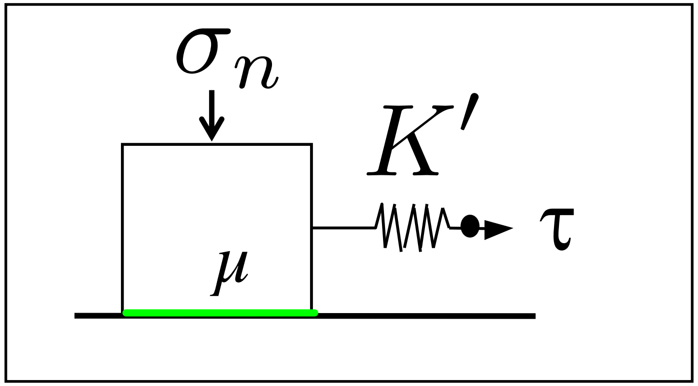
Mechanics of Earthquakes and Faulting

Lecture 10,   2 Mar. 2021

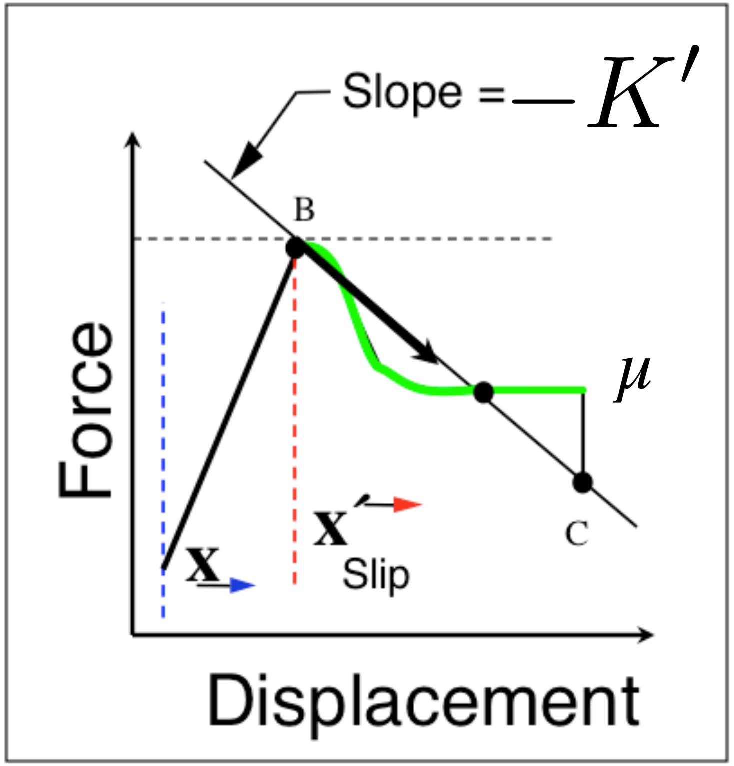
www.geosc.psu.edu/Courses/Geosc508

- Friction Constitutive Laws for Faulting
- Critical stiffness and stability transition from stable to unstable faulting
- SHS test to measure RSF parameters
- Earthquake Nucleation, Critical Fault Patch Size
- Stress Distribution for Propagating Rupture --(Crack Tip) Cohesive Zone
- Laboratory Observations of The transition from stable to unstable frictional sliding: Confirmation of the concept of a critical fault weakening rate with slip (kc)



Mechanics of Frictional Sliding: Stick-slip

$$\frac{\tau(\theta, v)}{\sigma_n} = \mu_o + a\; ln\left(\frac{v}{v_o}\right) + b\; ln\left(\frac{v_o\theta}{D_c}\right)$$

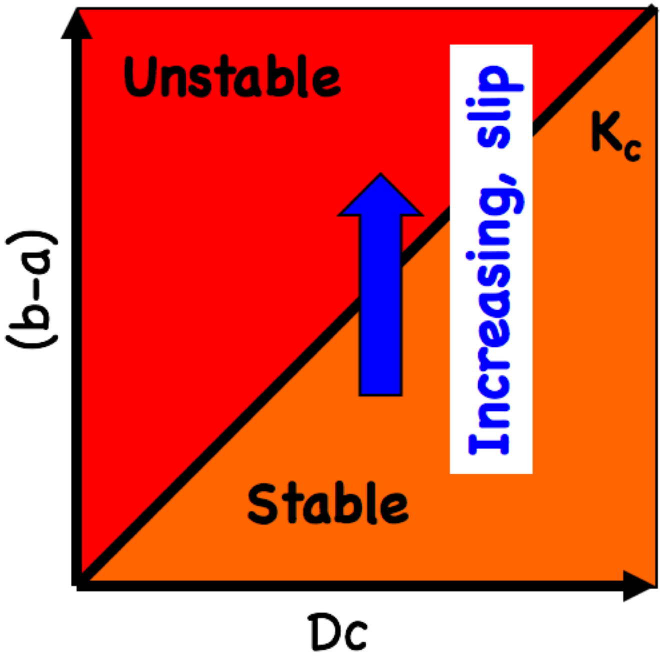


$$K_c = \frac{\sigma_n(b-a)}{D_c}[1 + \frac{m{V_o}^2}{\sigma_n a D_c}]$$

Rice & Ruina, 1983; Gu et al., 1984; Roy and Marone, 1996

$$K_c \approx \frac{\sigma_n(b-a)}{D_c}$$

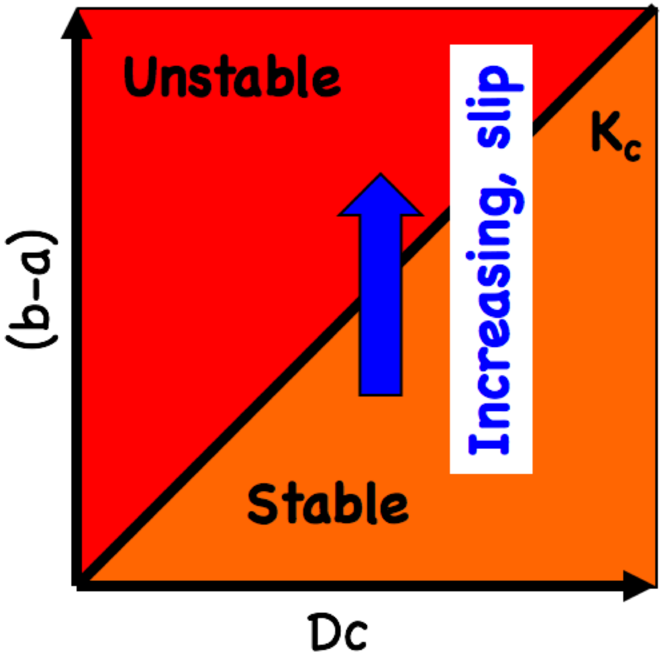
Unstable if $K < K_c$



Stability of sliding

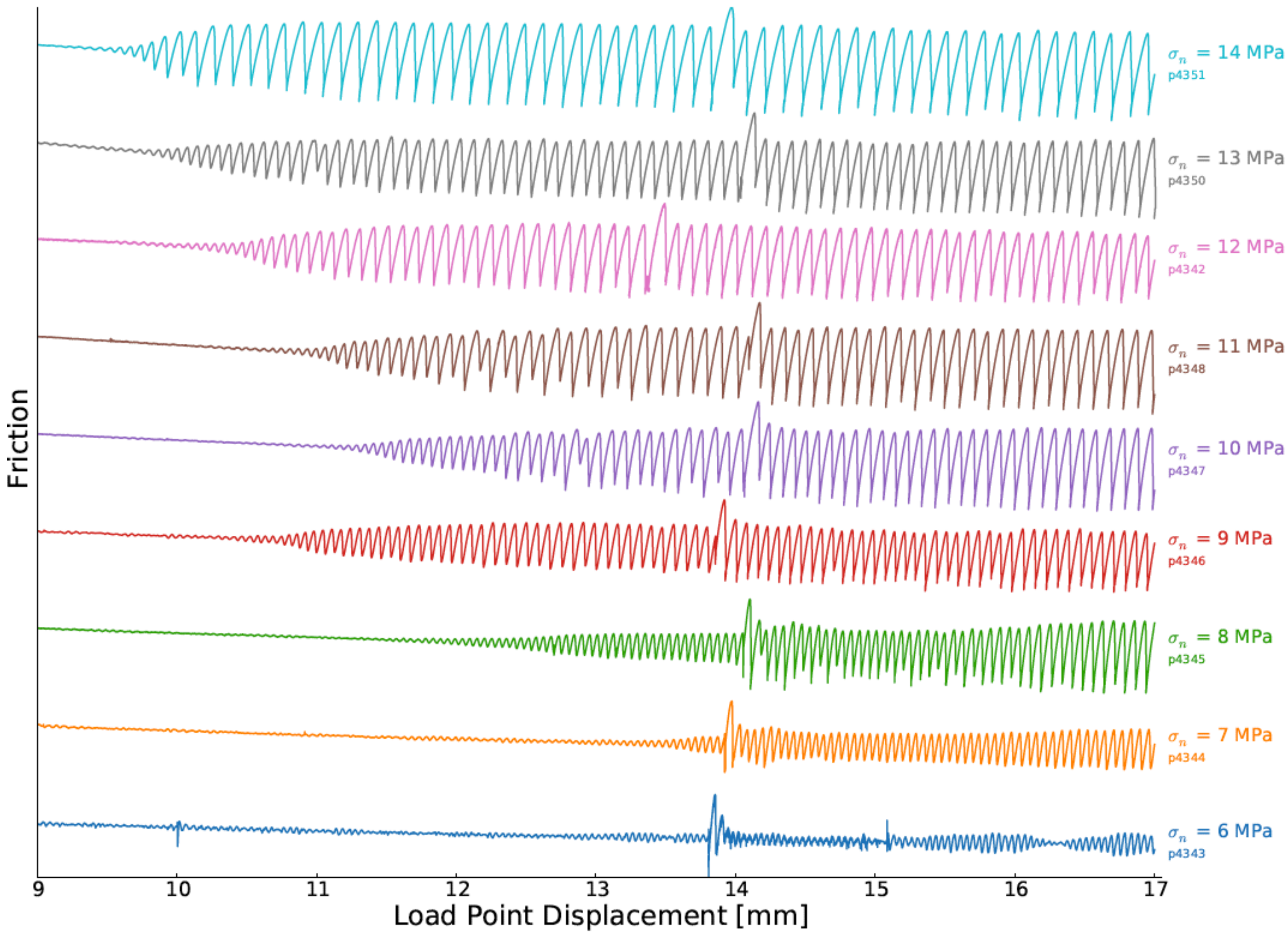
$$K_c = \frac{\sigma_n(b-a)}{D_c}[1 + \frac{mV_o{}^2}{\sigma_n a D_c}]$$

