupling**

\[ \frac{d\mu}{dt} = k(V_{lp} - V) \]

**Figure 13** Numerical simulations of SHS tests showing healing and relaxation predicted by the RSF laws and elastic interaction. Inset to (a) shows details of a single simulation for which steady sliding is prescribed prior to a hold that begins at a normalized slip of 0 (the friction level is arbitrary and chosen as 0). The RSF laws predict that friction decreases during the hold due to creep and elastic interaction. Each point in the main panels shows results from a simulation. Four cases are shown corresponding to two velocities for each of the friction laws. Panels (a) and (b) show variations in the coefficient of friction. The same results are shown in nondimensional form in panels (c) and (d). Both laws show that healing and relaxation scale with loading rate and hold time, consistent with the experimental results. Simulations were carried out using the constitutive parameters given in the figure.
Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution.

\[ \mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right) \]

\[ \frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c} \]

\[ \frac{d\mu}{dt} = k(V_{tp} - V) \]

**shs test:**

- 1 \( \mu \text{m/s} \)
- 10 \( \mu \text{m/s} \)
shs test:
1 $\mu$m/s
10 $\mu$m/s

Phase Plane Plots

steady-state sliding friction velocity dependence
slope = $b - a = 0.002$
Derivation of the healing rate

Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution.

\[
\mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right)
\]

\[
\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}
\]

\[
\frac{d\mu}{dt} = k(V_{lp} - V)
\]
Slide-hold-slide

Slip-reload-slip

Earthquake-interseismic healing and reloading-earthquake

The full seismic cycle of stick-slip, frictional restrengthening, and interseismic reloading
Rate ($v$) and State ($\theta$) Friction Constitutive Laws

1) $\mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right)$

2) $\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}$

3) $\frac{d\mu}{dt} = k(V_{lp} - V)$ \hspace{1cm} \text{Elastic Coupling}

Modeling experimental data

Coefficient of Friction

Frictional Healing, $\Delta\mu$

Hold Time (s)

\[
\frac{d\mu}{dt} = k(V_{lp} - V)
\]
Time dependent yield strength:

\[ \mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y} \]

Dieterich and Kilgore [1994]

Time dependent growth of contact (acrylic plastic) - true static contact

\[ \sigma_y = \sigma_o + f(t) \]

Modified from Beeler, 2003
How do fault/frictional surfaces heal (regain strength) after failure?

Earthquakes & Fault Mechanics: seismic cycle, fault reactivation. (friction and stick slip: doors, windows, machines, ships in dry dock, dancers...)

Stick-slip failure during shear at constant loading rate

Shear Stress

Time

0.5 MPa
Time dependence of “static” friction
Aging of frictional contacts

C. A. Coulomb (1736-1806)

Table 9.1

<table>
<thead>
<tr>
<th>T (time of repose, min)</th>
<th>$A + mT^p$ (static friction force, lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I^st observation</td>
<td>A = 502</td>
</tr>
<tr>
<td>II</td>
<td>790</td>
</tr>
<tr>
<td>III</td>
<td>866</td>
</tr>
<tr>
<td>IV</td>
<td>925</td>
</tr>
<tr>
<td>V</td>
<td>1,036</td>
</tr>
<tr>
<td>VI</td>
<td>1,186</td>
</tr>
<tr>
<td>VII</td>
<td>1,535</td>
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Static friction of two pieces of well-worn oak lubricated with tallow.
Time dependence of “static” friction
Aging of frictional contacts

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<th>T (time of repose, min)</th>
<th>A+mT* (static friction force, lbf)</th>
</tr>
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<tbody>
<tr>
<td>I* observation</td>
<td>0</td>
</tr>
<tr>
<td>II*</td>
<td>2</td>
</tr>
<tr>
<td>III*</td>
<td>4</td>
</tr>
<tr>
<td>IV*</td>
<td>9</td>
</tr>
<tr>
<td>V*</td>
<td>26</td>
</tr>
<tr>
<td>VI*</td>
<td>60</td>
</tr>
<tr>
<td>VII*</td>
<td>960</td>
</tr>
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static friction of two pieces of well-worn oak lubricated with tallow.

C. A. Coulomb (1736-1806)
Time dependence of friction in rocks; Macroscopic frictional aging


Steady state friction & the rate of healing vary with sliding velocity.
Fault surface

Load point

Stress relaxation via creep

Coefficient of Friction

Displacement (mm)

1044 s hold, $V_{s/r} = 10 \, \mu m/s$

Hold

Reload

Load point
Fault surface

Stress relaxation via creep
Weaker at higher slip velocity: velocity weakening friction behavior

Frictional aging.
How does it work?
What is the role of shear velocity?
Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions

In-situ Particle Comminution; Production of Fresh Surface Area

Frye and Marone, JGR 2002
Hydrolytic Weakening causes enhanced rate of strengthening.
Hydrolytic Weakening causes enhanced rate of strengthening, but base level frictional strength is unchanged.

Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions

Frye and Marone, JGR 2002
Granite Surfaces: 

- Solid Surfaces: Base level of frictional strength decreases with increasing water content (cf. Dieterich & Conrad, 1984)

- Interpretation: Contact junctions subject to time dependent strengthening or growth, which inhibits sliding, but particle rolling is not affected by these factors.

Granular Materials: Frictional strength is independent of water content

Frye and Marone, JGR 2002
Empirical laws, based on laboratory friction data

Velocity dependence of steady state friction varies changes from positive to negative. (cf. Tullis and co-workers)

Chemically-assisted creep at adhesive contact junctions

Frye and Marone, JGR 2002
Measuring the velocity dependence of friction

Frictional Instability
Requires \((a-b) < 0\)

\[
(a-b) = \frac{\Delta \mu_{ss}}{\Delta \ln V}
\]

Constitutive Modelling
Rate and State Friction Law
Elastic Interaction, Testing Apparatus

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\[
\frac{d \mu}{dt} = k' \left( v_{lp} - v \right)
\]
Results: Velocity stepping
Measuring the velocity dependence of friction

Frictional Instability
Requires $K < K_c$

$$K_c = \frac{\sigma_n (b - a)}{D_c}$$

This example shows steady-state velocity strengthening: $(a-b) > 0$
Stresses v. Unstressed Aging

Load-up sequence  C-SHS: Partial unload SHS  Zero load SHS

Friction \( \mu = \frac{\tau}{\sigma_n} \)

Shear Displacement (mm)

Karner & Marone (GRL 1998, JGR 2001)
100 s holds, Healing rate varies systematically with shear stress

Zero-load SHS

Partial unload

Conventional SHS

Hold time 100s
Loading rate 10 µm/s

• Unload rate 300 µm/s

△ Unload rate 100 µm/s

Frictional healing, Δµ

Normalized shear load (η)

Karner & Marone (GRL 1998 JGR 2001)