Mechanics of Earthquakes and Faulting

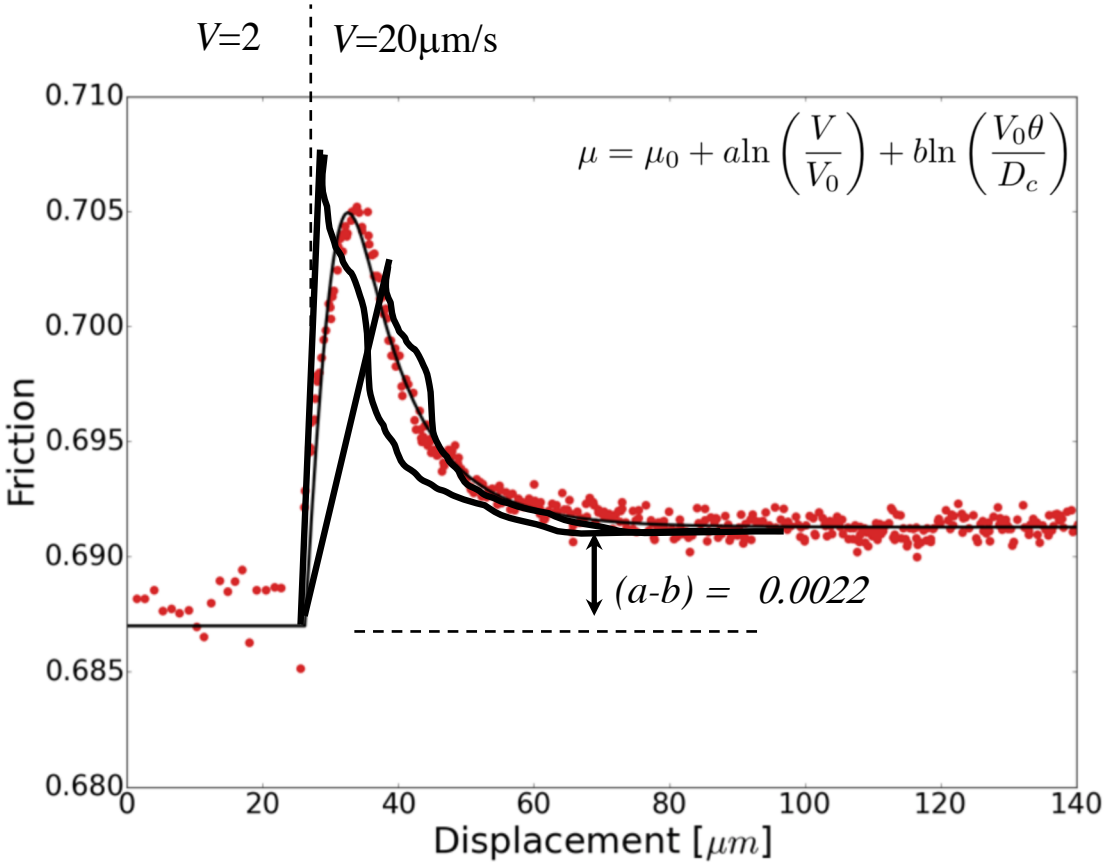
Lecture 11, 4 Mar. 2021

www.geosc.psu.edu/Courses/Geosc508

- Friction Constitutive Laws for Faulting
- Critical stiffness and stability transition from stable to unstable faulting
- SHS test to measure RSF parameters
- Earthquake Nucleation, Critical Fault Patch Size
- Stress Distribution for Propagating Rupture --(Crack Tip) Cohesive Zone
- Laboratory Observations of The transition from stable to unstable frictional sliding: Confirmation of the concept of a critical fault weakening rate with slip (kc)

Rate/State Friction
Measuring the friction constitutive parameters

Velocity strengthening frictional behavior



Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

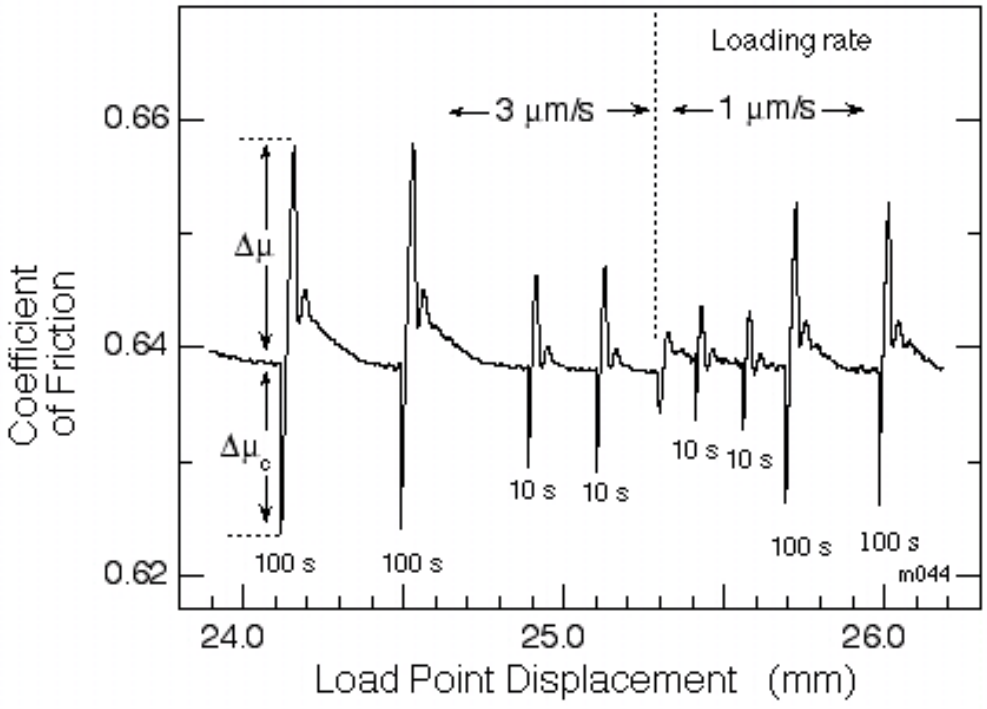
$$\mu(\theta,v) = \mu_0 + a\ln\left(\frac{v}{v_o}\right) + b\ln\left(\frac{v_o\theta}{D_c}\right)$$

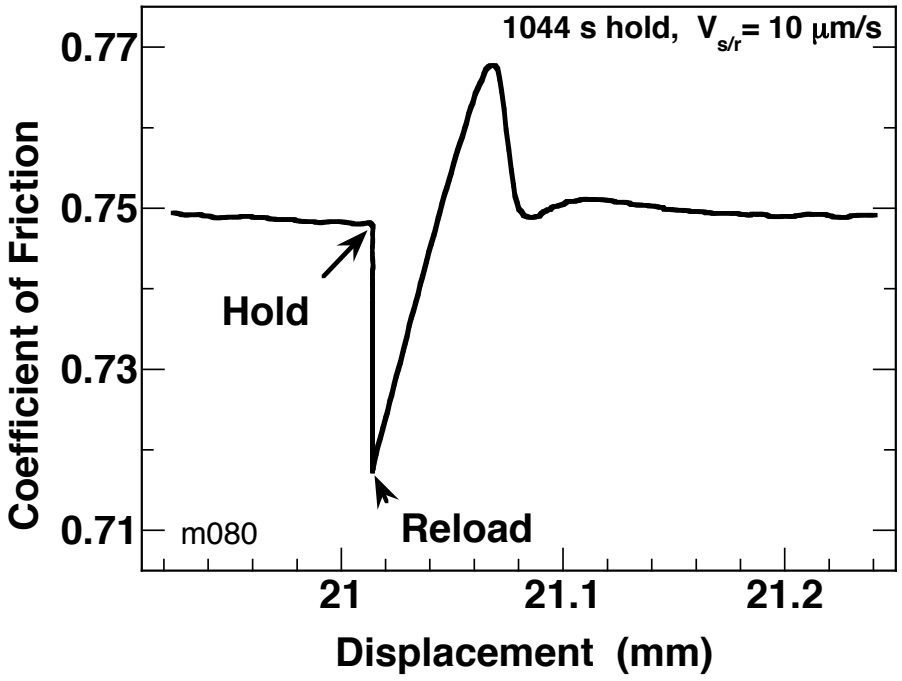
$$\frac{d\theta}{dt} = 1 - \frac{v\theta}{D_c}$$

$$\theta_{ss} = \frac{D_c}{v}$$

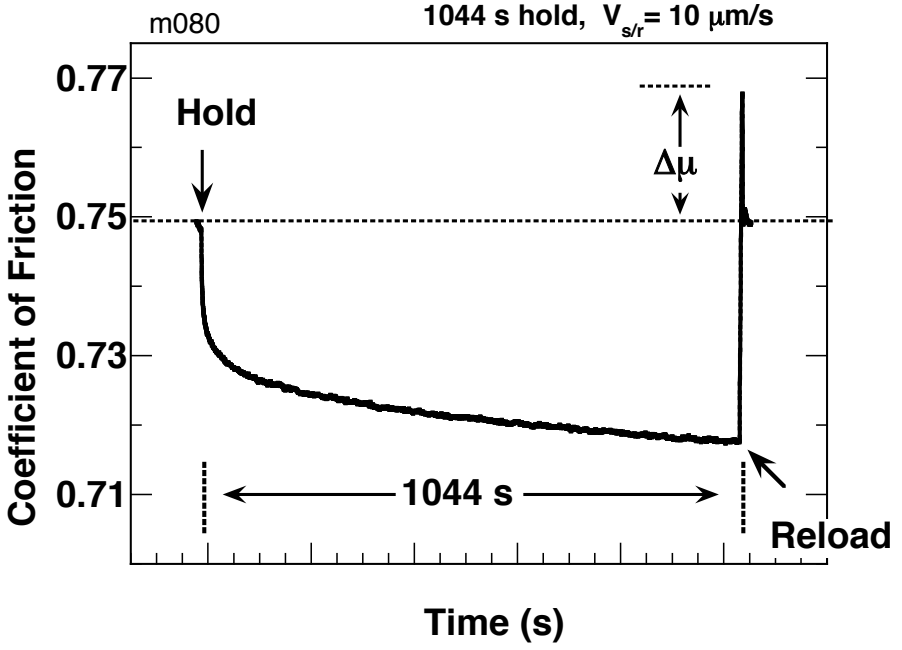
$$\Delta\mu_{ss} = (a-b)\ln\left(\frac{v}{v_o}\right)$$

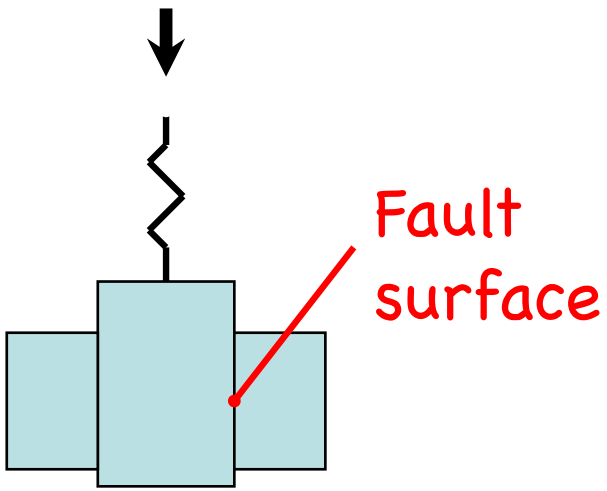
$$\frac{d\mu}{dt} = k'\left(v_{lp} - v\right)$$