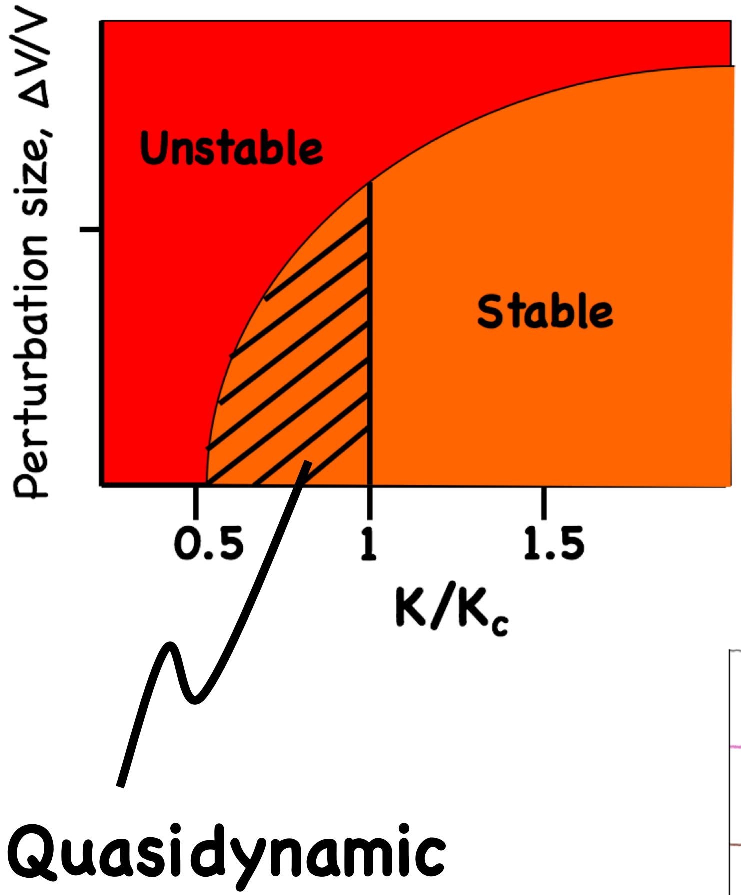
Stability line: K = Kc

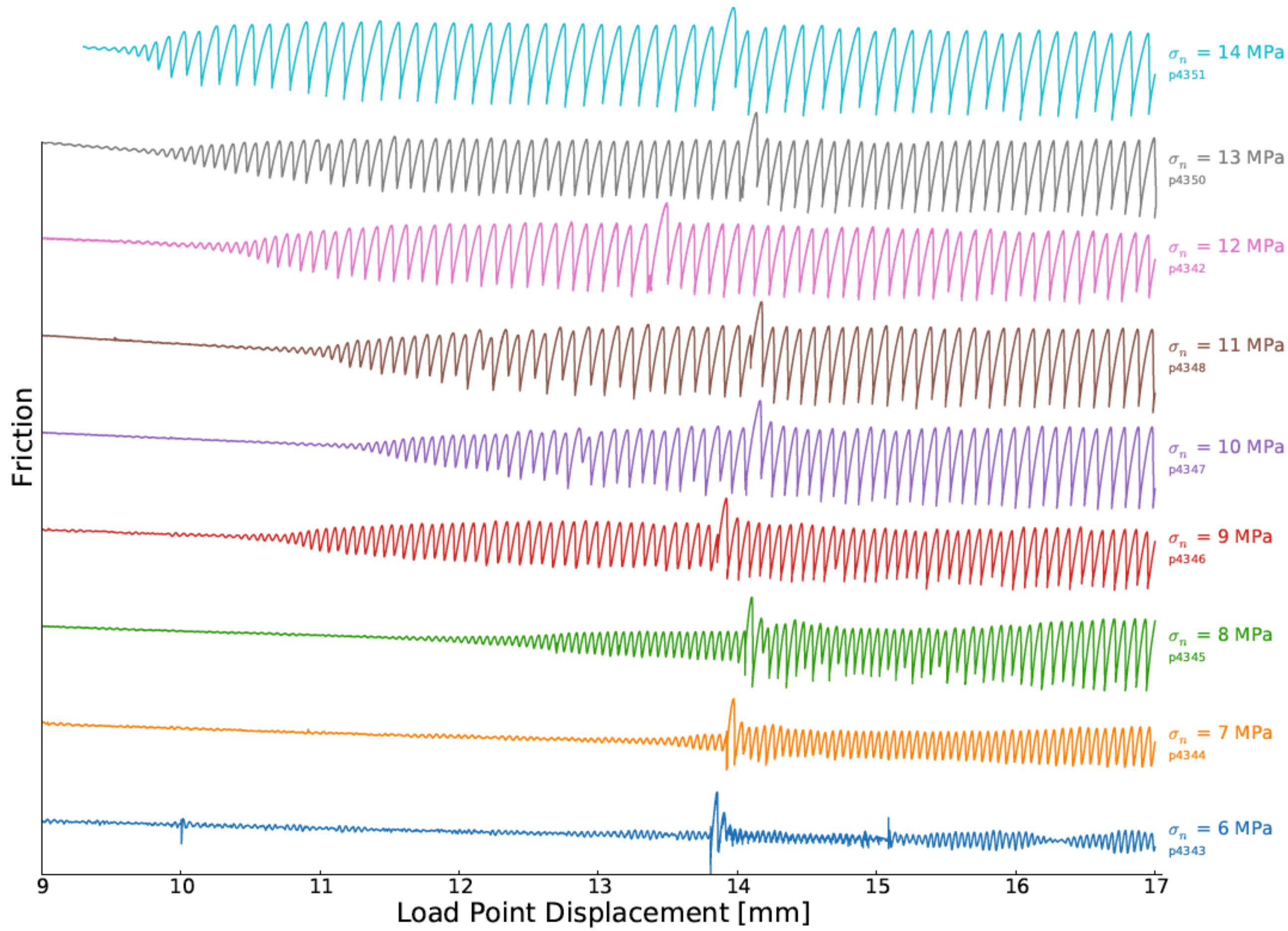


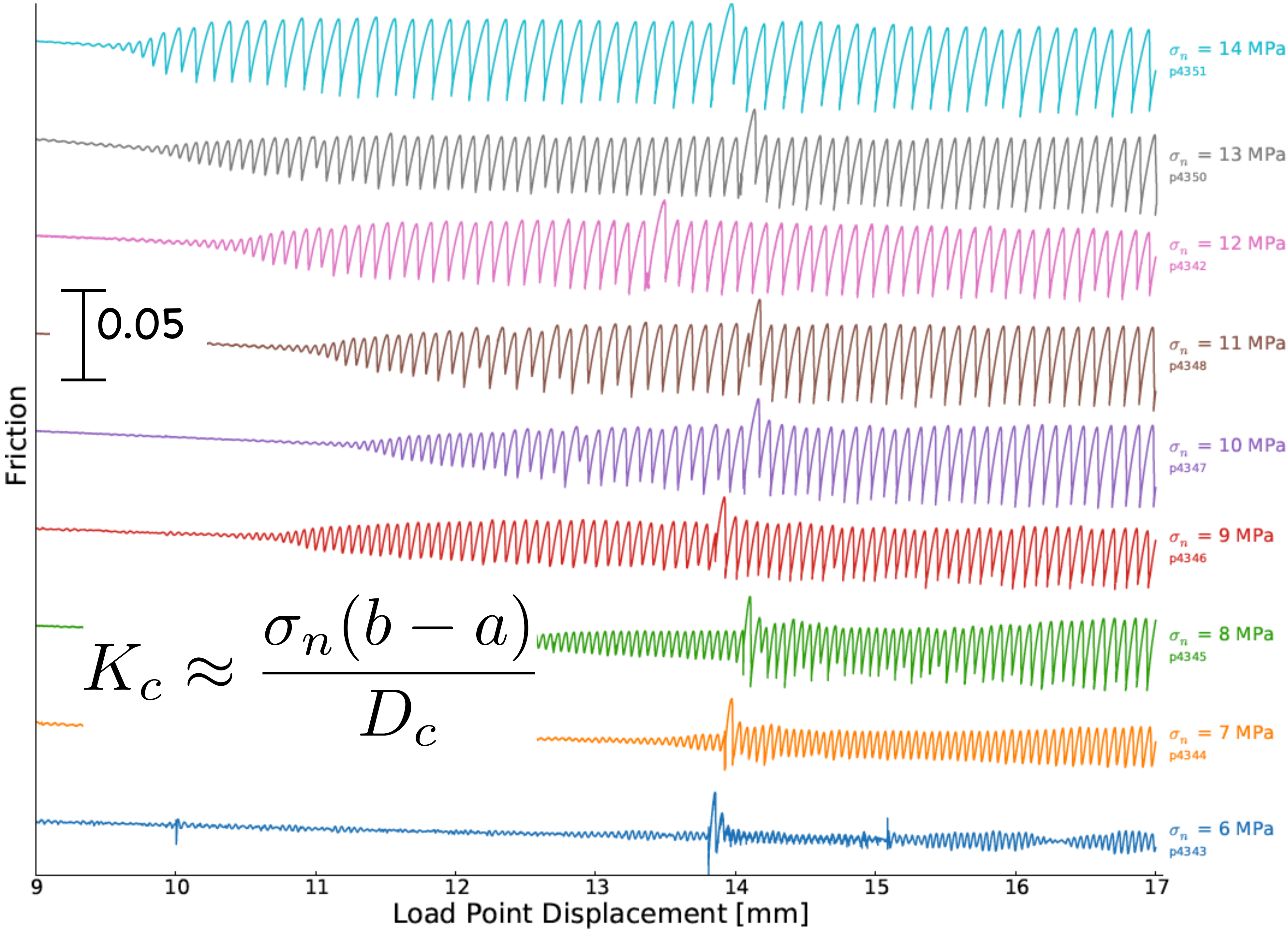
Stability of sliding

$$K_c = \frac{\sigma_n(b-a)}{D_c}\left[1 + \frac{mV_o^{\,2}}{\sigma_n a D_c}\right]$$




Perturbation size, ΔV/V
Unstable
Stable
0.5
1
1.5
K/K_c
Quasidynamic


σ_n = 14 MPa
p4351
σ_n = 13 MPa
p4350
σ_n = 12 MPa
p4342
σ_n = 11 MPa
p4348
σ_n = 10 MPa
p4347
σ_n = 9 MPa
p4346
σ_n = 8 MPa
p4345
σ_n = 7 MPa
p4344
σ_n = 6 MPa
p4343
Friction
9
10
11
12
13
14
15
16
17
Load Point Displacement [mm]


