• Stability of frictional slip, Stick-slip dynamics
• Static and dynamic friction and the nature of the transition between them
• Slip weakening distance
• Time and velocity dependent friction

• Rabinowicz, 1951 & 1956
• Read Chapter 2 of Scholz
A model for Rate/State Friction and the critical slip distance, $D_c$

Adhesive asperity contacts

- "Static" friction is higher than "Dynamic" friction because contacts are older (larger)
- Contact size decreases as velocity increases

Bowden & Tabor, 1950
Rabinowicz, 1951
Adhesive asperity contacts

But what about a single contact? It can grow with time, but what makes it smaller? (only slip?)

Rabinowicz, 1951
Time dependent yield strength:

\[ \mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y} \]

Dieterich and Kilgore [1994]

Time dependent growth of contact (acrylic plastic)- true static contact
Stick-slip sliding and earthquakes

Frictional Aging (Healing) of Seismogenic Faults:
Earthquake stress drop is $\approx 10\%$ of total shear strength.

Stick-slip failure during shear at constant loading rate
What is the connection between aging and stick slip?

Mair, Frye, and CM, JGR, 2002

Shear Stress/Normal Stress

Shear Displacement (mm)

Spherical beads

$\sigma_n = 5$ MPa

$V = 10$ $\mu$m/s

$V = 100$ $\mu$m/s

$\tau_{\text{max}}$

$\tau_{\text{min}}$

$\log V$

Log time

$A_r$

Shear Stress/Normal Stress

Shear Displacement (mm)

Log time

$A_r$
Rate of frictional healing:
\( \Delta \mu \sim 0.005 \) to \( 0.01 \) per decade in time
\( \Delta \mu \sim 0.01 \) per decade in slip (time*V)

What is the connection between aging and stick slip?

Mair, Frye, and CM, JGR, 2002
Frictional aging is due to Chemically-assisted plastic deformation at highly stressed contacts.

Frictional aging is due to Chemically-assisted plastic deformation at highly stressed contacts.

Mair, Frye, and CM, JGR, 2002

Shear Stress/Normal Stress

Shear Displacement (mm)

V=10 µm/s  V=100 µm/s

σ_n = 5 MPa

Spherical beads

m349

τ_{max}  τ_{min}
What is the connection between aging and stick slip?
Effects of relative humidity on stick-slip in a granular layer

- Normal stress (5 - 25 MPa)
- Layer thickness (4 vs. 3 mm)
- Also: Scuderi et al., 2011 Fall AGU

J. Robertson, Geosc 508: Mechanics of Earthquakes and Faulting
Scuderi et al., submitted to JGR, 2013
Effects of relative humidity on stick-slip in a granular layer.
Effects of relative humidity on stick-slip in a granular layer

Fig. 12. Idealized representation of a typical metallic junction in elevation and plan, the shaded region being the true area of contact. (a) and (c) — as formed during static loading. (b) and (d) — after sliding a distance $s$. Rabinowicz (1951)
Stick-Slip Instability Requires Some Form of Weakening: Velocity Weakening, Slip Weakening, Thermal/hydraulic Weakening

**Slip Weakening Friction Law**

\[ \mu_s - \mu_d \neq \mu_d^{(v)} \]

**Rate and State Dependent Friction Law**

\[ \mu = e^{V_1} V_0 \]

**Velocity Weakening**

\[ b > a > 0 \]

**Stability Criterion**

\[ K_c = \frac{\sigma_n(\mu_s - \mu_d)}{L} \]

- \( K < K_c \); Unstable, stick-slip
- \( K > K_c \); Stable sliding

\[ K_c = \frac{\sigma_n(b - a)}{D_c} \]

- \( K < K_c \); Unstable, stick-slip
- \( K > K_c \); Stable sliding
Rate (v) and State (θ) Friction Constitutive Laws

Recall (as motivation for going beyond other friction laws)
- Time-dependent static friction
- Velocity dependent sliding friction
- Memory effects, state dependence
- Repetitive stick-slip instability

Key Observations
- log-time strengthening
- log-velocity dependence

Application to earthquakes
- One set of constitutive relations to describe ‘entire’ seismic cycle
Rate ($v$) and State ($\theta$) Friction Constitutive Laws

$$\mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right)$$

- state variable, characterizes physical state of surface or shearing region
- critical slip distance
- reference velocity
- reference value of base friction

$$\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}$$

Dieterich, aging law

$$\frac{d\theta}{dt} = - \frac{V \theta}{D_c} \ln \left( \frac{V \theta}{D_c} \right)$$

Ruina, slip law

![Graph showing coefficient of friction over load point displacement with loading rates indicated.](image)
Rate (v) and State (θ) Friction Constitutive Laws

1) \( \mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right) \)

2) \( \frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c} \)

Implies:

\[
\mu \propto \exp\left(-\frac{u}{D_c}\right)
\]

Fading memory of past state

Direct Effect

Evolution Effect

Steady-state sliding:

\( \theta_{ss} = \frac{D_c}{V_1} \) \implies \( \frac{d\theta}{dt} = 0 \)

then (1) becomes:

\[ \mu_1 - \mu_o = (a - b) \ln \left( \frac{V_1}{V_o} \right) \]

\[ (a - b) = \frac{\Delta \mu}{\Delta \ln V} \]
Rate (v) and State (θ) Friction Constitutive Laws

1) \[ \mu(\theta, V) = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0\theta}{D_c} \right) \]