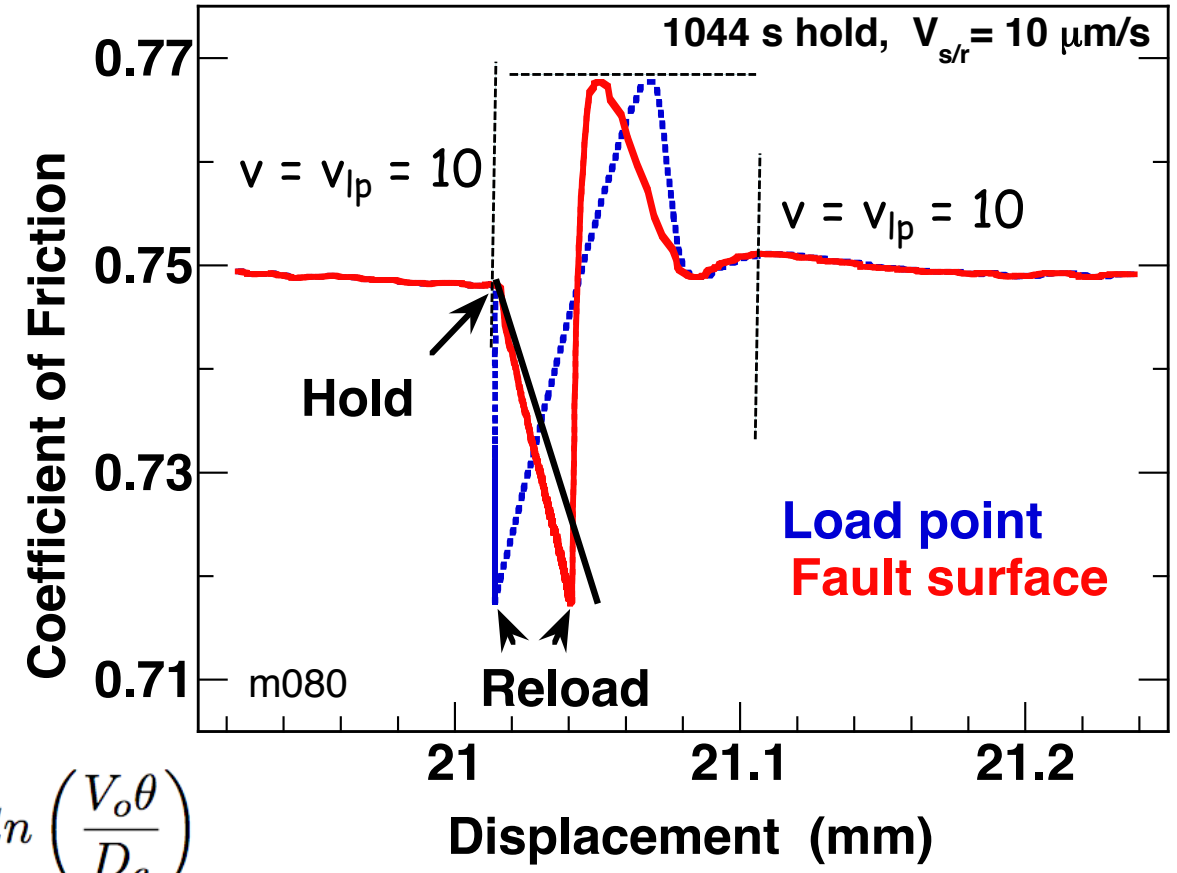




Sheared layer of quartz particles (100-150 μm), 25 MPa normal stress . Marone, 1998







1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

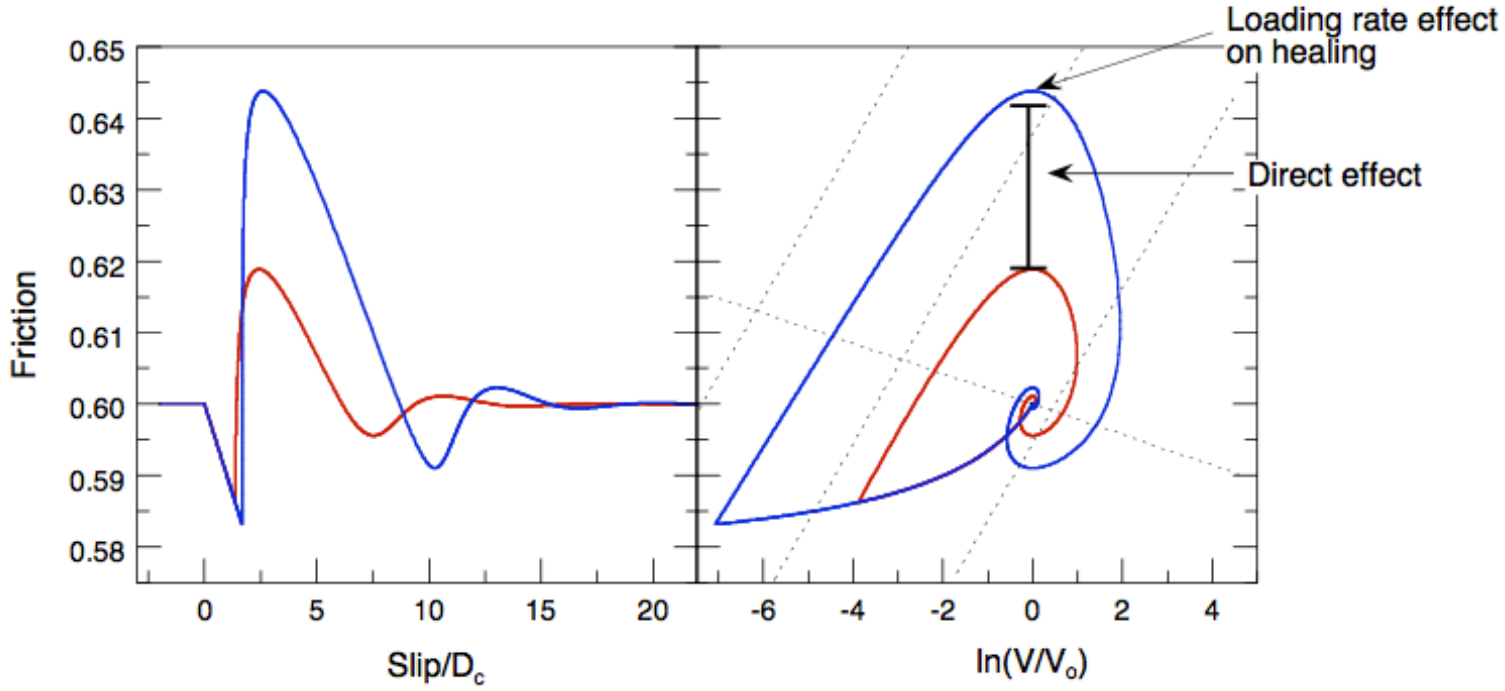
2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling

$$\frac{d\mu}{dt} = k\left(V_{lp} - V_o\, exp\left[\frac{\mu - \mu_o - b\, ln(\frac{V_o\theta}{D_c})}{a}\right]\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Derivation of the healing rate



Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution

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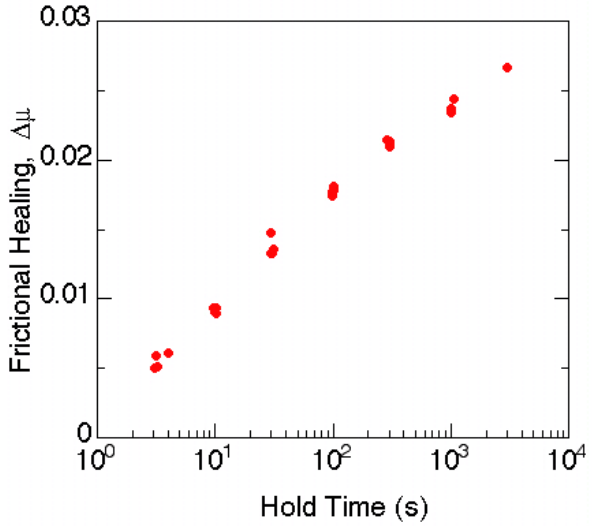

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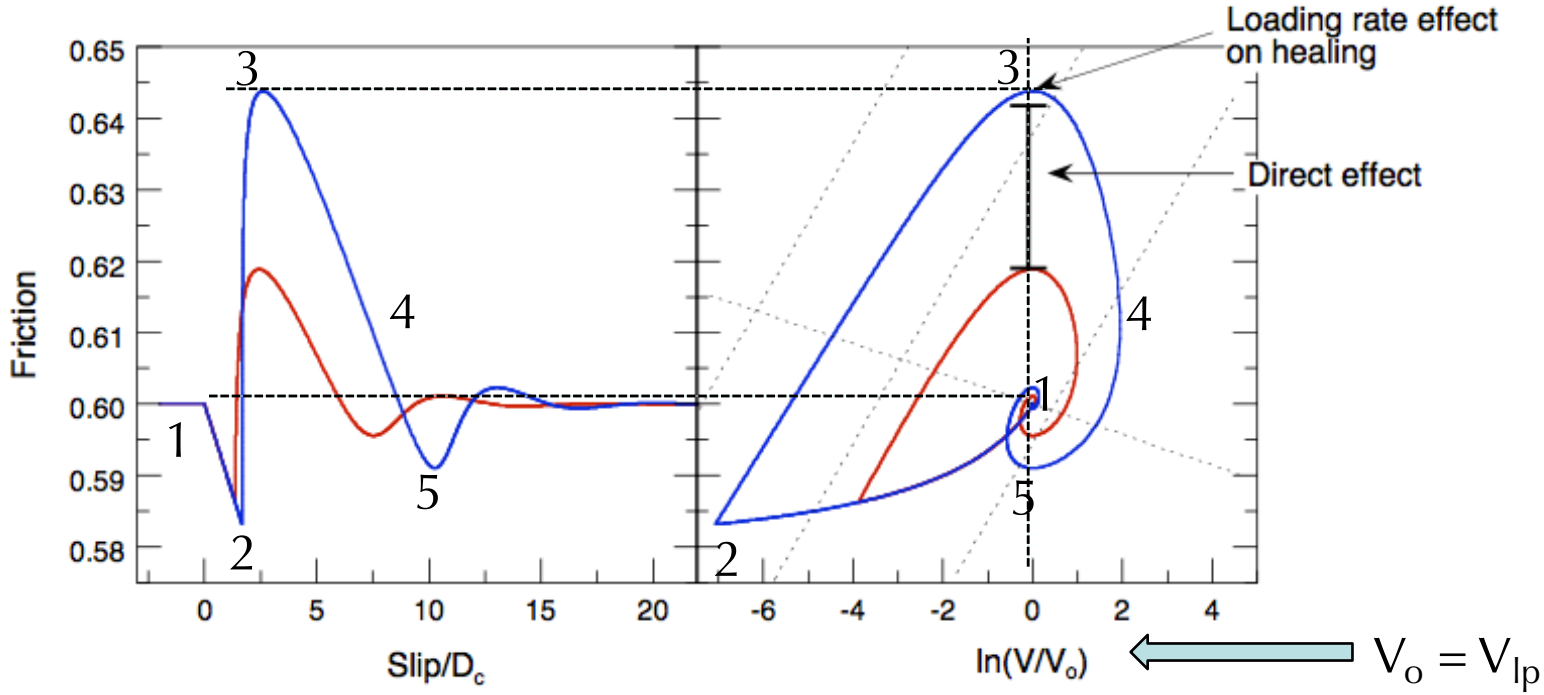

But let's focus on just one velocity, so that we can see what the healing rate tells us.