σn = 14 MPa
p4351
σn = 13 MPa
p4350
σn = 12 MPa
p4342
0.05
σn = 11 MPa
p4348
σn = 10 MPa
p4347
Friction
σn = 9 MPa
p4346
$K_c \approx \frac{\sigma_n(b-a)}{D_c}$
σn = 8 MPa
p4345
σn = 7 MPa
p4344
σn = 6 MPa
p4343
9
10
11
12
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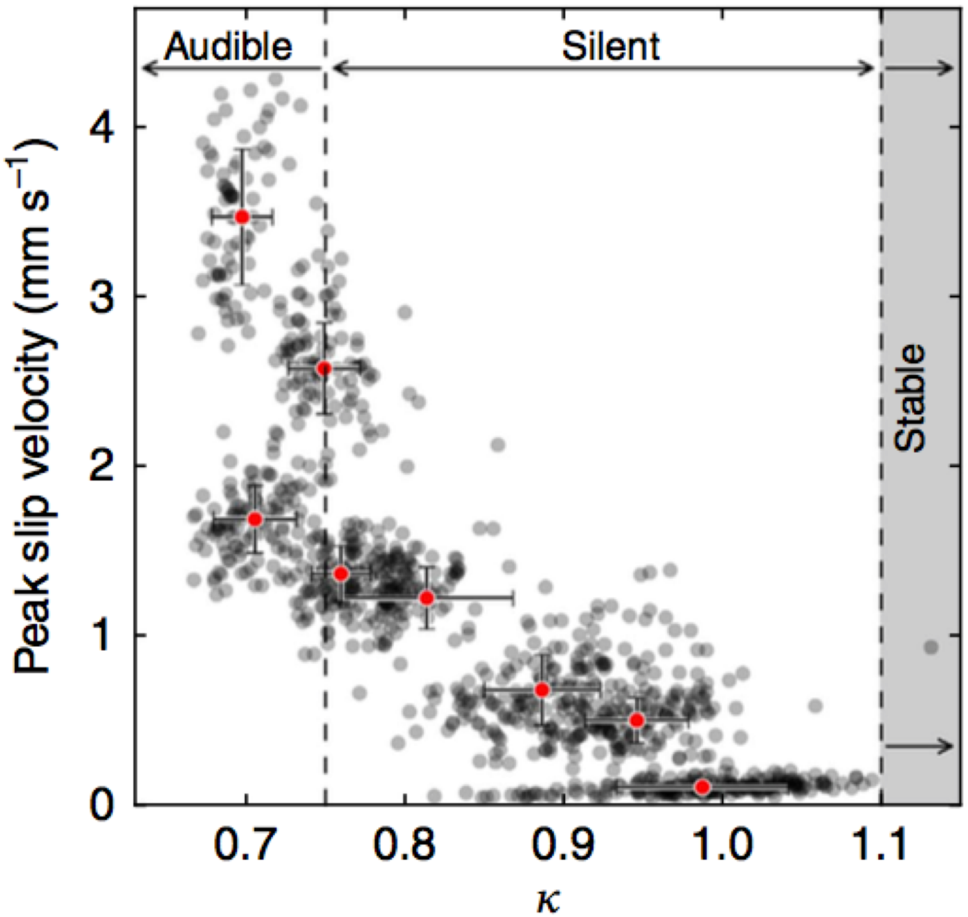
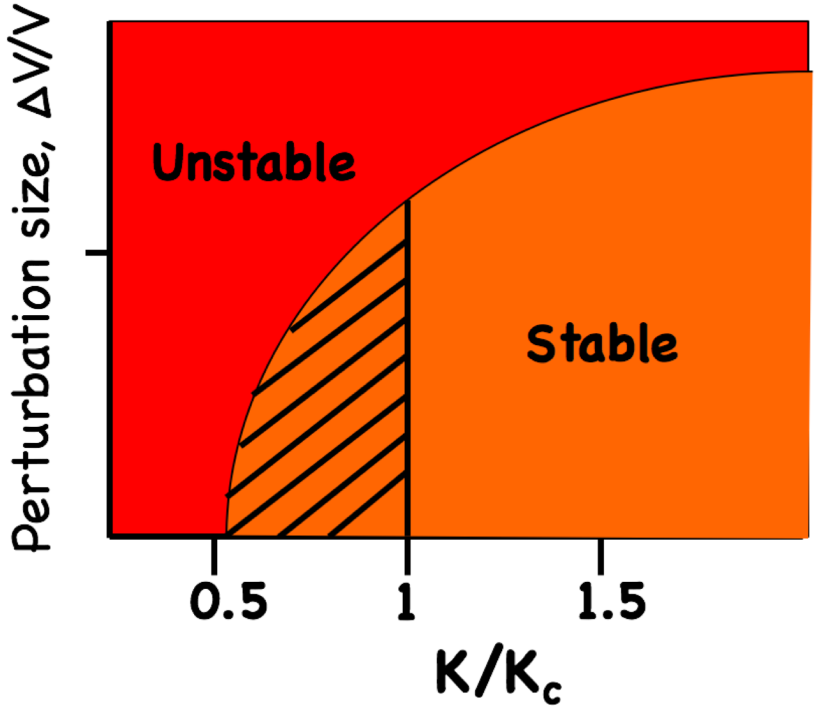
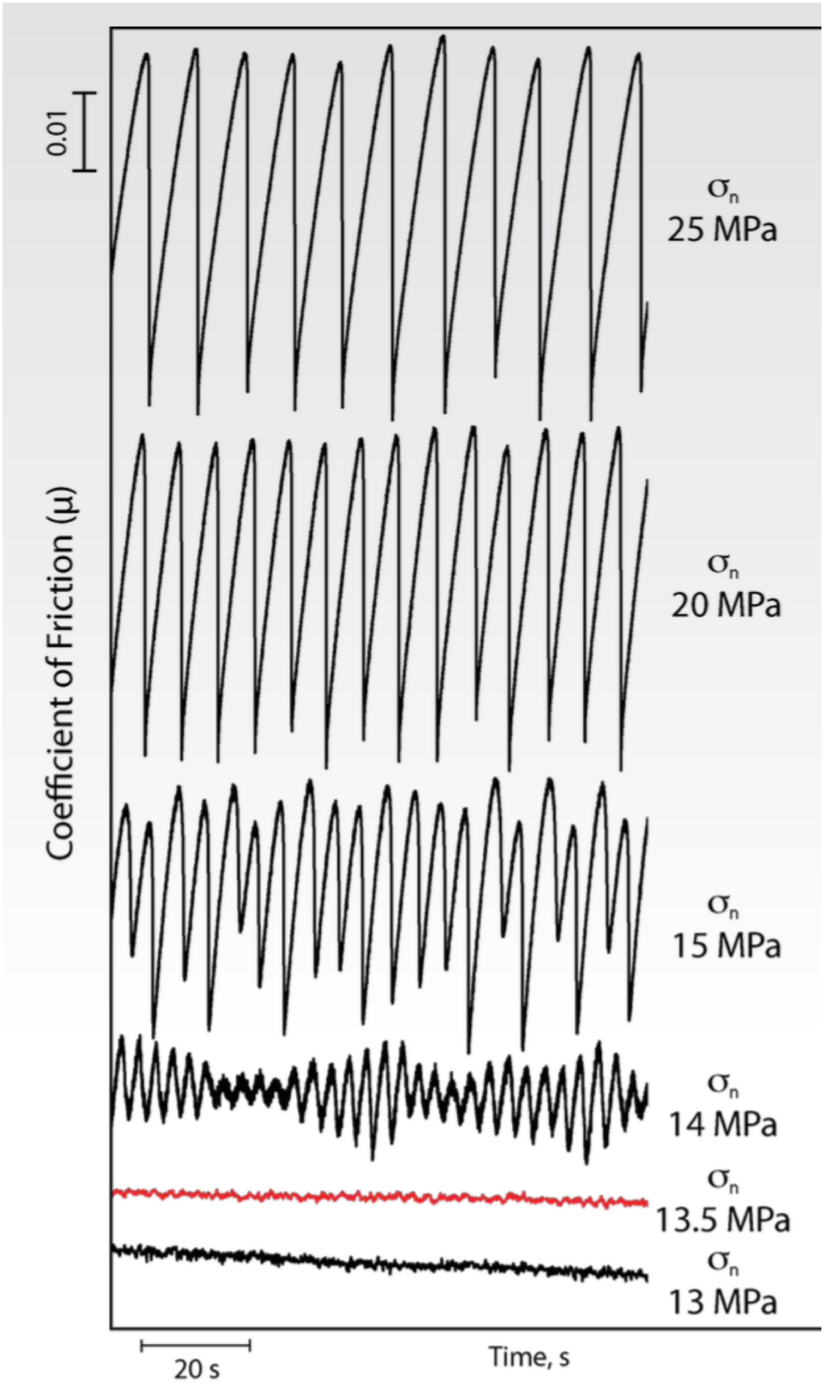
14
15
16
17
Load Point Displacement [mm]





Unstable slip if

$$K < K_c \approx \frac{\sigma_n (b - a)}{D_c}$$



Complex Behavior (Period Doubling) Near The Stability Boundary

Scuderi, Marone, Tinti, Di Stefano, & Collettini, *Nature Geosc.* 2016

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

reference value of base friction

reference velocity

critical slip distance

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Dieterich, aging law

$$\frac{d\theta}{dt} = -\frac{V\theta}{D_c} ln(\frac{V\theta}{D_c})$$

Ruina, slip law

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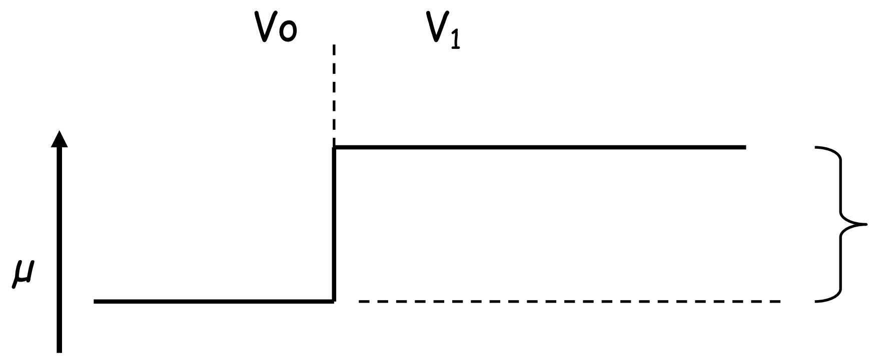
Velocity Step test



$$a\, ln\left(\frac{V_1}{V_o}\right)$$

Direct Effect

Friction vs.
shear displacement

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o \theta}{D_c}\right)$$

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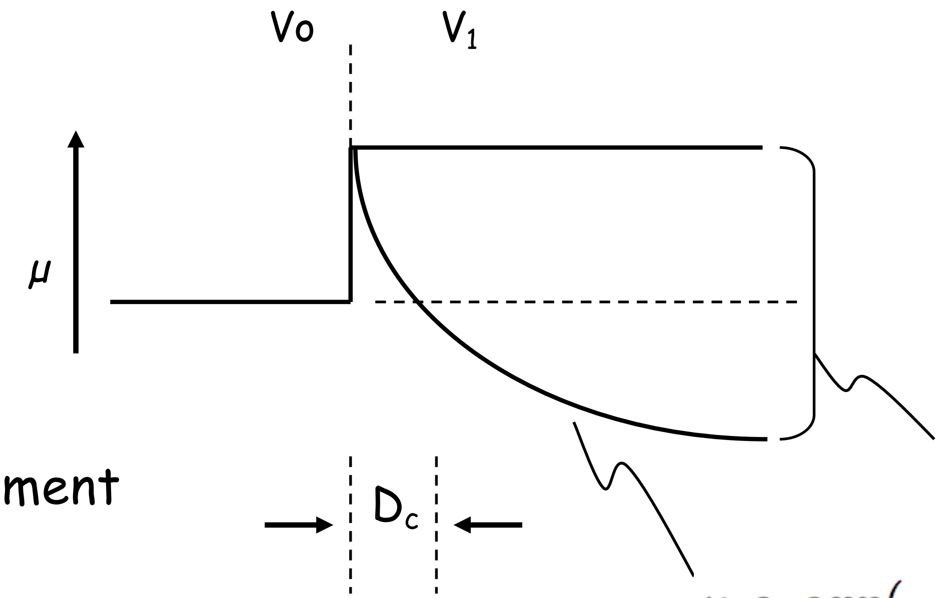


$$b\, ln\left(\frac{V_1}{V_o}\right)$$

Evolution Effect

$$\mu\ \alpha\ exp(-\frac{u}{D_c})$$

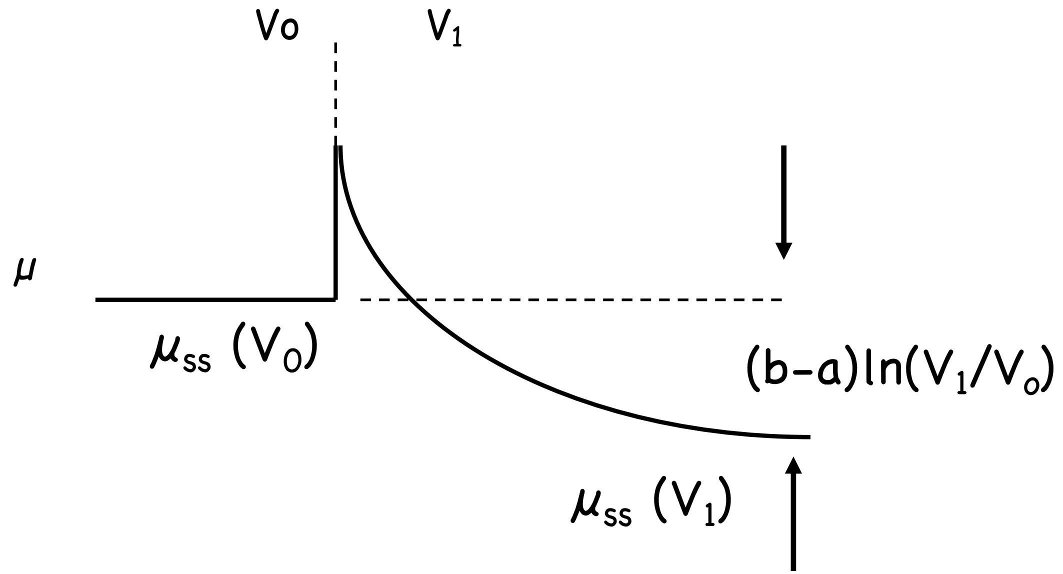
Fading memory of past state

Friction vs.
shear displacement

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Dieterich law



$$\frac{d\theta}{dt} = -\frac{V\theta}{D_c} ln(\frac{V\theta}{D_c})$$

Ruina law

Steady-state sliding:

$$\frac{d\theta}{dt} = 0$$

Thus, for both laws
$V\theta/D_c = 1$, which means: $\theta_{ss} = \frac{D_c}{V_1}$

(1) $\mu(\theta, V) = \mu_o + a\,ln\left(\frac{V}{V_o}\right) + b\,ln\left(\frac{V_o\theta}{D_c}\right)$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Dieterich law