2) \[ \frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c} \]

Convention is to use \( a, b \) for friction and \( A, B \) for Stress

\[ \tau(\theta, v) = \tau_0 + A \ln \left( \frac{V}{V_0} \right) + B \ln \left( \frac{V_0\theta}{D_c} \right) \]

\[ A - B = \frac{\Delta \tau}{\Delta \ln V} \]

Steady-state velocity strengthening if \( a-b > 0 \),
velocity weakening if \( a-b < 0 \)

- velocity strengthening
- velocity weakening
Rate (v) and State (θ) Friction Constitutive Laws

1) \[ \mu(\theta, V) = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \theta}{D_c} \right) \]

2) \[ \frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c} \]

Steady-state velocity strengthening if \( a-b > 0 \),
velocity weakening if \( a-b < 0 \)

\[ \mu \]

\[ \text{velocity strengthening} \]

\[ \text{velocity weakening} \]

\[ \log V \]

Modeling experimental data

3) \[ \frac{d\mu}{dt} = k(V_{lp} - V) \]

Elastic Coupling

a & b are small, dimensionless constants determined from experiments

\( D_c \) has units of length
Rate ($v$) and State ($\theta$) Friction Constitutive Laws

1) \[ \mu(\theta, V) = \mu_o + a \ln \left( \frac{V}{V_o} \right) + b \ln \left( \frac{V_o \theta}{D_c} \right) \]

2) \[ \frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c} \quad \text{Modeling experimental data} \]

3) \[ \frac{d\mu}{dt} = k(V_{lp} - V) \quad \text{Elastic Coupling} \]

\[ V = V_o \exp \left[ \frac{\mu - \mu_o - b \ln \left( \frac{V_o \theta}{D_c} \right)}{a} \right] \]

Solve:
\[ \frac{d\mu}{dt} = k \left( V_{lp} - V_o \exp \left[ \frac{\mu - \mu_o - b \ln \left( \frac{V_o \theta}{D_c} \right)}{a} \right] \right) \]
\[ \frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c} \]
Sheared layer of quartz particles (100-150 μm), 25 MPa normal stress. Marone, 1998
Fault surface

Load point

Fault surface

1044 s hold, \( V_{s/r} = 10 \, \mu m/s \)

Coefficient of Friction

Hold

Reload

Displacement (mm)
Time Dependence of “static” friction

Stressed Aging

Monodisperse, angular quartz particles
Time Dependence of “static” friction

Effect of loading velocity

[Graph showing coefficient of friction vs. hold time and loading rate]
Stressed Aging

Aging rate depends on the rate of shearing

(Marone, 1998, Nature)
Stresses v. Unstressed Aging

Load-up sequence
C-SHS
Partial unload
SHS
Zero load
SHS

Friction ($\mu = \tau / \sigma_n$)

Shear Displacement (mm)

Loading rate 10 $\mu$m/s
Unloading rate 300 $\mu$m/s

Karner & Marone (GRL 1998, JGR 2001)
100 s holds, Healing rate varies systematically with shear stress

Frictional healing, $\Delta \mu$

Normalized shear load ($\eta$)

Zero-load SHS

Partial unload

Conventional SHS

Hold time 100s
Loading rate 10 $\mu$m/s

- Unload rate 300 $\mu$m/s
- Unload rate 100 $\mu$m/s

Karner & Marone (GRL 1998 JGR 2001)
Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions

In-situ Particle Comminution; Production of Fresh Surface Area

Frye and Marone, JGR 2002
Granular quartz

Hydrolytic Weakening causes enhanced rate of strengthening
Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions

Hydrolytic Weakening causes enhanced rate of strengthening, but base level frictional strength is unchanged

Frye and Marone, JGR 2002
Granite Surfaces

Granular Materials

Solid Surfaces: Base level of frictional strength decreases with increasing water content (cf. Dieterich & Conrad, 1984)

Interpretation: Contact junctions subject to time dependent strengthening or growth, which inhibits sliding, but particle rolling is not affected by these factors.

Granular Materials: Frictional strength is independent of water content

Frye and Marone, JGR 2002
Empirical laws, based on laboratory friction data

Velocity weakening frictional behavior in granular fault gouge

Velocity dependence of steady state friction varies changes from positive to negative. (cf. Tullis and co-workers)

Chemically-assisted creep at adhesive contact junctions

Frye and Marone, JGR 2002
Measuring the velocity dependence of friction

Frictional Instability
Requires \((a-b) < 0\)

Constitutive Modelling
Rate and State Friction Law
Elastic Interaction, Testing Apparatus

\[
\mu(\theta,v) = \mu_0 + a \ln\left(\frac{v}{v_0}\right) + b \ln\left(\frac{v_0 \theta}{D_c}\right)
\]

\[
\frac{d\theta}{dt} = 1 - \frac{v \theta}{D_c}
\]

\[
\theta_{ss} = \frac{D_c}{v}
\]

\[
\Delta \mu_{ss} = (a-b) \ln\left(\frac{v}{v_0}\right)
\]

\[
\frac{d\mu}{dt} = k' \left(v_{lp} - v\right)
\]
Results: Velocity stepping
Measuring the velocity dependence of friction

Frictional Instability
 Requires $K < K_c$

$$K_c = \frac{\sigma_n(b - a)}{D_c}$$

This example shows steady-state velocity strengthening: $(a-b) > 0$