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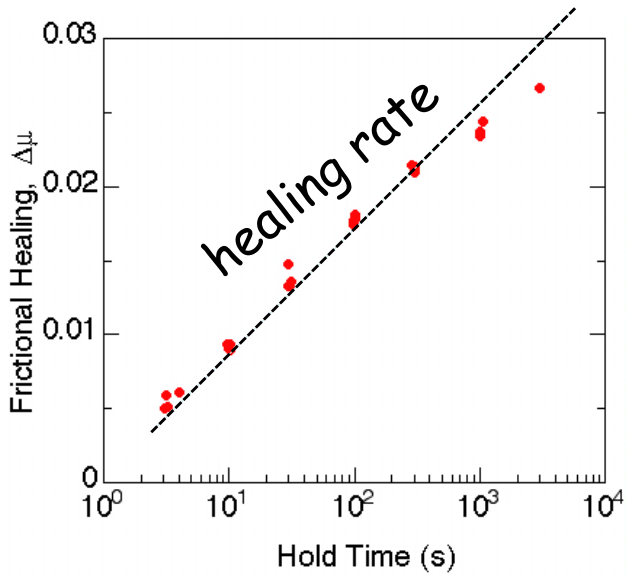

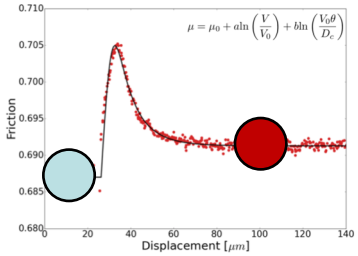

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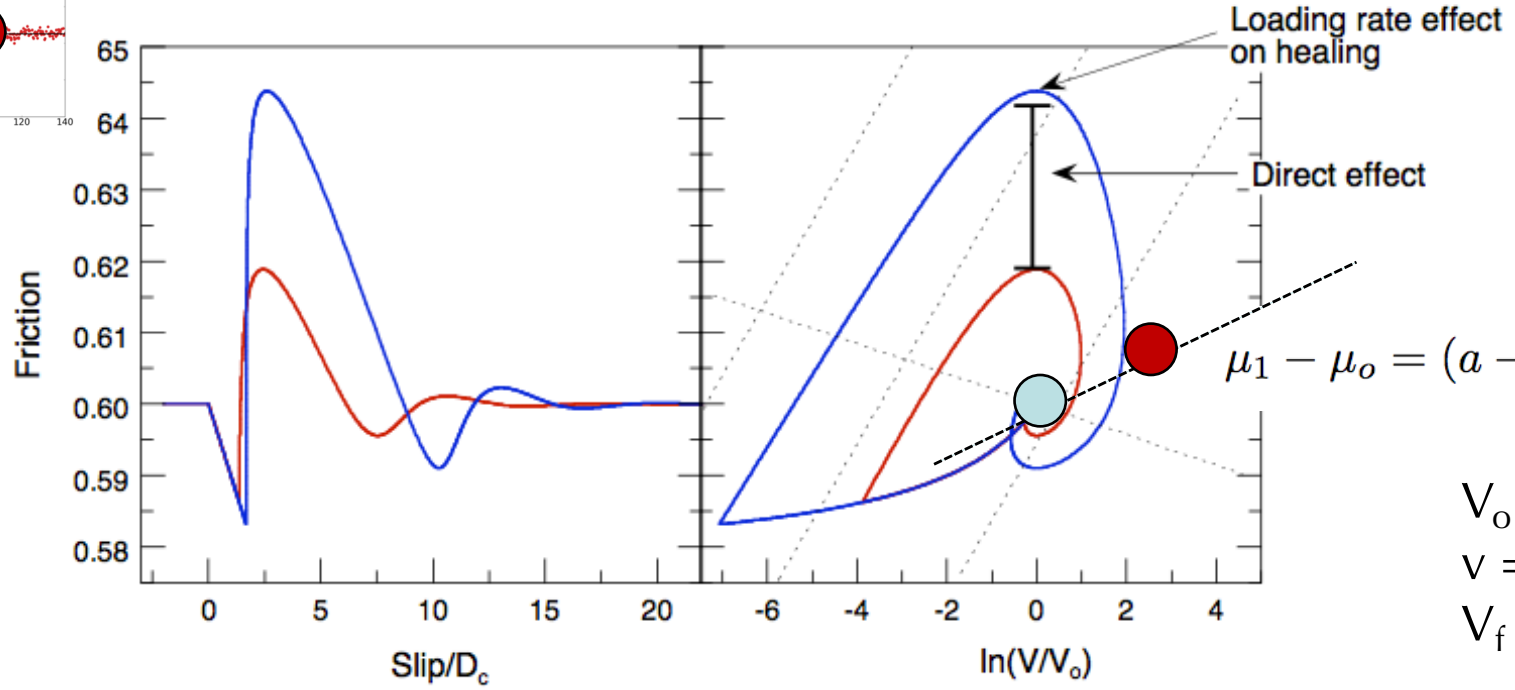


$$\mu_1 - \mu_o = (a-b)ln(\frac{V_1}{V_o})$$

$V_o = 1$
$v = 2$
$V_f = 20$

But let's focus on just one velocity, so that we can see what the healing rate tells us.

(1) $$\mu(\theta, V) = \mu_o + a\ ln\left(\frac{V}{V_o}\right) + b\ ln\left(\frac{V_o\theta}{D_c}\right)$$

(2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

(3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$

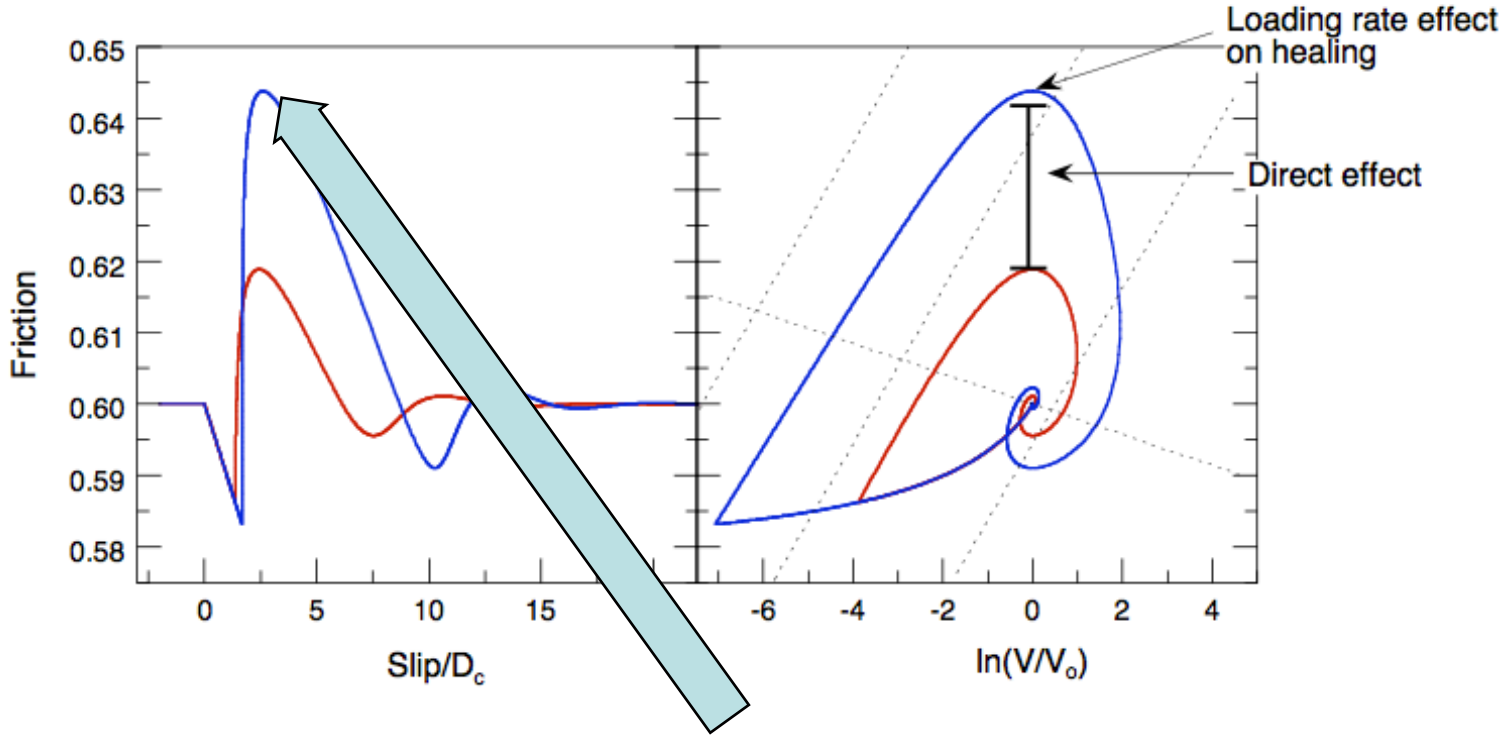
The healing rate involves measurements of Δμ -look back at notes from Lec. 7

At steady state, we have

$$\mu_1 - \mu_o = (a-b)ln(\frac{V_1}{V_o})$$

but for the peak ("static") friction, we need to think about both terms on the RSH of eq'n 1

Derivation of the healing rate



What do we know about V at the peak?

(1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

(2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

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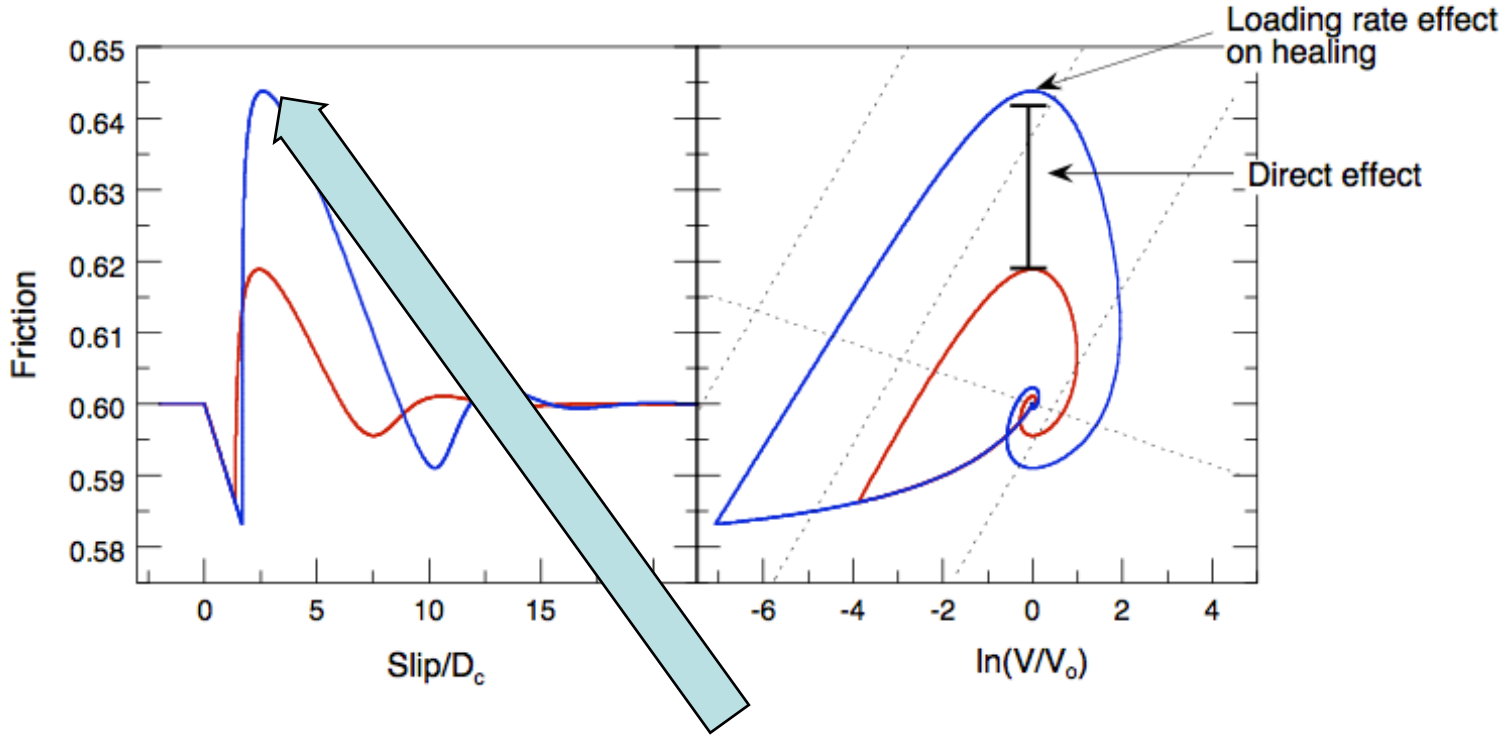
Derivation of the healing rate



What do we know about V at the peak?