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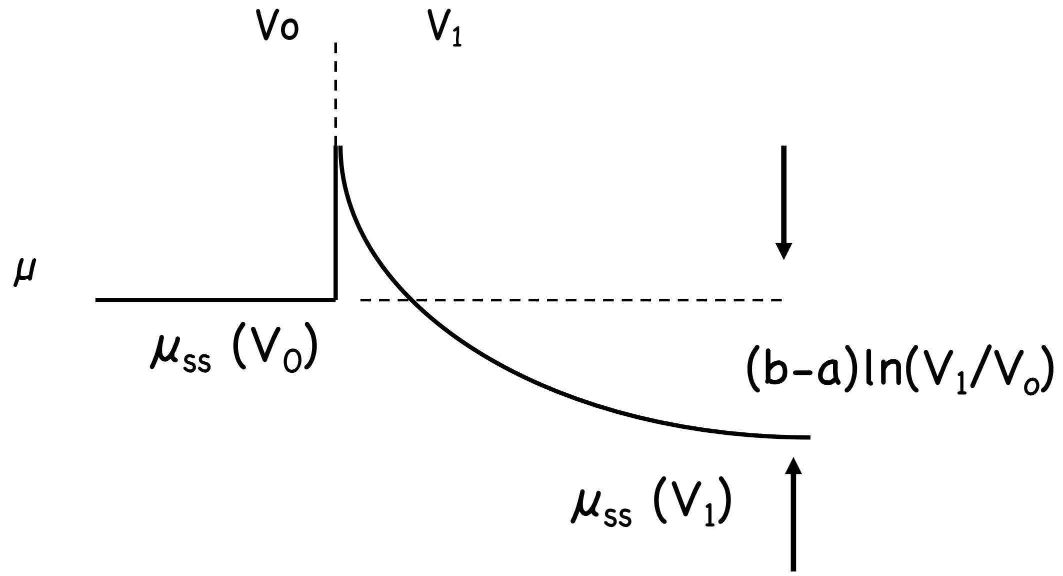
Ruina law



Steady-state sliding:

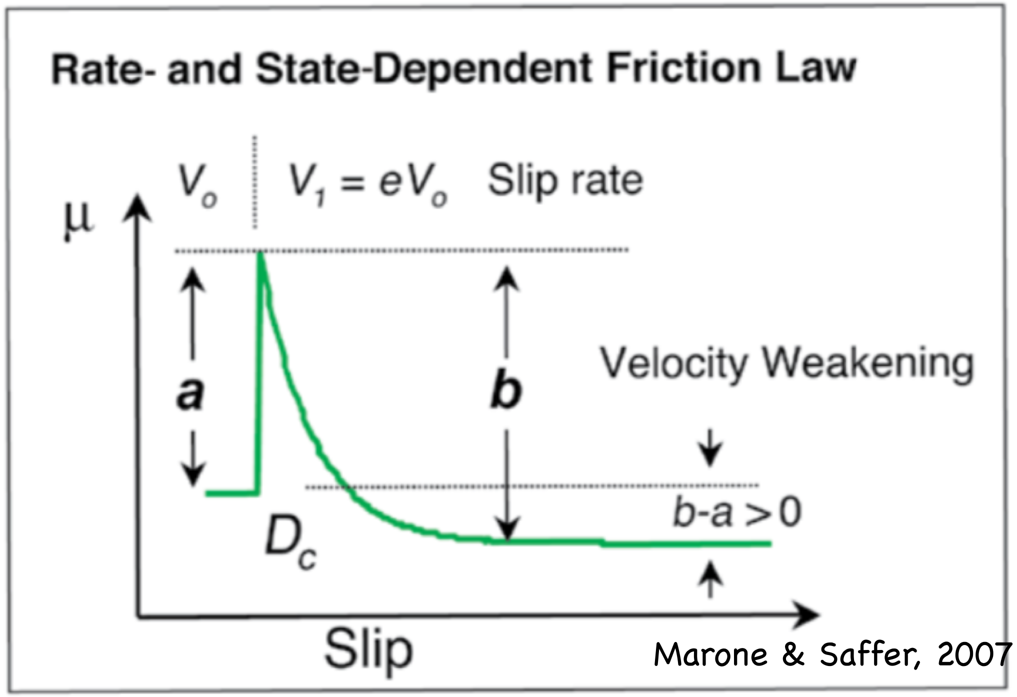
$$\theta_{ss} = \frac{D_c}{V_1}$$

then (1) becomes: $\mu_1 - \mu_o = (a-b)ln(\frac{V_1}{V_o})$

or $$(a-b) = \frac{\Delta\mu}{\Delta lnV}$$

(1) $$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

(b)



Marone & Saffer, 2007

Steady-state sliding:

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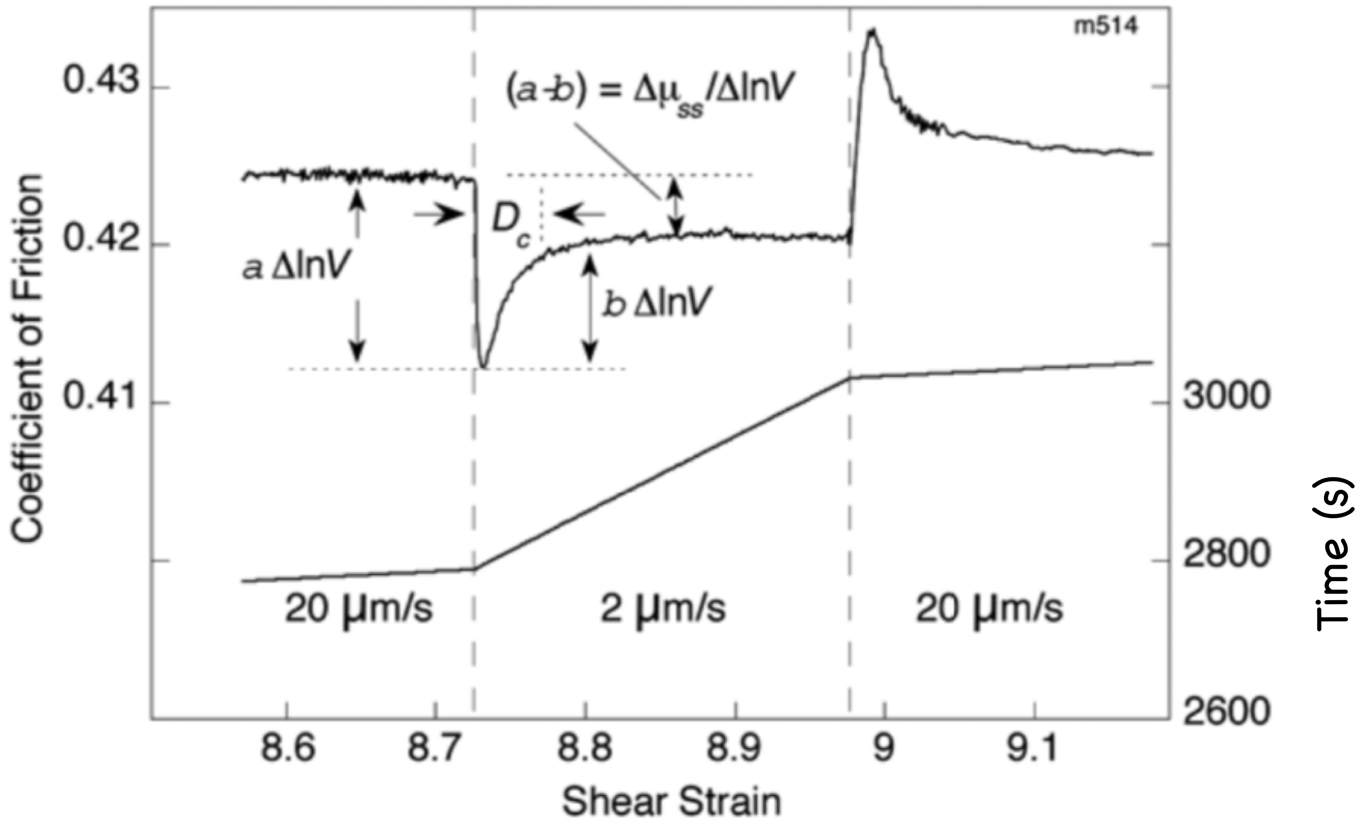

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Steady-state sliding:

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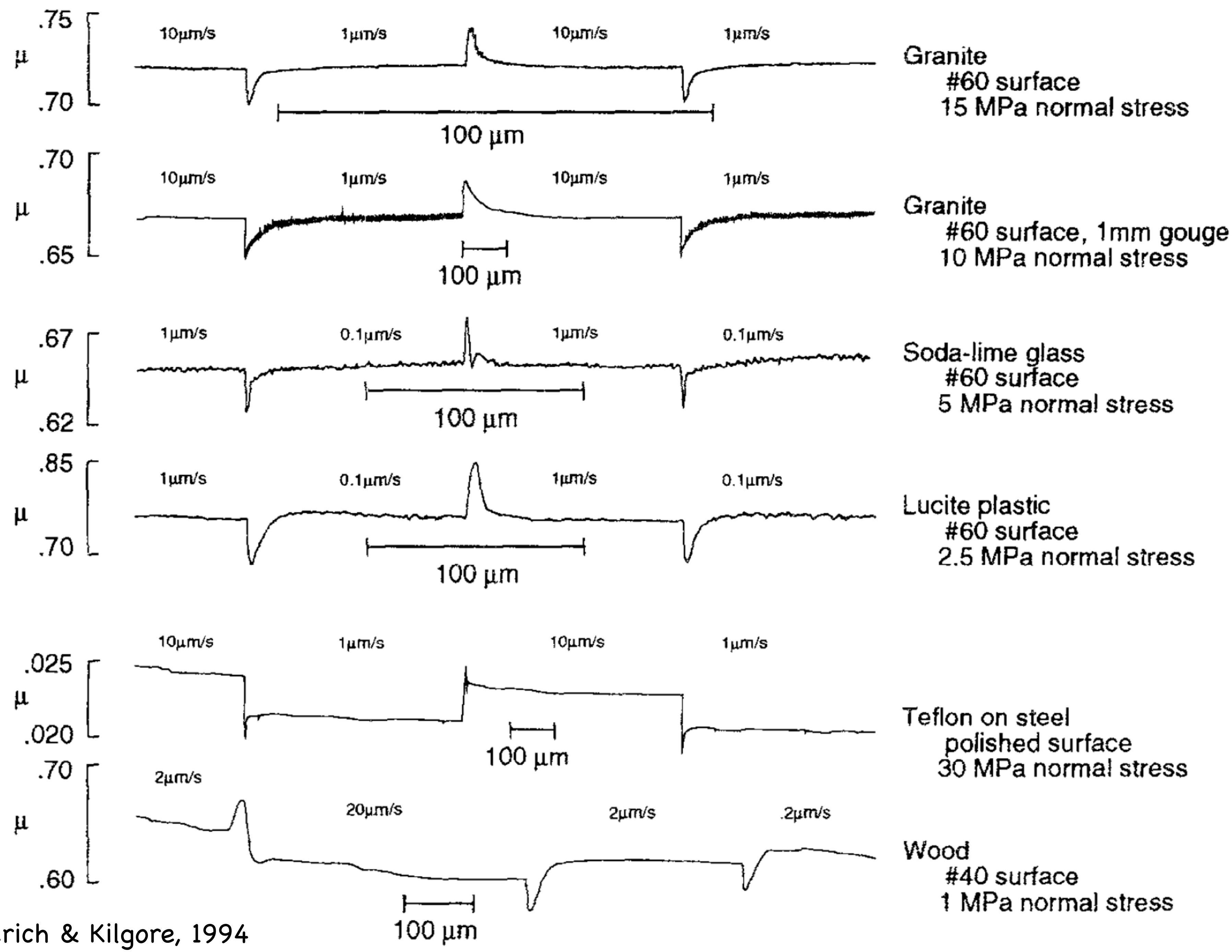
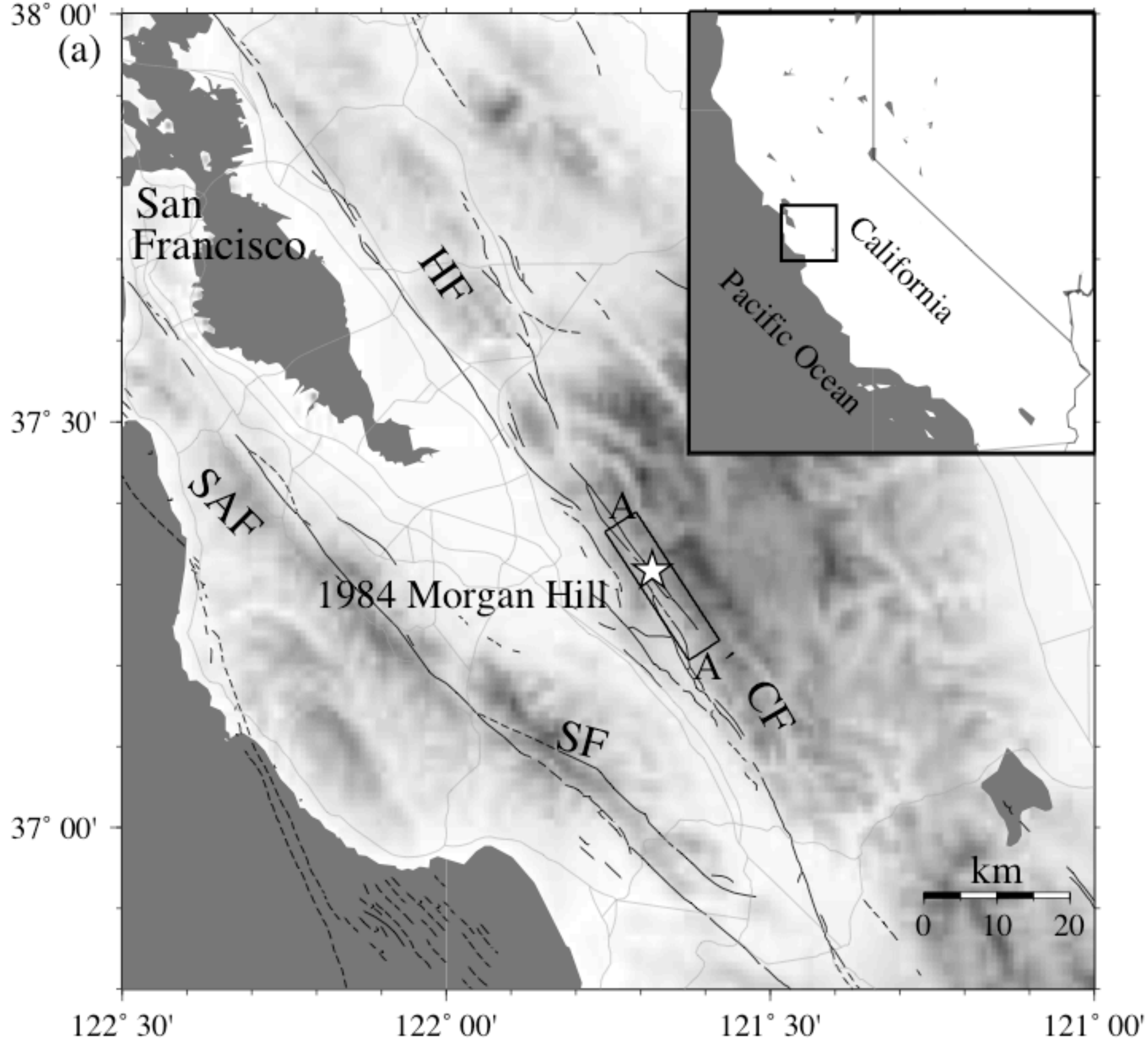
Universality of rate and state friction



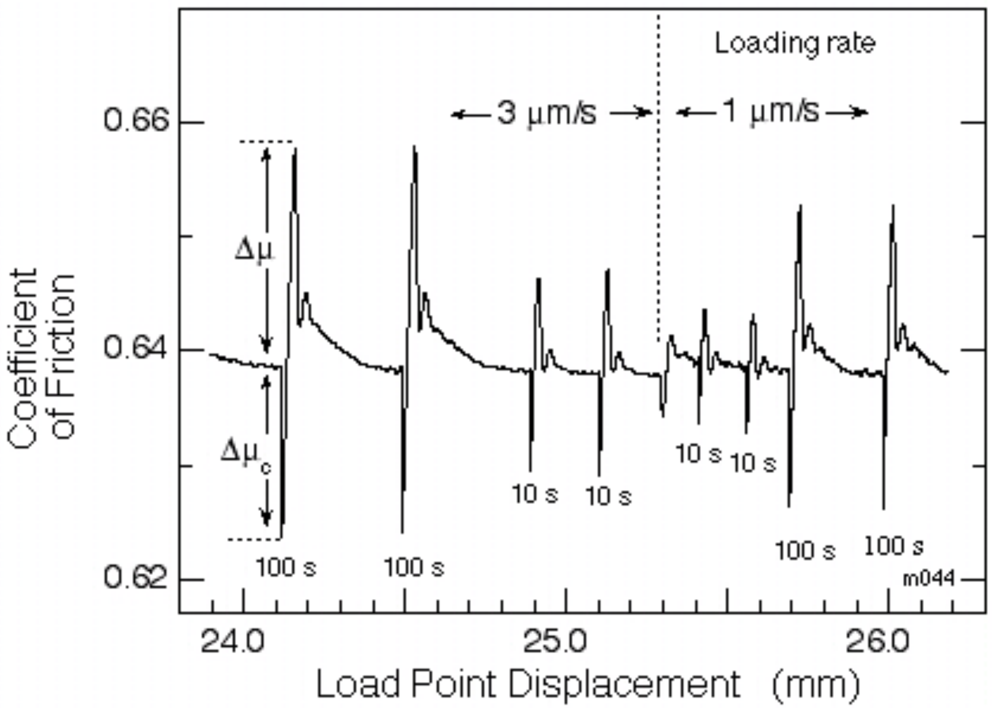
Dieterich & Kilgore, 1994

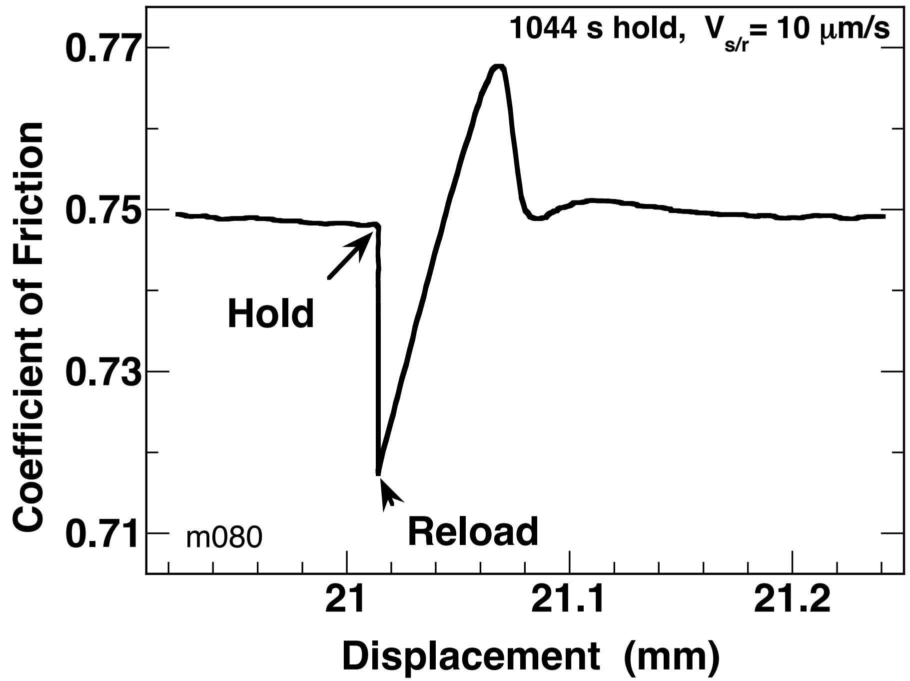
Fault Healing and the Seismic Cycle: Repeating Earthquakes

How do faults regain strength between earthquakes?

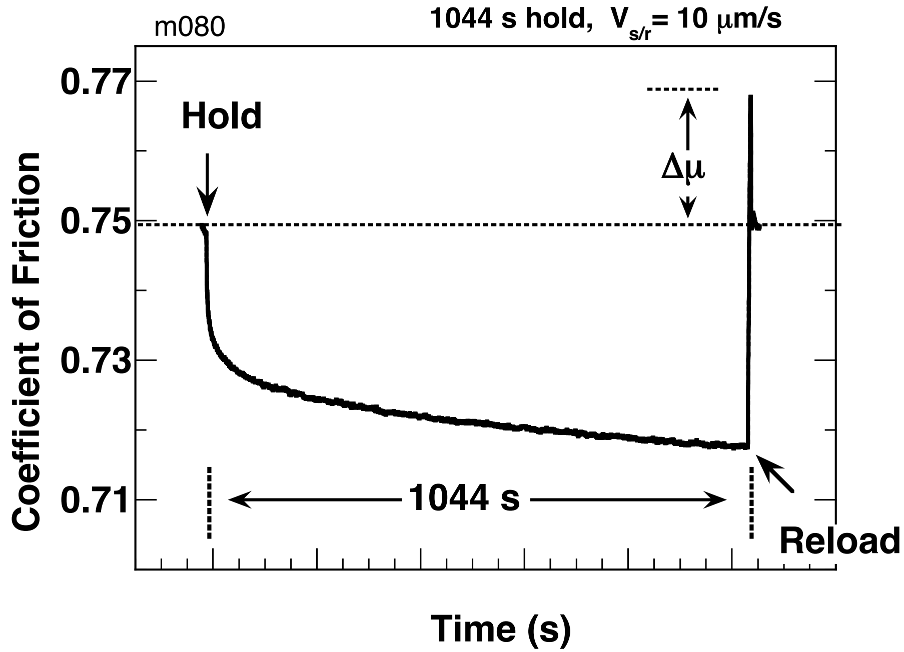


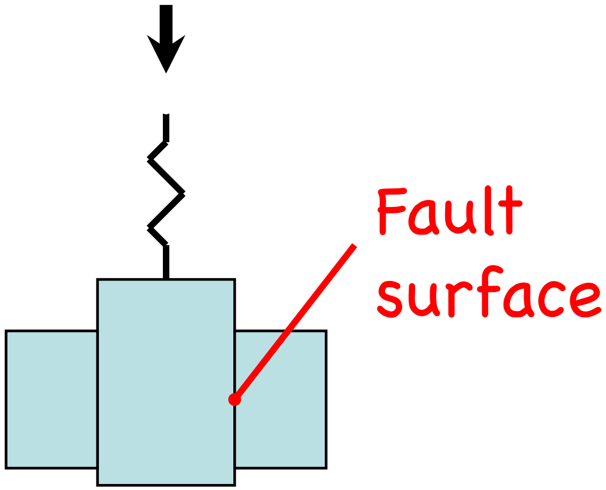
Vidale et al., 1994; Peng, Vidale, Marone & Rubin, GRL 2005