V = V$_o$ at the peak --right, that's what we see in the phase plane.

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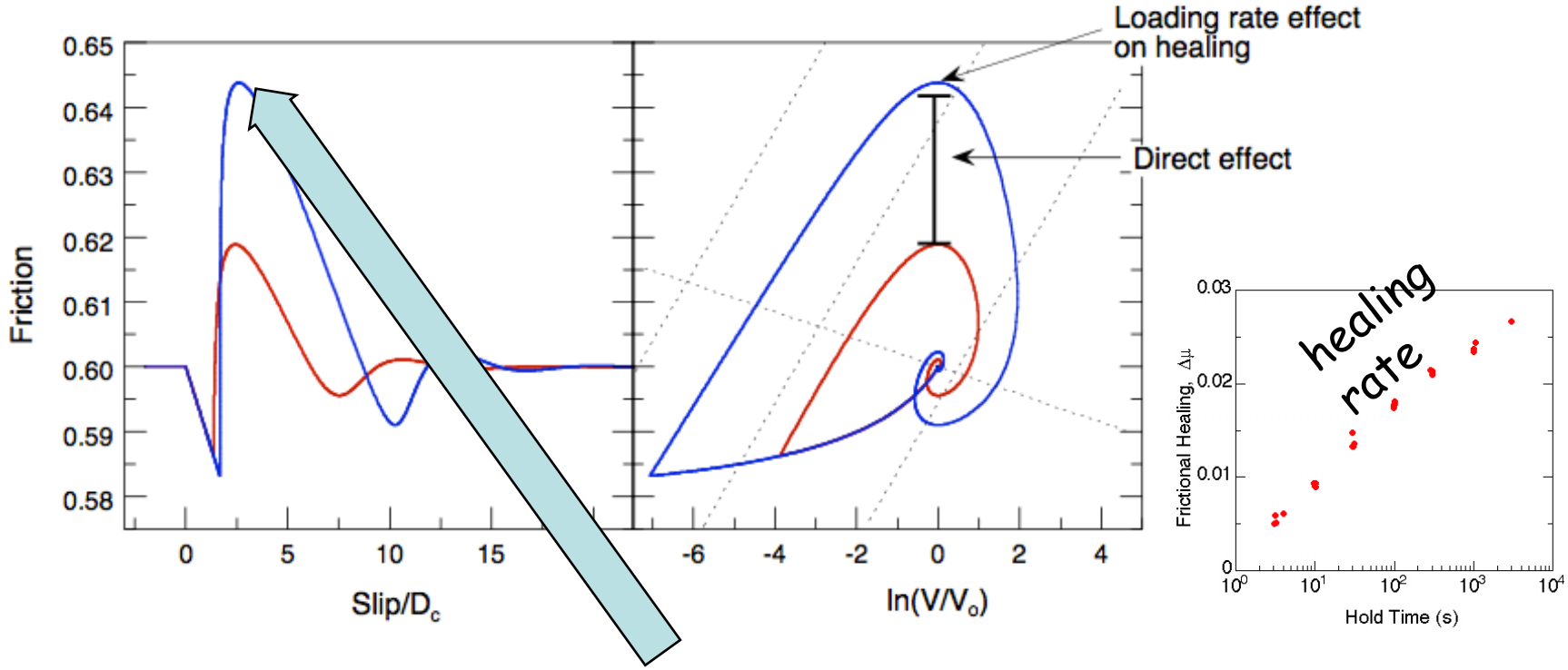

What do we know about V at the peak?

V = V_o at the peak --right, that's what we see in the phase plane.

Hmmm, that means that the a ln(V/V_o) term is zero

So $\Delta\mu = b\ ln\ (V_o\ \theta/D_c)$

0

(1) $$\mu(\theta, V) = \mu_o + a\ ln\left(\frac{V}{V_o}\right) + b\ ln\left(\frac{V_o\theta}{D_c}\right)$$

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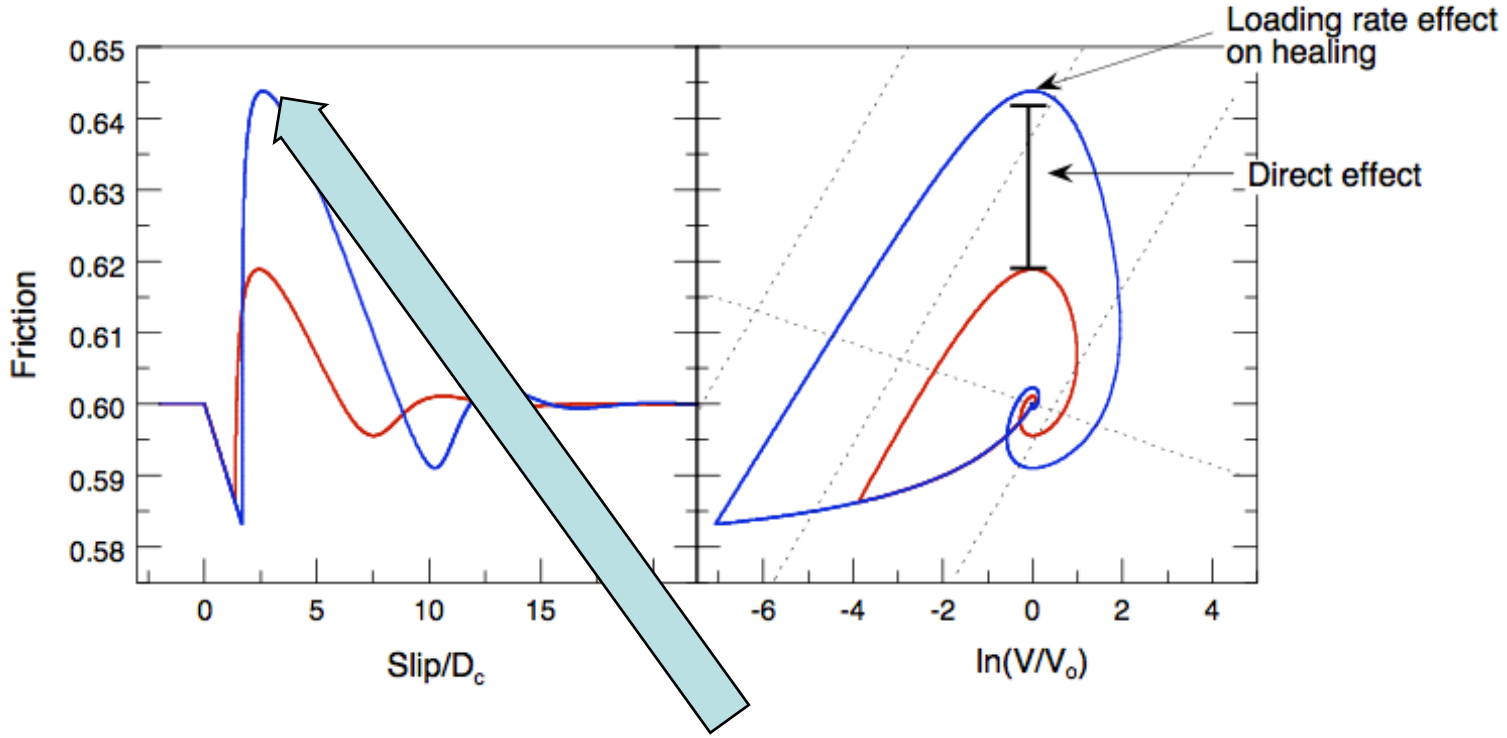
Derivation of the healing rate



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(1) $\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$

Hmmm, that means that the a ln(V/V$_o$) term is zero

So $\Delta\mu = b\ ln\ (V_o\ \theta/D_c)$

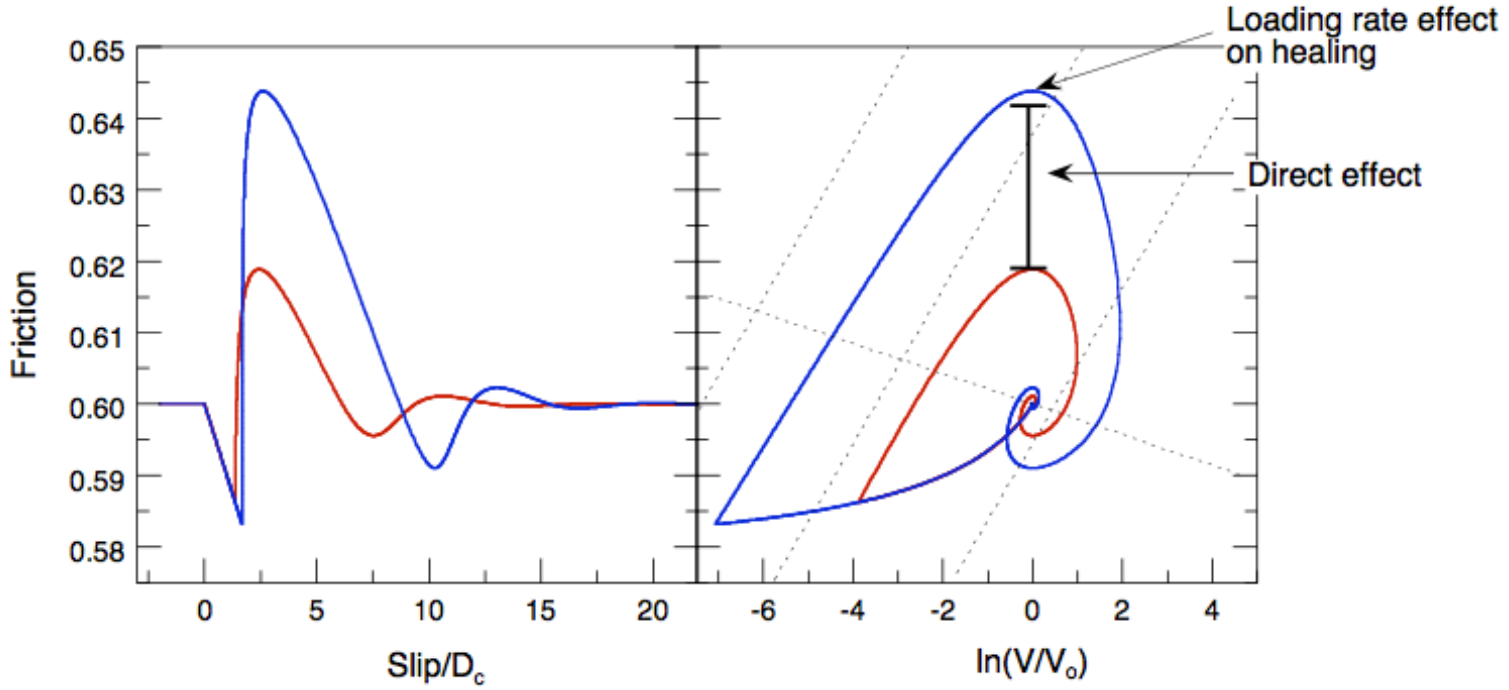
(2) $\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$

and how does θ vary with SHS time?