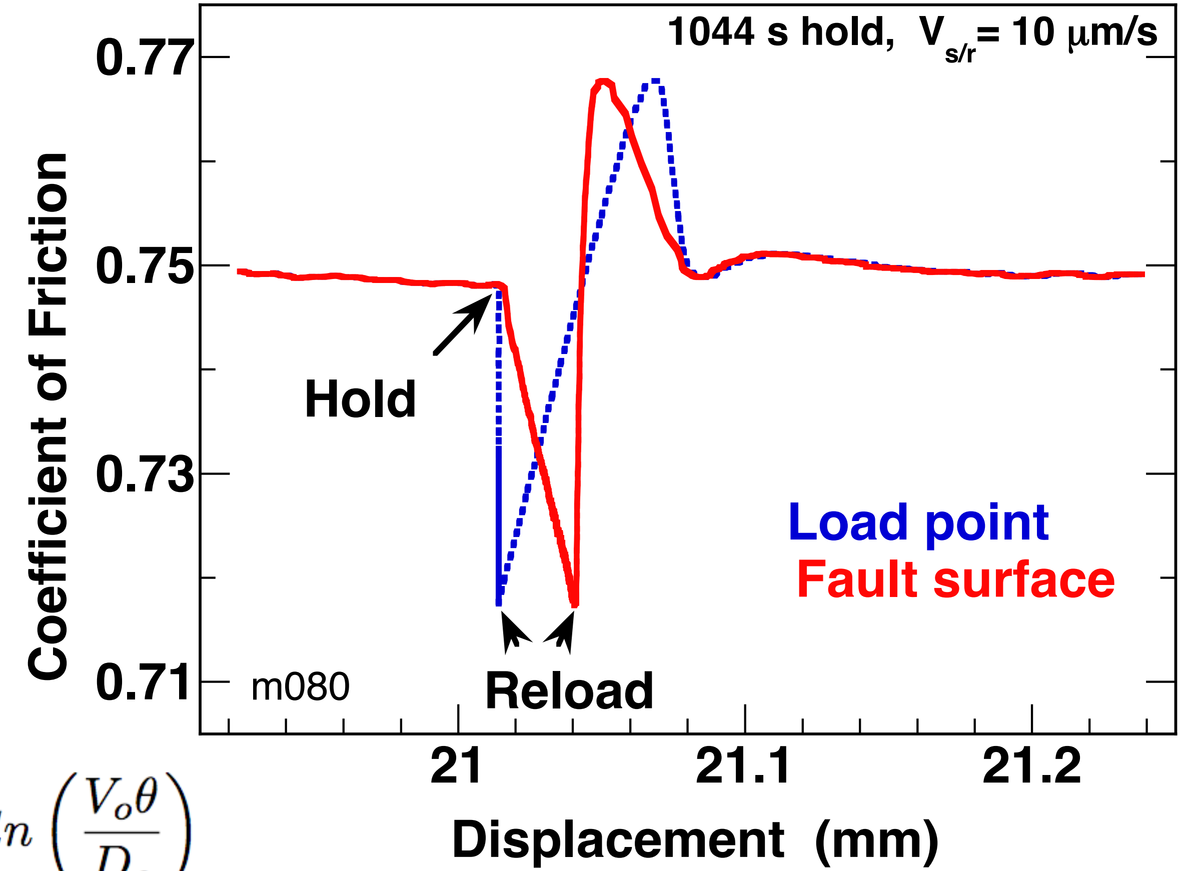




Sheared layer of quartz particles (100-150 μm), 25 MPa normal stress . Marone, 1998







1) $$\mu(\theta, V) = \mu_o + a\,ln\left(\frac{V}{V_o}\right) + b\,ln\left(\frac{V_o\theta}{D_c}\right)$$

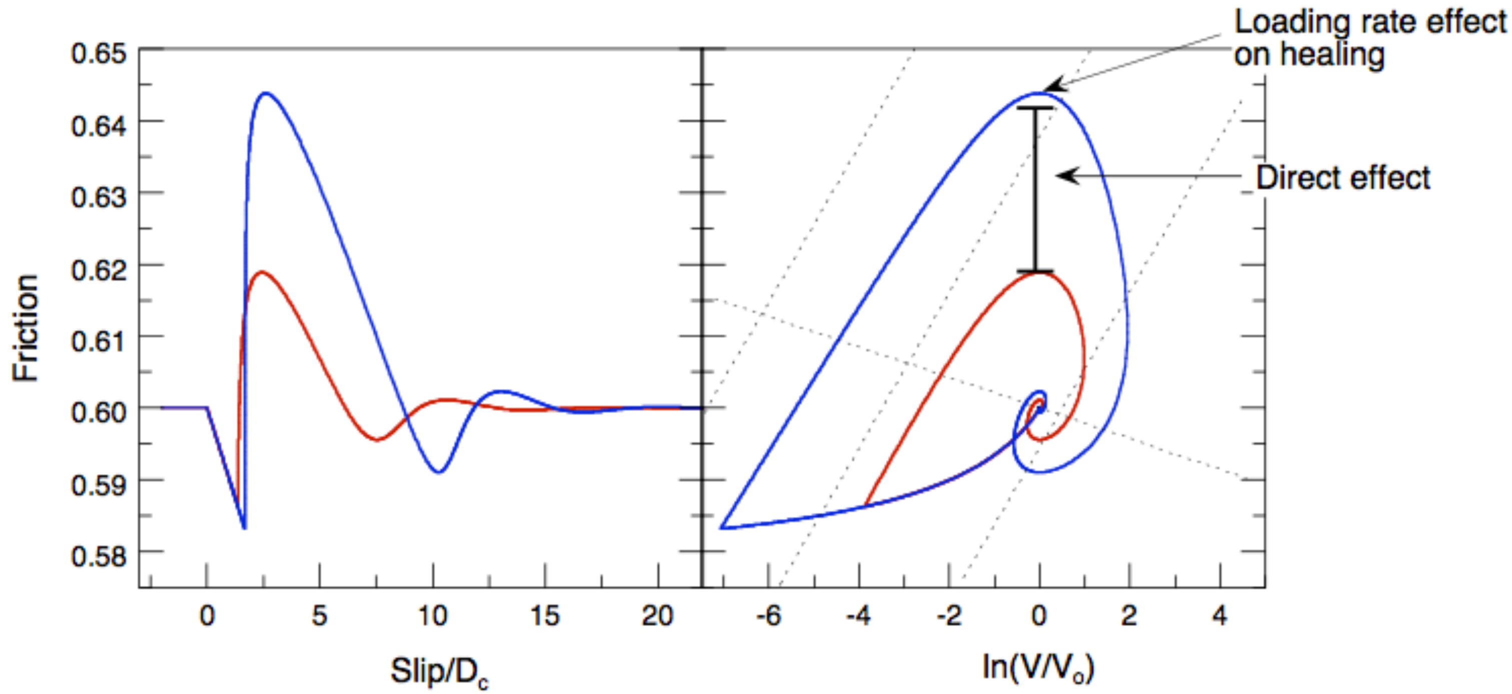
2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling

$$\frac{d\mu}{dt} = k\left(V_{lp} - V_o\,exp\left[\frac{\mu - \mu_o - b\,ln(\frac{V_o\theta}{D_c})}{a}\right]\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Derivation of the healing rate

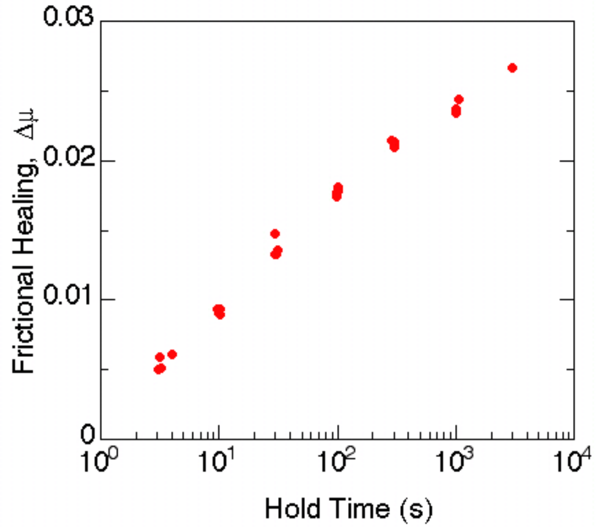


Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution

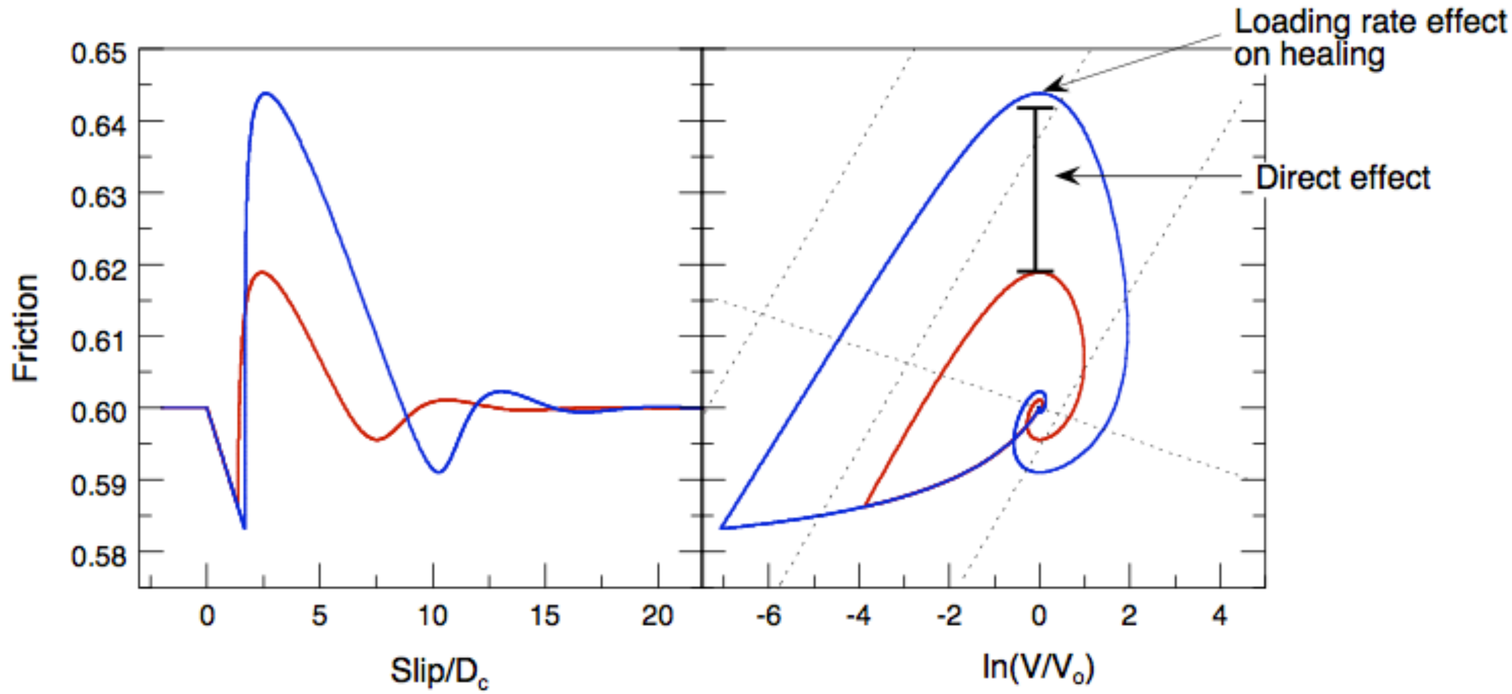
$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

$$\frac{d\mu}{dt} = k(V_{lp} - V)$$



Derivation of the healing rate



But let's focus on just one velocity, so that we can see what the healing rate tells us.