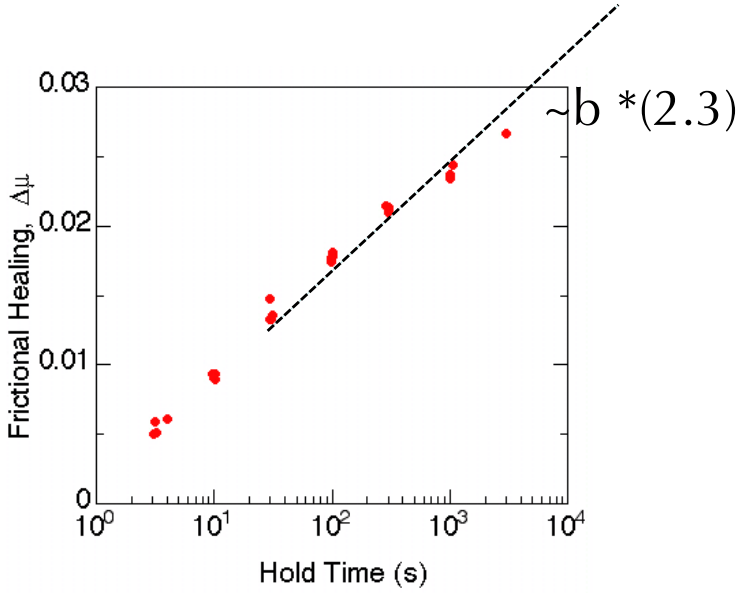
(3) $\frac{d\mu}{dt} = k(V_{lp} - V)$

according to (2), as V goes to 0, $d\theta = dt$

Derivation of the healing rate



so, if we plot Δμ vs ln(t) we should have the healing rate



~b *(2.3)

(1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

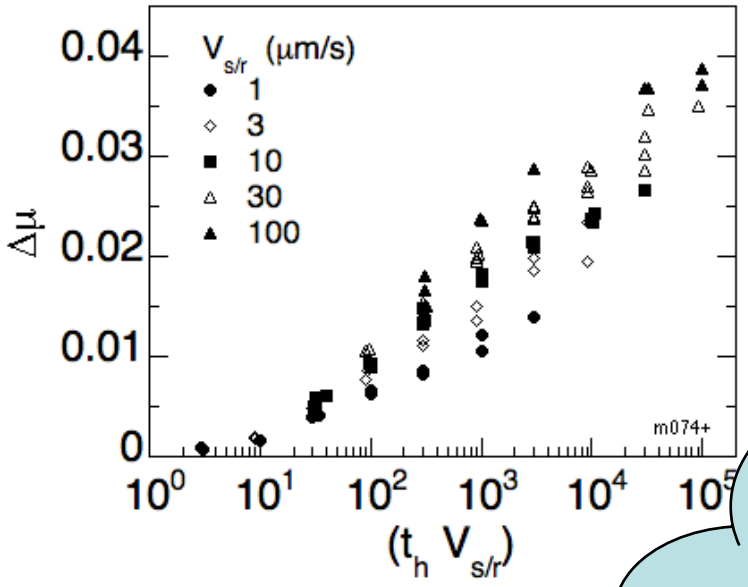
(2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

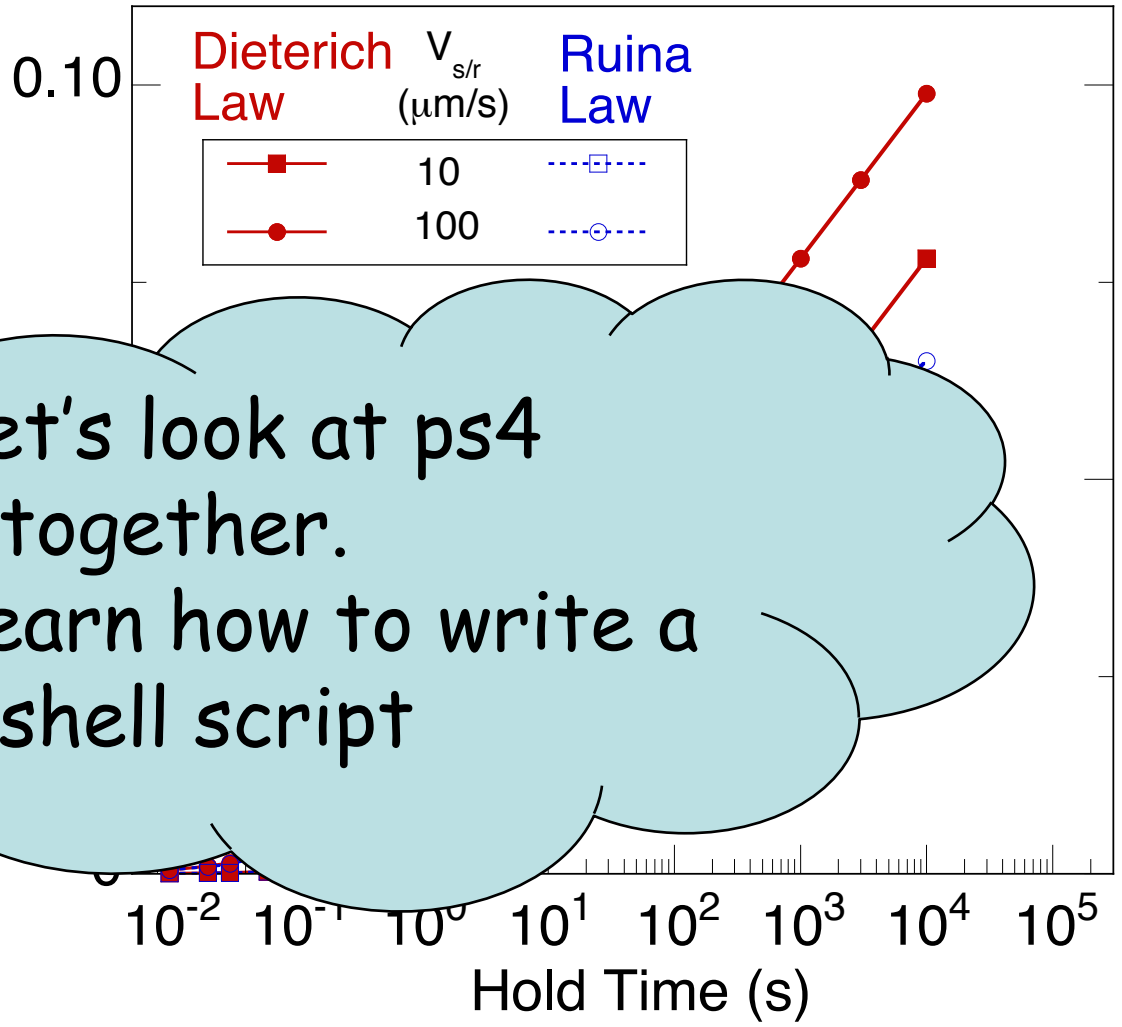
(3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$

So $\Delta\mu = b\ ln\ (V_o\ \theta/D_c)$

$\Delta\mu/d(ln\theta) = b$

as V goes to 0, $d\theta = dt$





Let's look at ps4 together.
Learn how to write a shell script

Friction Law

$\mu = \mu_o + a \ln(V/V_o) + b \ln(V_o\theta/D_c)$

State Evolution

$d\theta/dt = 1 - V\theta/D_c$

$d\theta/dt = - V\theta/D_c \ln(V\theta/D_c)$

Elastic Coupling

$d\mu/dt = k(V_{lp} - V)$

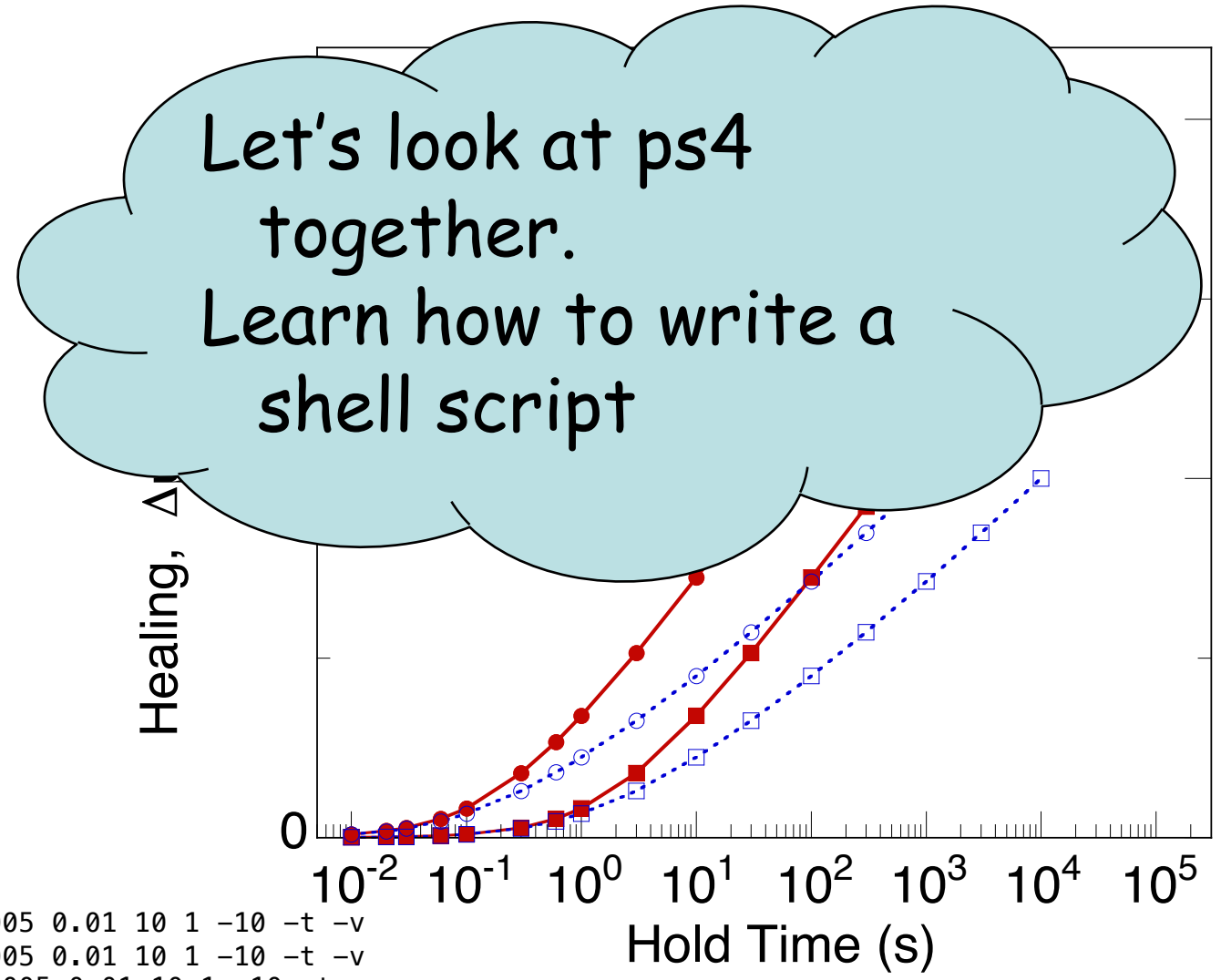
The rate of frictional healing depends on the rate of shearing (Marone, 1998, *Nature*)