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

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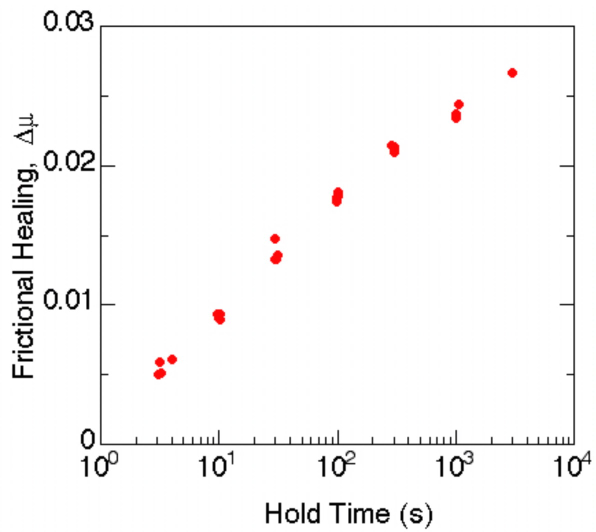
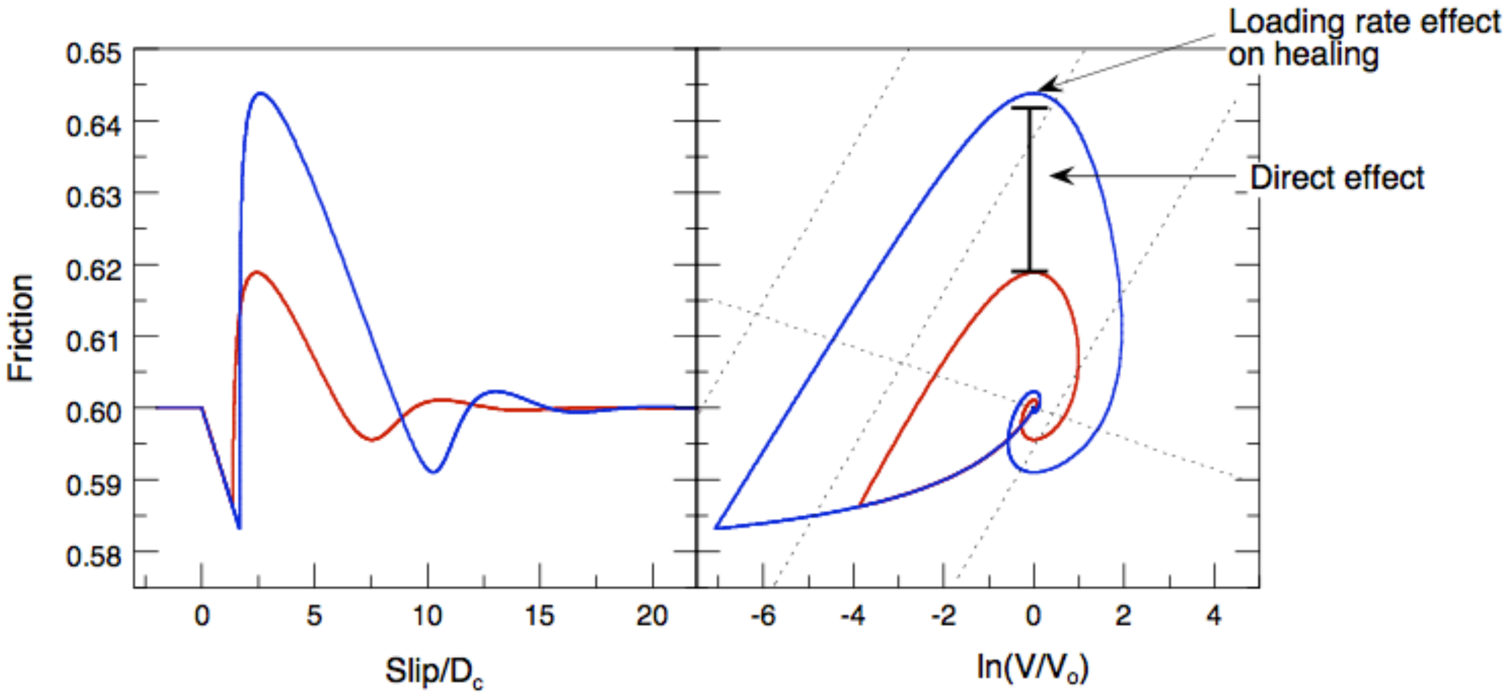


Derivation of the healing rate



But let's focus on just one velocity, so that we can see what the healing rate tells us.

The healing rate involves measurements of Δμ -look back at notes from Lec. 7

(1) $\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$

(2) $\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$

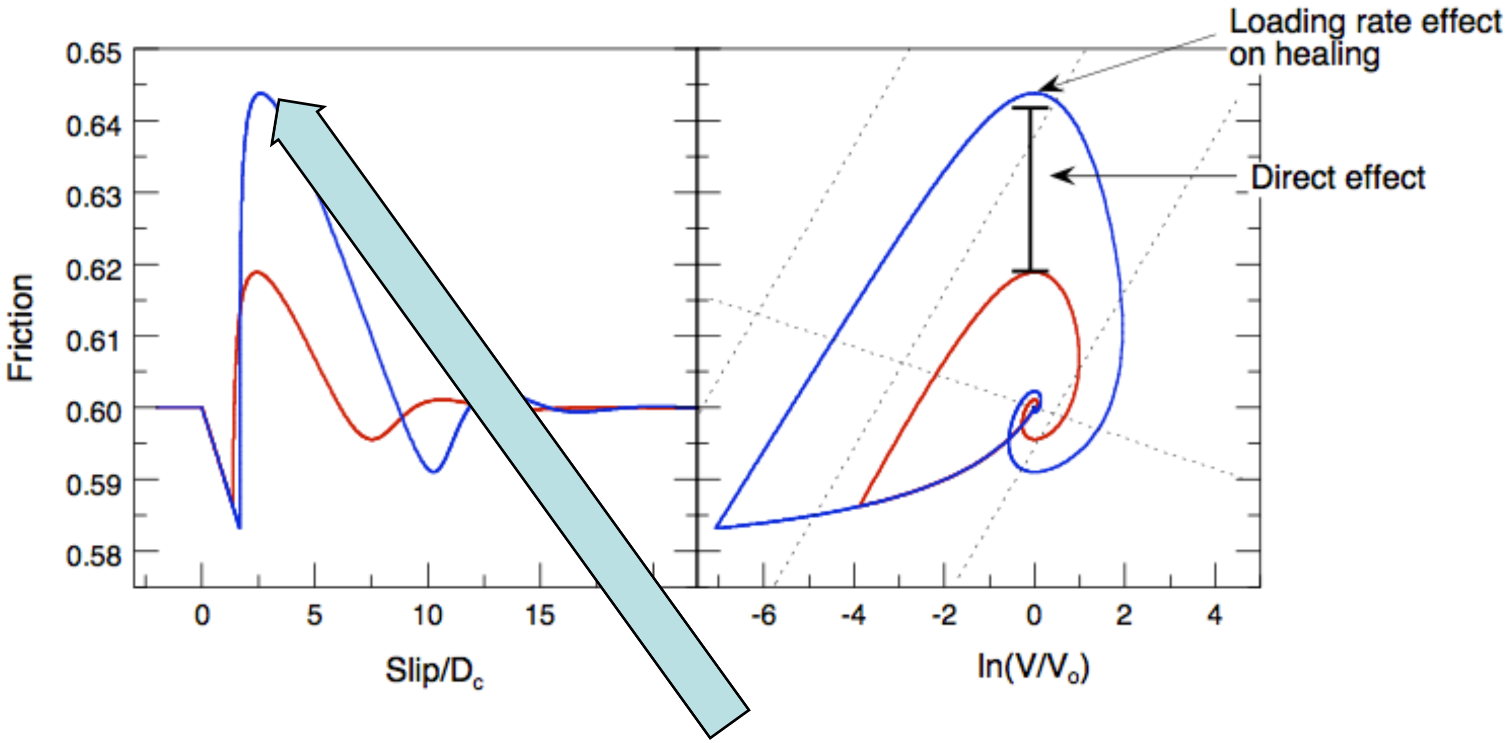
(3) $\frac{d\mu}{dt} = k(V_{lp} - V)$

At steady state, we have

$$\mu_1 - \mu_o = (a-b)ln(\frac{V_1}{V_o})$$

but for the peak ("static") friction, we need to think about both terms on the RSH of eq'n 1

Derivation of the healing rate



What do we know about V at the peak?

$V = V_o$ at the peak --right, that's what we see in the phase plane.

(1) $\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$

Hmmm, that means that the a $\ln(V/V_o)$ term is zero

(2) $\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$

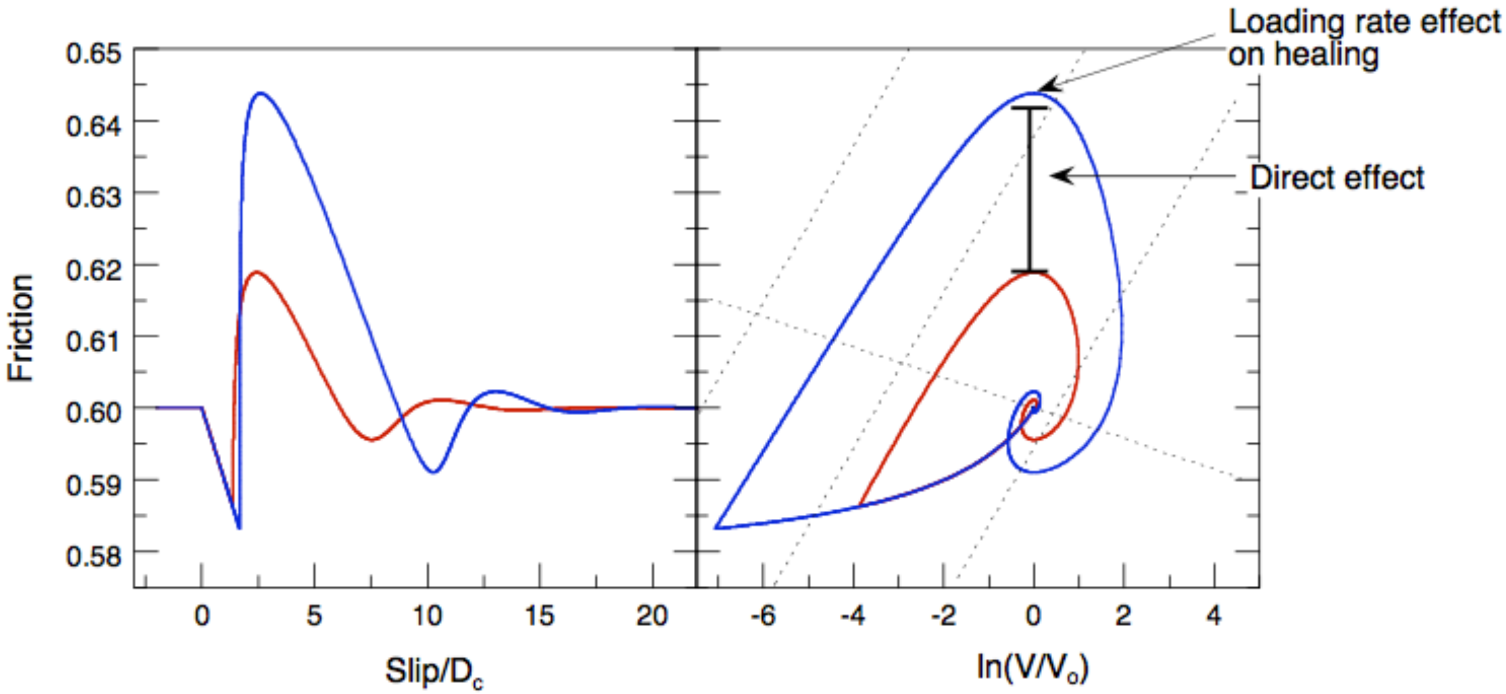
So $\Delta\mu = b\ ln\ (V_o\ \theta/D_c)$

and how does θ vary with SHS time?