Rate State Friction Laws predict this behavior

```
 cjm.laptop.14 more junk22
 rsfs junk d 1 1 1 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
 rsfs junk d 1 1 3 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
 rsfs junk d 1 1 10 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
 rsfs junk d 1 1 30 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
 rsfs junk d 1 1 100 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
 rsfs junk d 1 1 300 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
 rsfs junk d 1 1 1000 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
 cjm.laptop.15 chmod +x junk22
```

```
cjm.laptop.18 more output22
1, 0.000666797, 1, 1, 0.00090648, 0.600667, 0.600667, 10.0908, 10.6891, 1.00008, 0.005, 0.01, 10, 0, 0, 0.001, junk.dis, d,
15.0747, 13.507, 0.903112
3, 0.00190981, 1, 1, 0.00226292, 0.60191, 0.60191, 10.6893, 12.1073, 0.999508, 0.005, 0.01, 10, 0, 0, 0.001, junk.dis, d,
15.9628, 11.8027, 2.25335
10, 0.00540296, 1, 1, 0.0045354, 0.605403, 0.605403, 14.9105, 17.1718, 0.999223, 0.005, 0.01, 10, 0, 0, 0.001, junk.dis, d,
18.7662, 8.86459, 4.51034
30, 0.0113824, 1, 1, 0.00603286, 0.611382, 0.611382, 31.8245, 31.2058, 1.00045, 0.005, 0.01, 10, 0, 0, 0.001, junk.dis, d,
24.0801, 6.68911, 6.00022
100, 0.0204361, 1, 1, 0.00666746, 0.620436, 0.620436, 98.5141, 77.1104, 1.00192, 0.005, 0.01, 10, 0, 0, 0.001, junk.dis, d,
32.6832, 5.59862, 6.63347
300, 0.0300862, 1, 1, 0.00684581, 0.630086, 0.630086, 295.632, 202.429, 1.00163, 0.005, 0.01, 10, 0, 0, 0.001, junk.dis, d,
42.143, 5.22621, 6.81171
1000, 0.0414662, 1, 1, 0.00690619, 0.641466, 0.641466, 992.559, 632.55, 0.998898, 0.005, 0.01, 10, 0, 0, 0.001, junk.dis, d,
53.4299, 5.06943, 6.87207
cjm.laptop.19
```

junk22 > output22
more output22




```
cjm.laptop.14 more junk22
rsfs junk d 1 1 1 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
rsfs junk d 1 1 3 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
rsfs junk d 1 1 10 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
rsfs junk d 1 1 30 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
rsfs junk d 1 1 100 .1 .1 200 1e-3 0.005 0.01 10 1 -10 -t -v
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cjm.laptop.15 chmod +x junk22
```
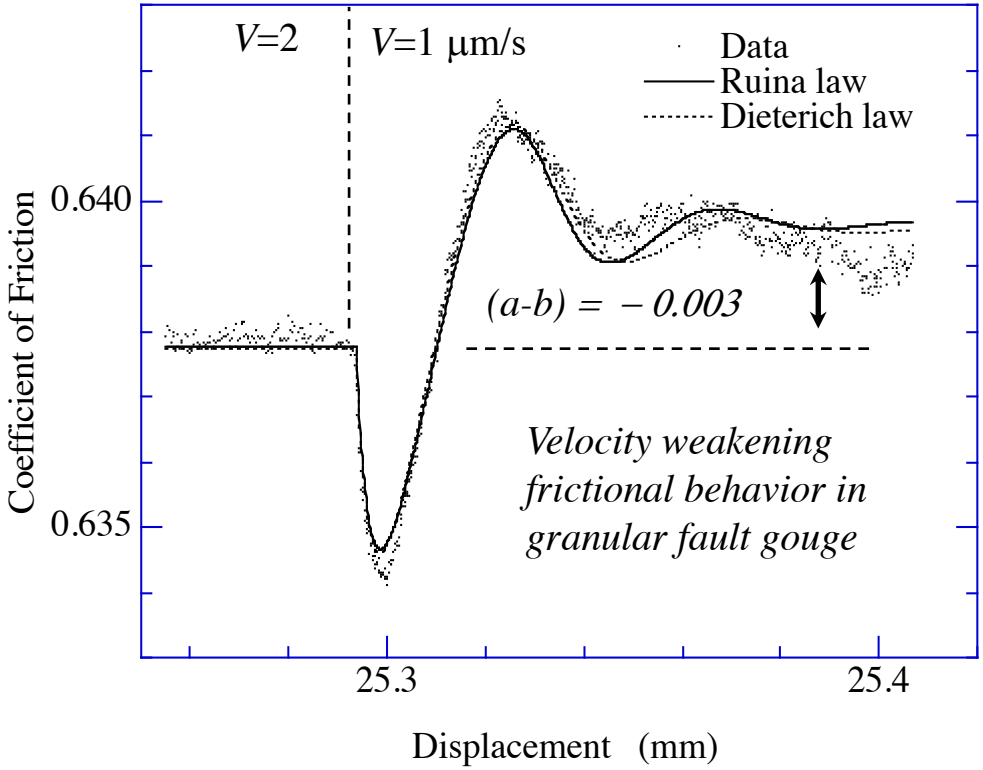

Rate/State Friction
Measuring the friction constitutive parameters

Empirical laws, based on laboratory friction data



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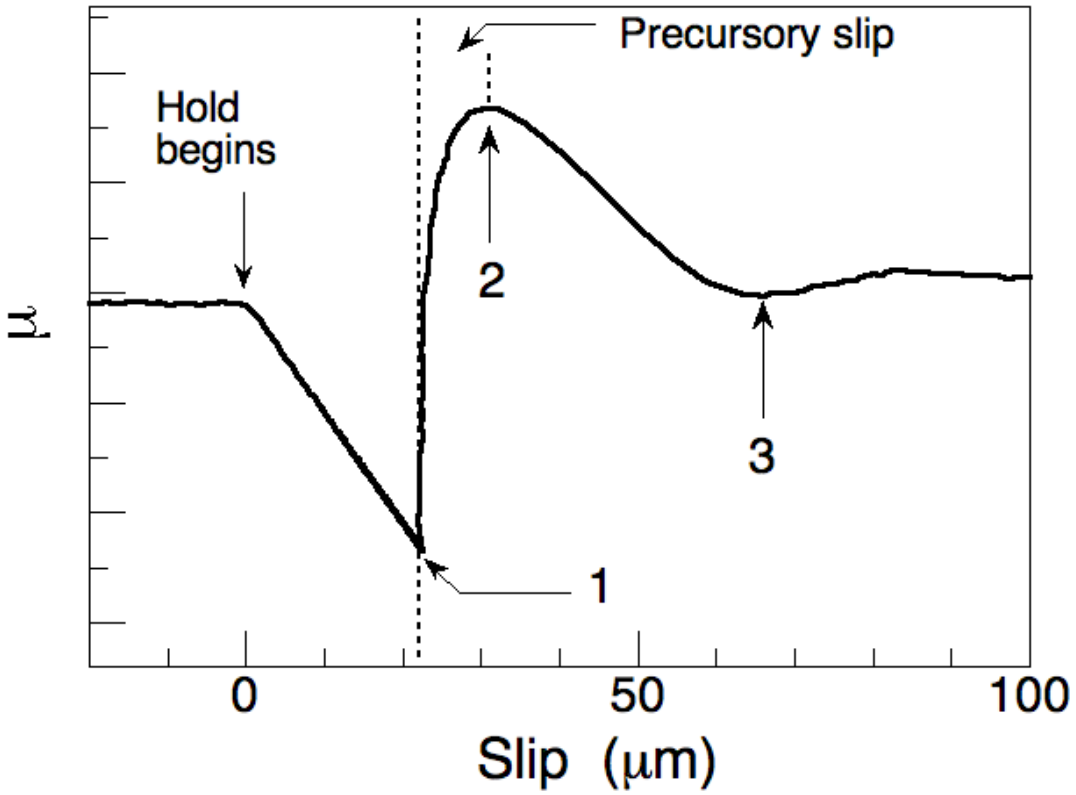

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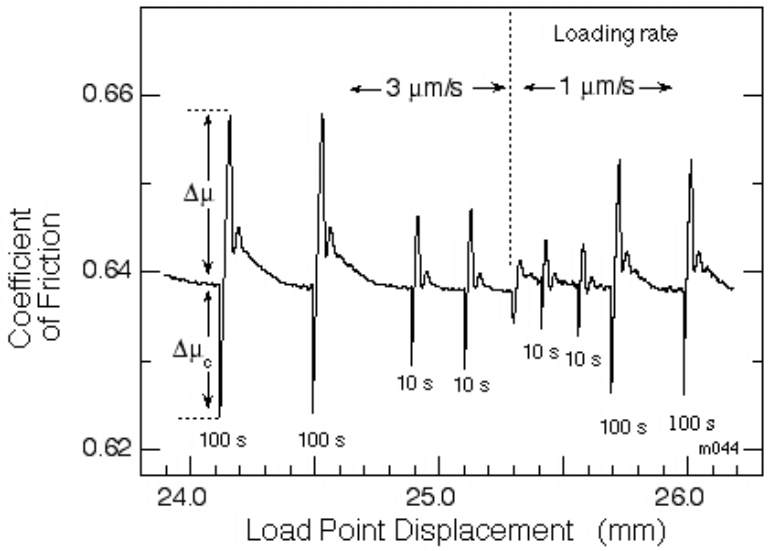
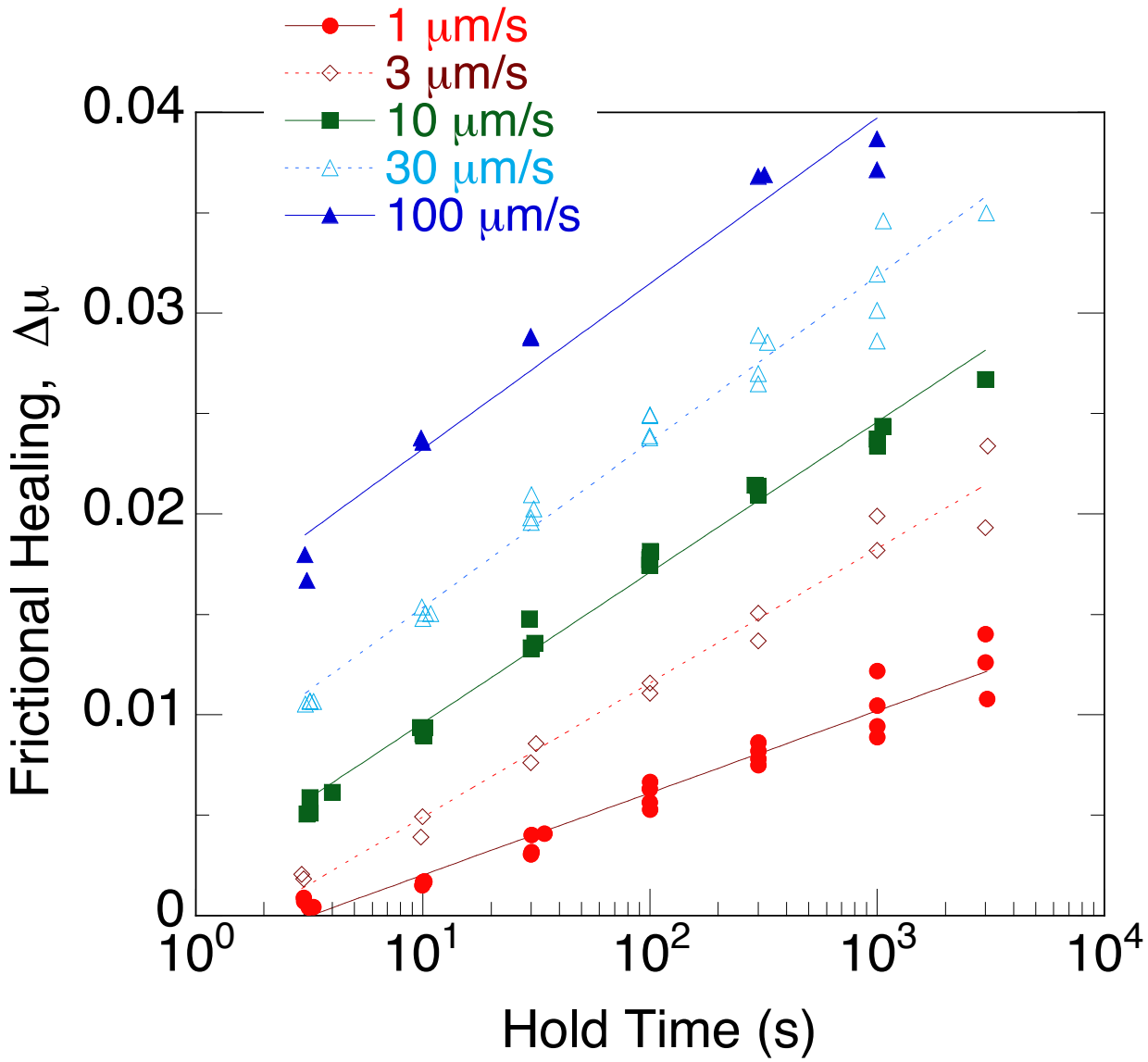


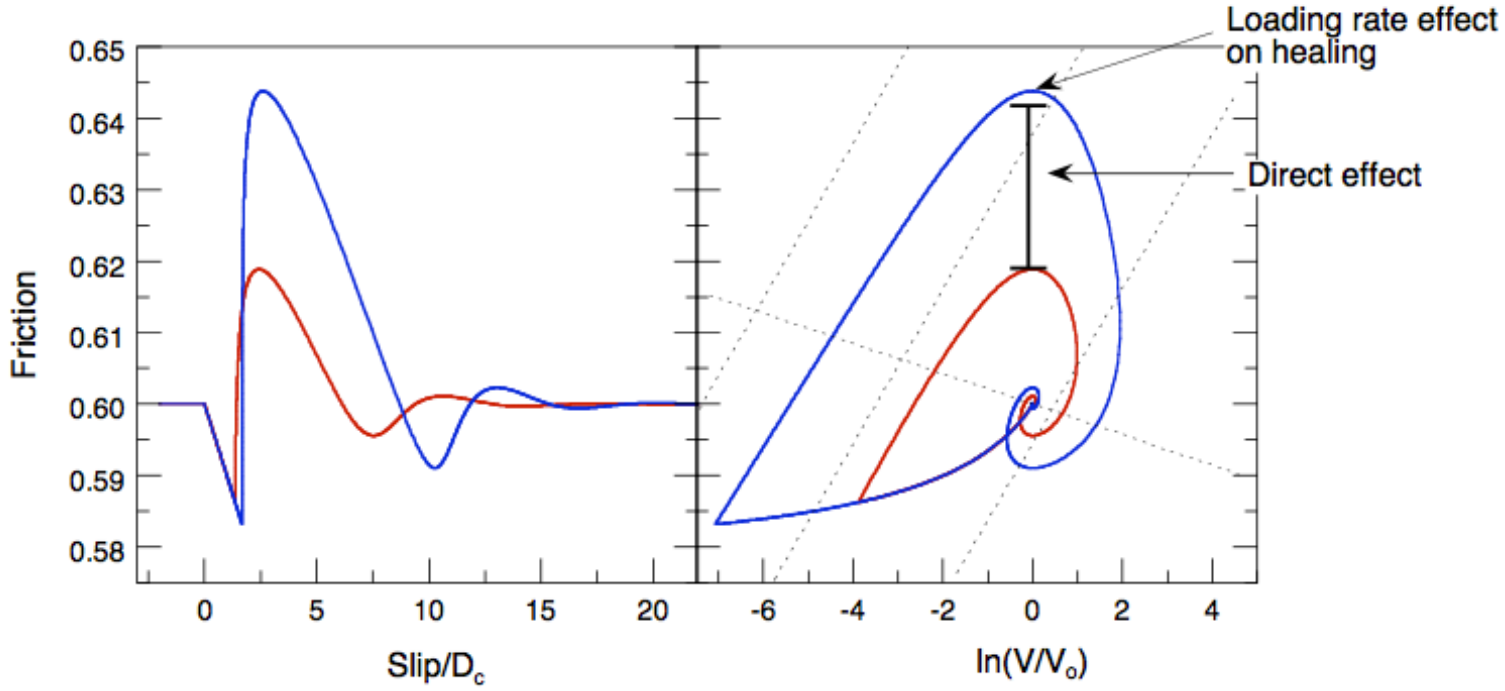
Stressed Aging

Aging rate depends on the rate of shearing



(Marone, 1998, *Nature*)

Phase Plane Plots



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shs test:
1 μm/s
10 μm/s

Phase Plane Plots

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shs test:
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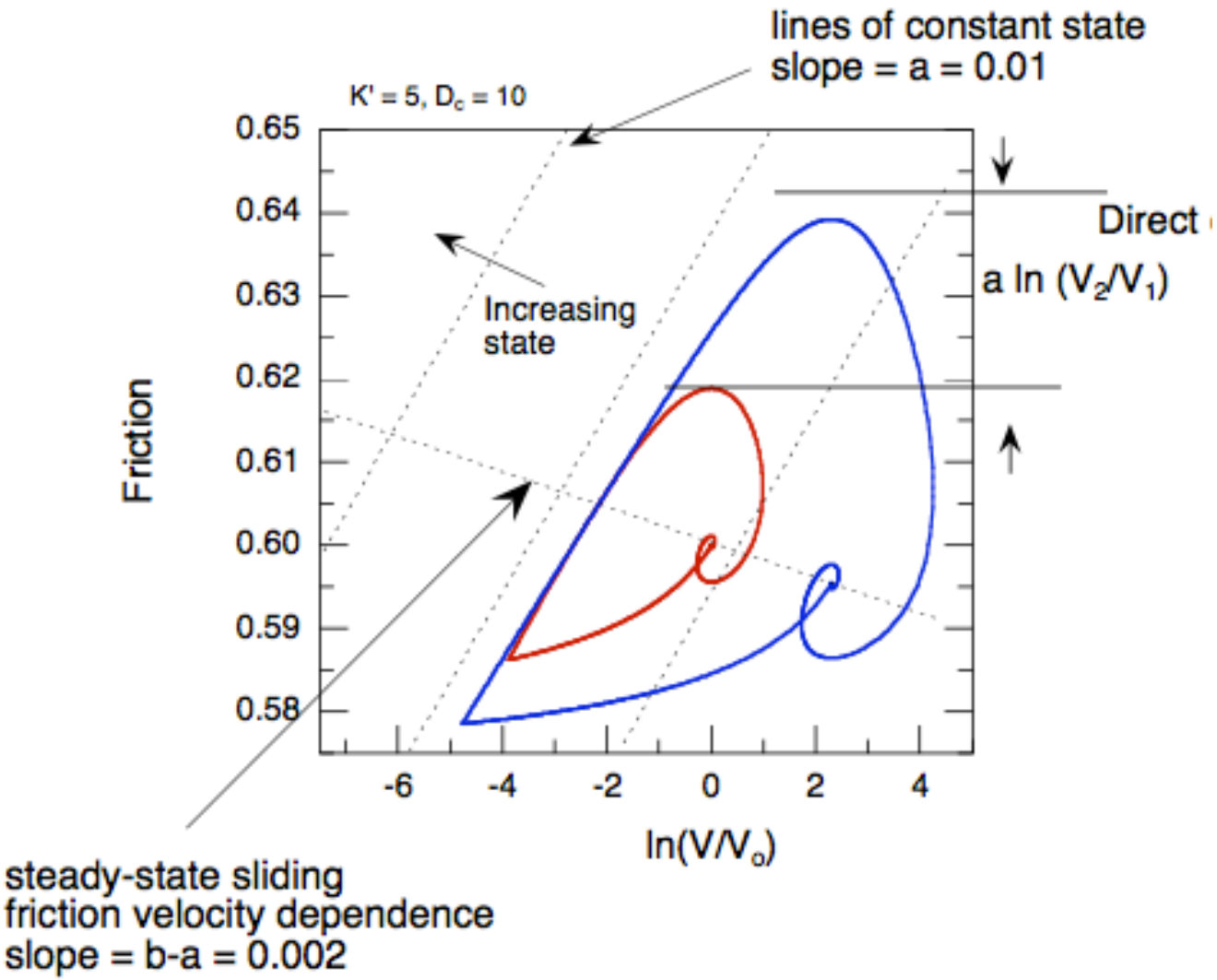
10 μm/s



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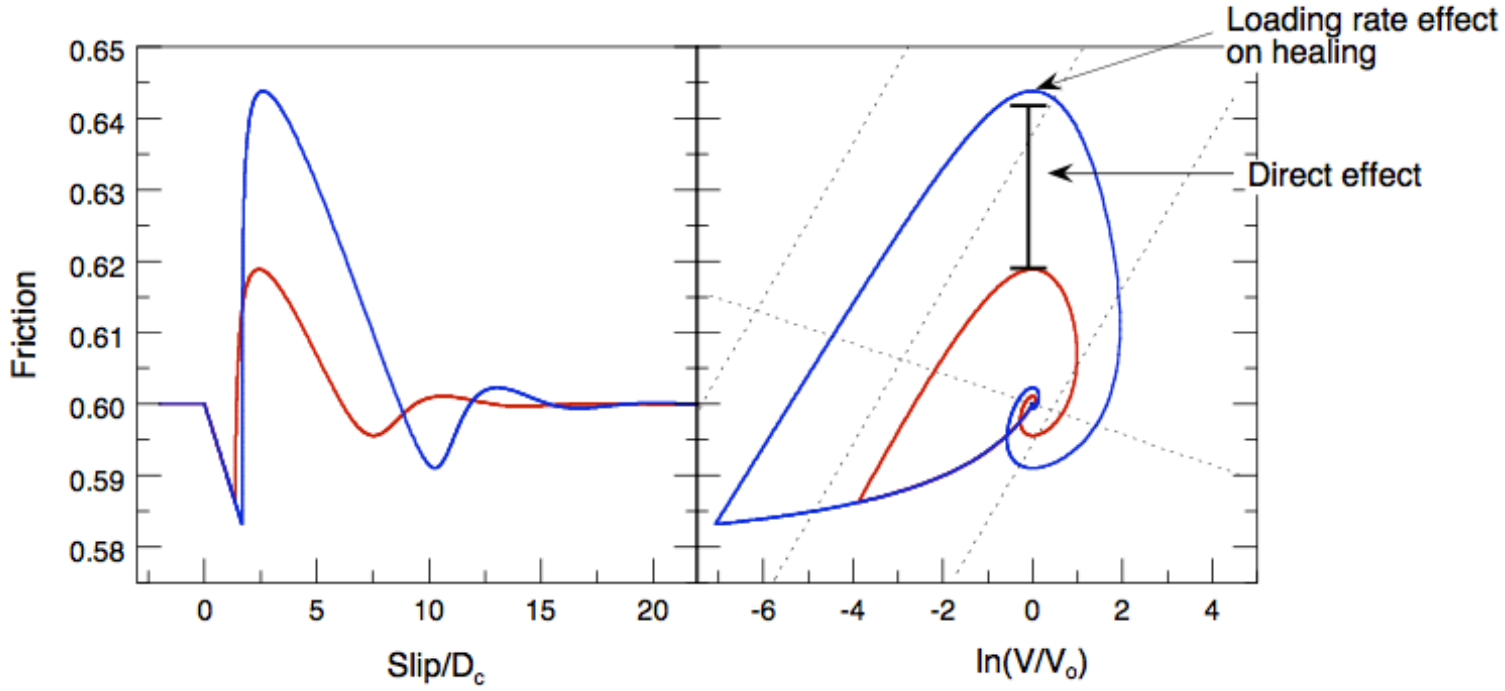

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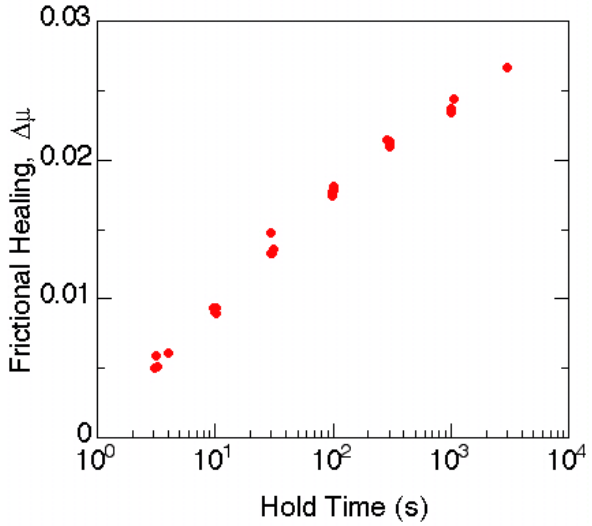
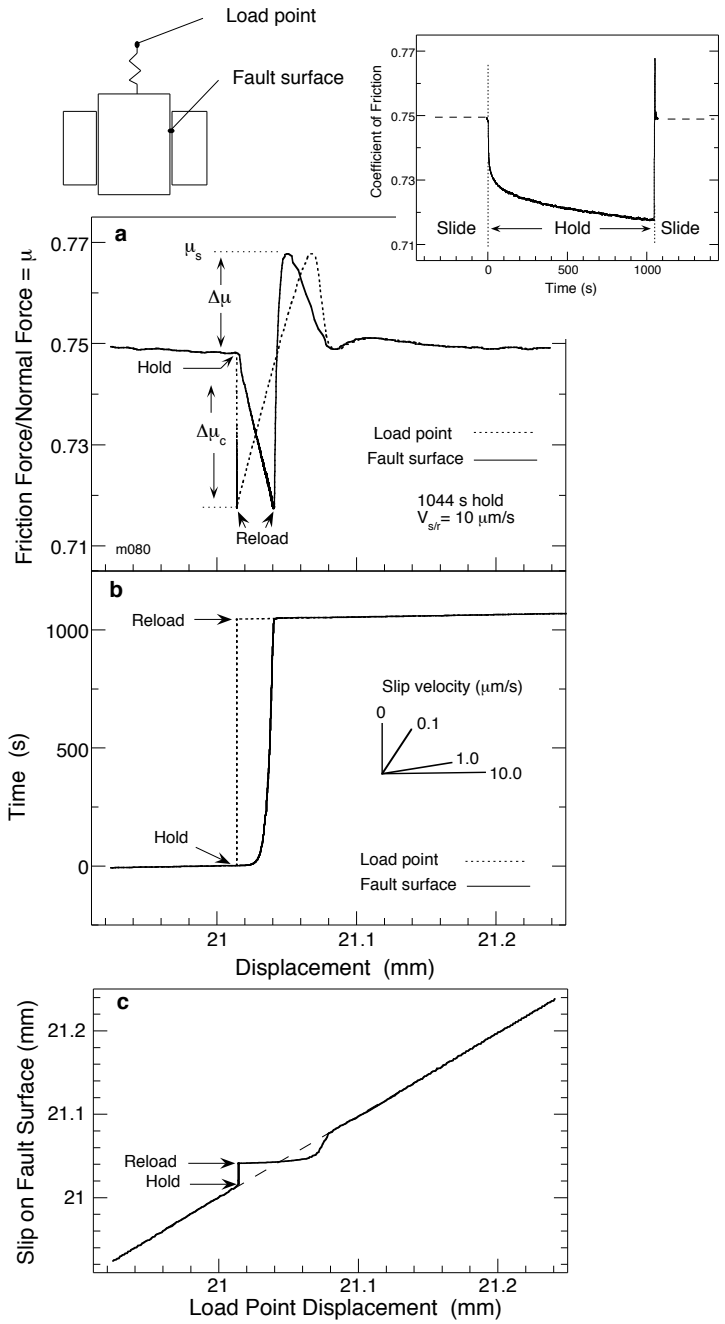


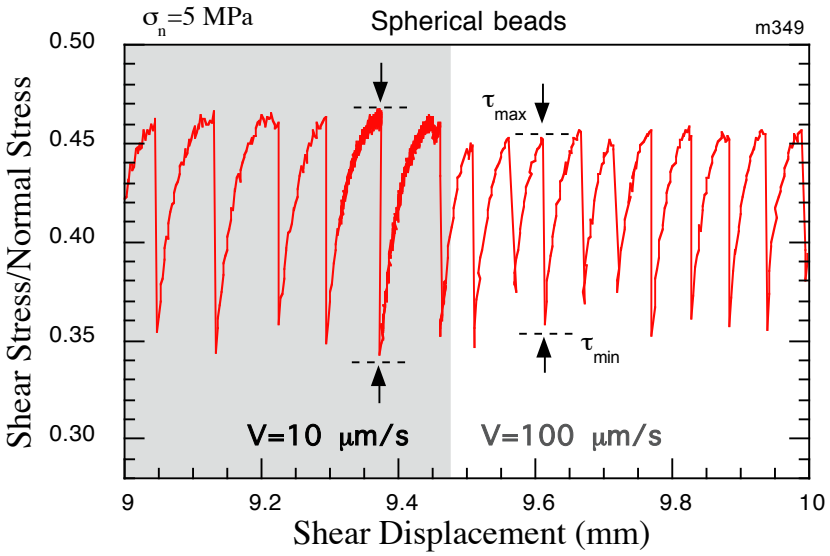


Slide-hold-slide

Slip-reload-slip

Earthquake-interseismic healing and reloading-earthquake

The full seismic cycle of stick-slip, frictional restrengthening, and interseismic reloading



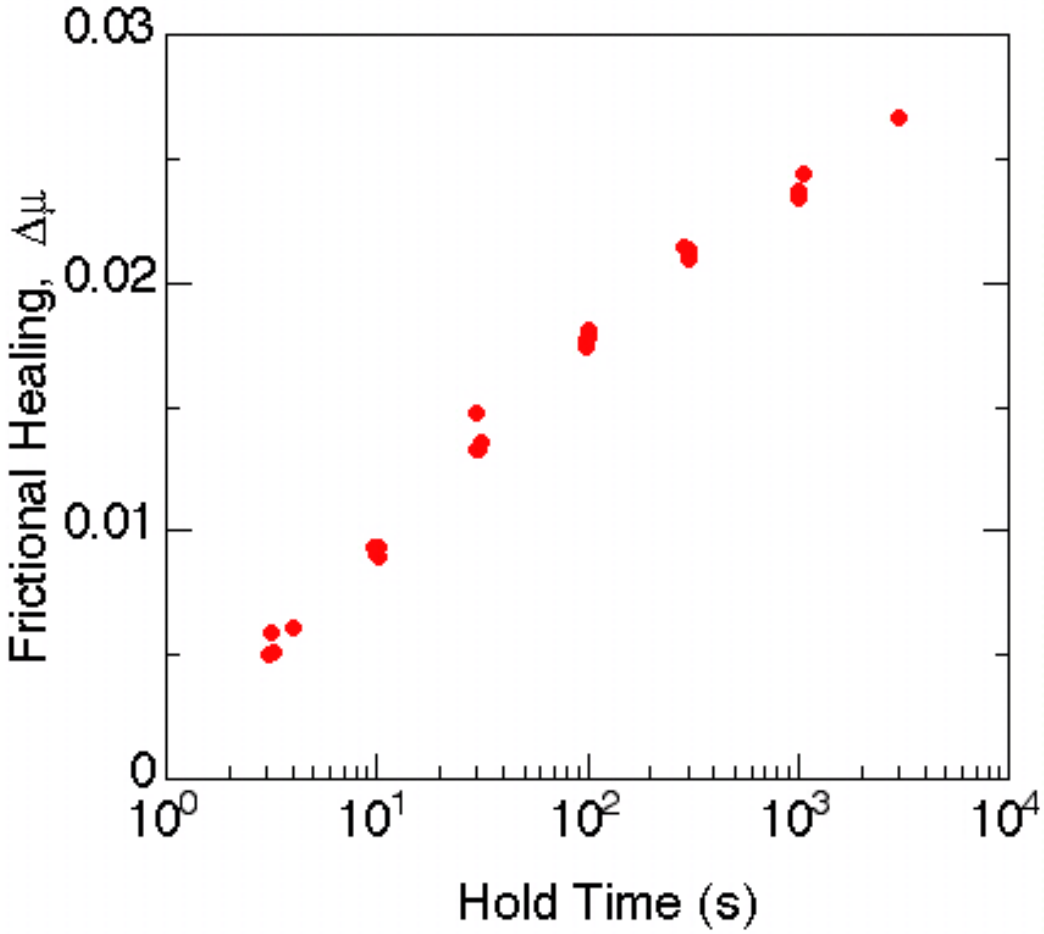
Marone and Saffer, 2013 Figure 4

Rate (v) and State (θ) Friction Constitutive Laws

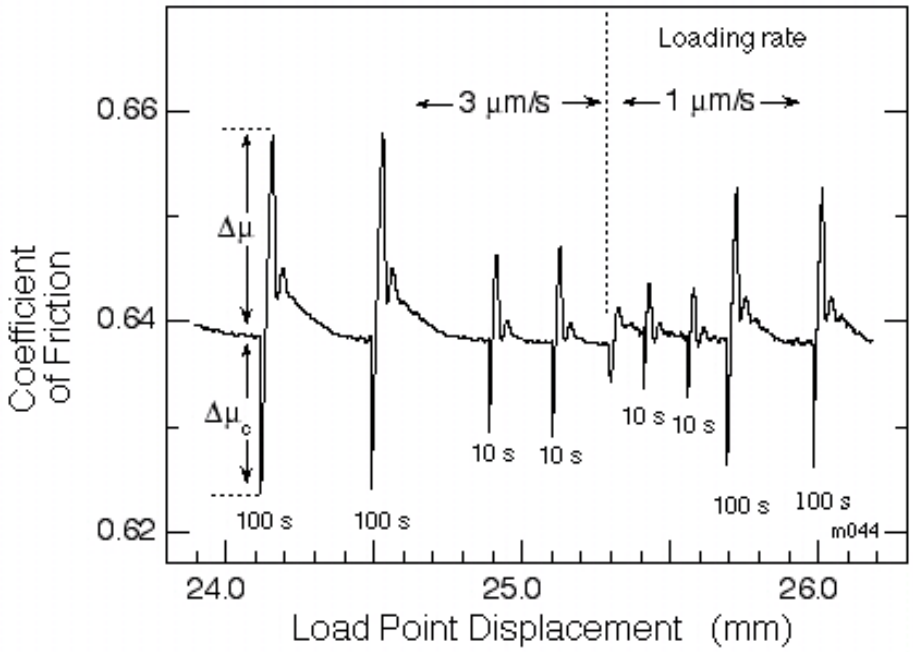
1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling



Modeling experimental data



$$\frac{d\mu}{dt} = k(V_{lp} - V)$$