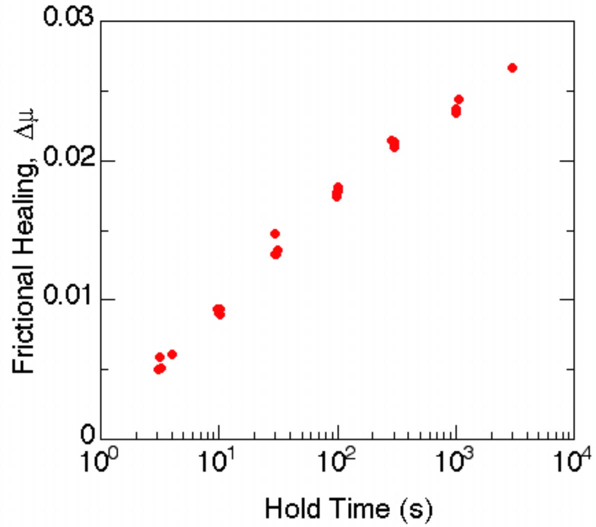
(3) $\frac{d\mu}{dt} = k(V_{lp} - V)$

according to (2), as V goes to 0, $d\theta = dt$

Derivation of the healing rate



so, if we plot Δμ vs ln(t) we should have the healing rate



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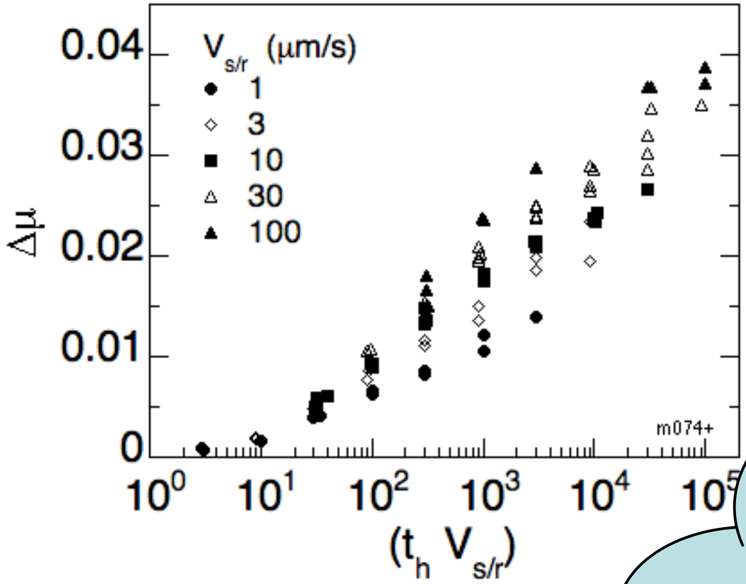
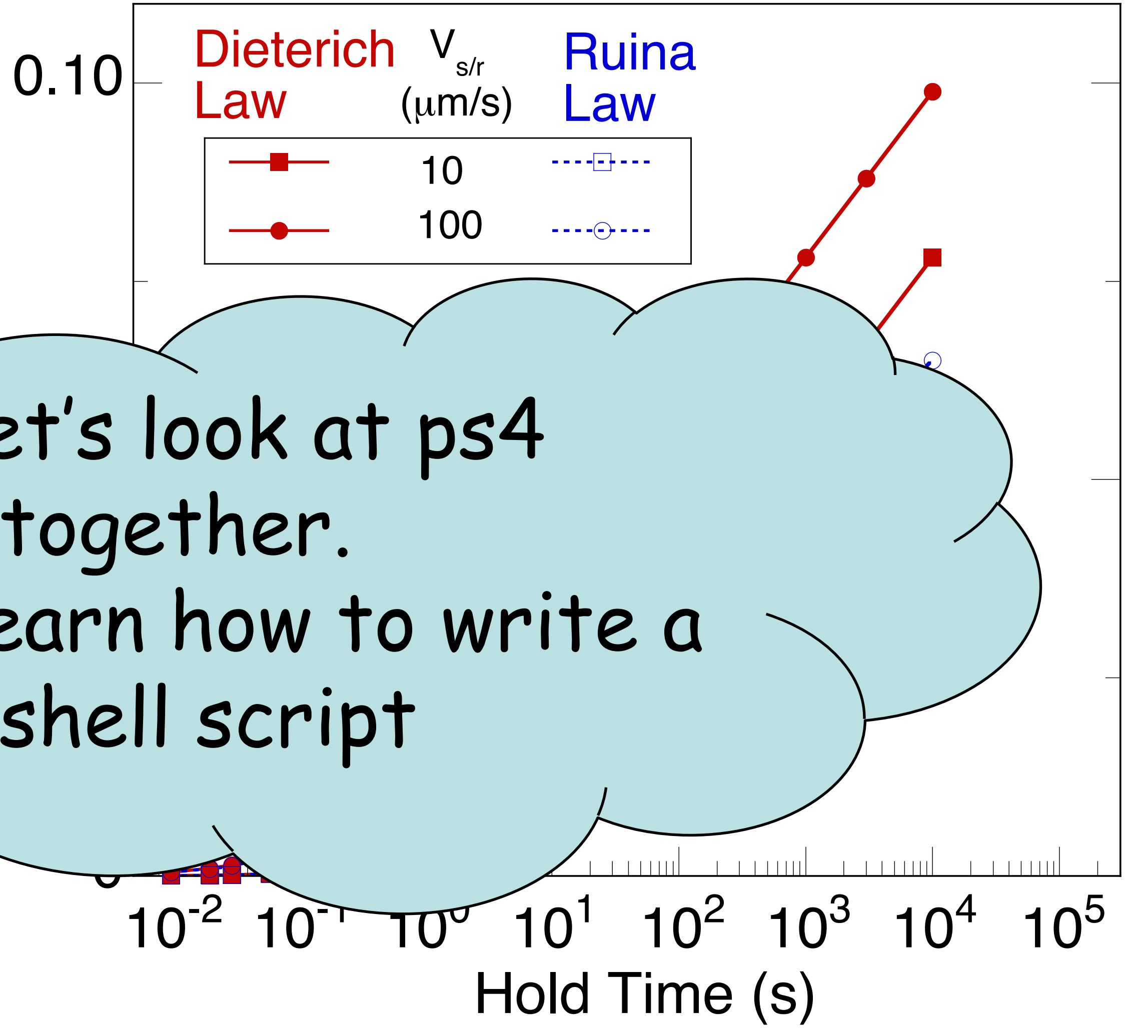




Let's look at ps4 together.
Learn how to write a shell script

Friction Law

$$\mu = \mu_o + a \ln(V/V_o) + b \ln(V_o \theta / D_c)$$

State Evolution

$$d\theta/dt = 1 - V\theta/D_c$$

$$d\theta/dt = - V\theta/D_c \ln(V\theta/D_c)$$

Elastic Coupling

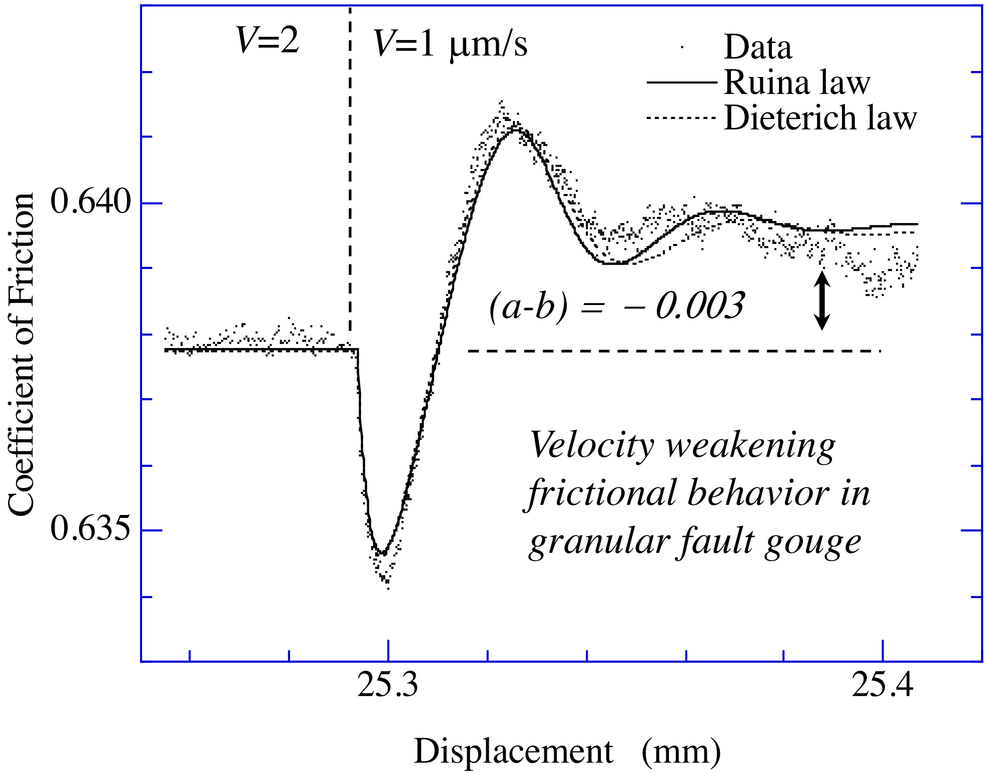
$$d\mu/dt = k(V_{lp} - V)$$

The rate of frictional healing depends on the rate of shearing (Marone, 1998, *Nature*)

Rate State Friction Laws predict this behavior

Rate/State Friction
Measuring the friction constitutive parameters

Empirical laws, based on laboratory friction data



Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

$$\mu(\theta, v) = \mu_0 + a \ln\left(\frac{v}{v_o}\right) + b \ln\left(\frac{v_o \theta}{D_c}\right)$$

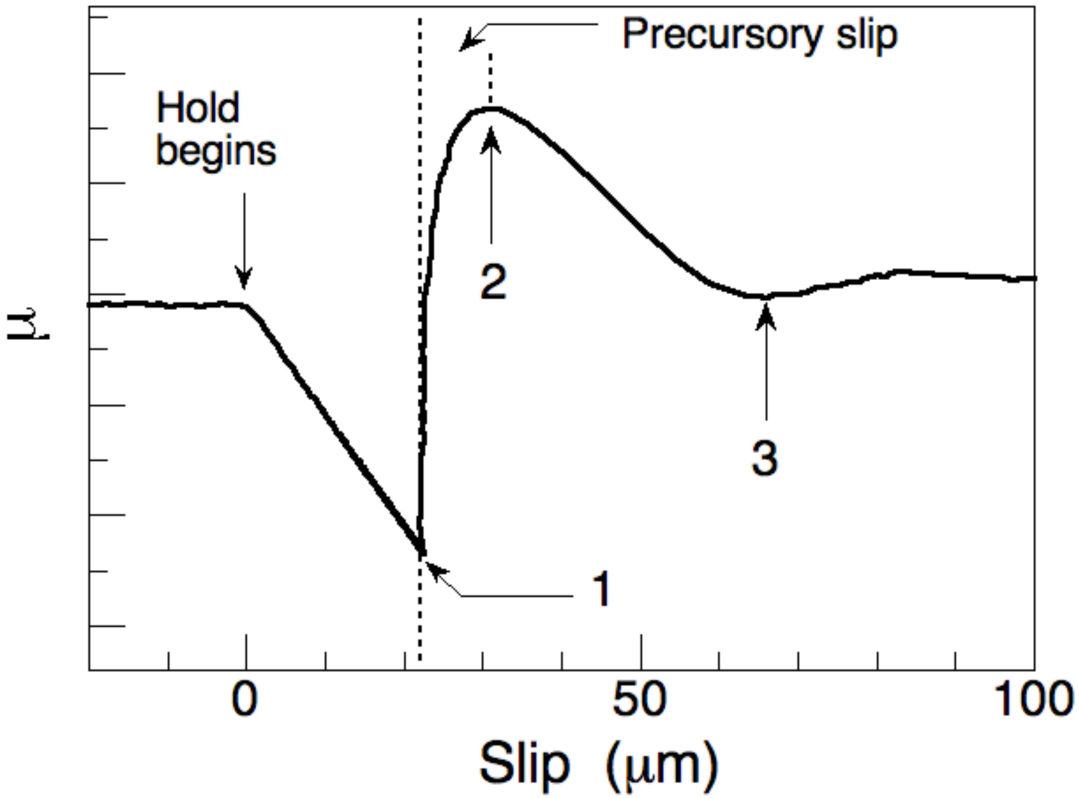
$$\frac{d\theta}{dt} = 1 - \frac{v\,\theta}{D_c}$$

$$\theta_{ss} = \frac{D_c}{v}$$

$$\Delta\mu_{ss} = (a-b)\ln\left(\frac{v}{v_o}\right)$$

$$\frac{d\mu}{dt} = k' \left(v_{lp} - v\right)$$

Rate/State Friction
Measuring the friction constitutive parameters



Constitutive Modelling

Rate and State Friction Law

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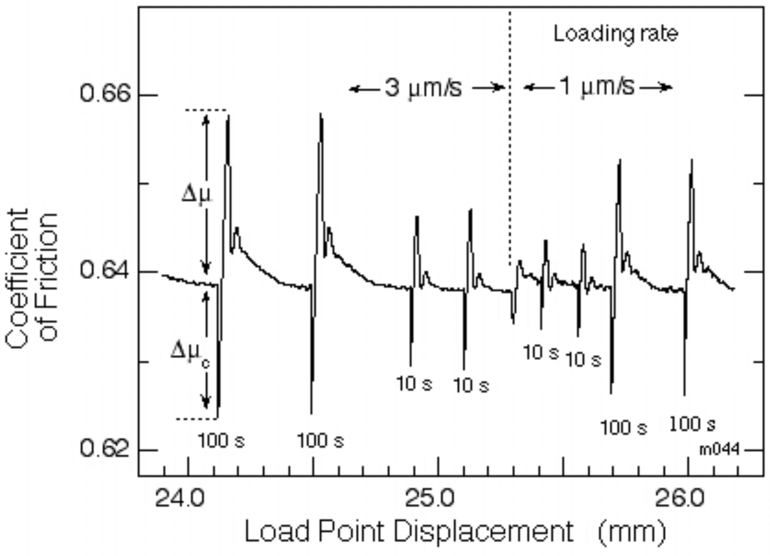
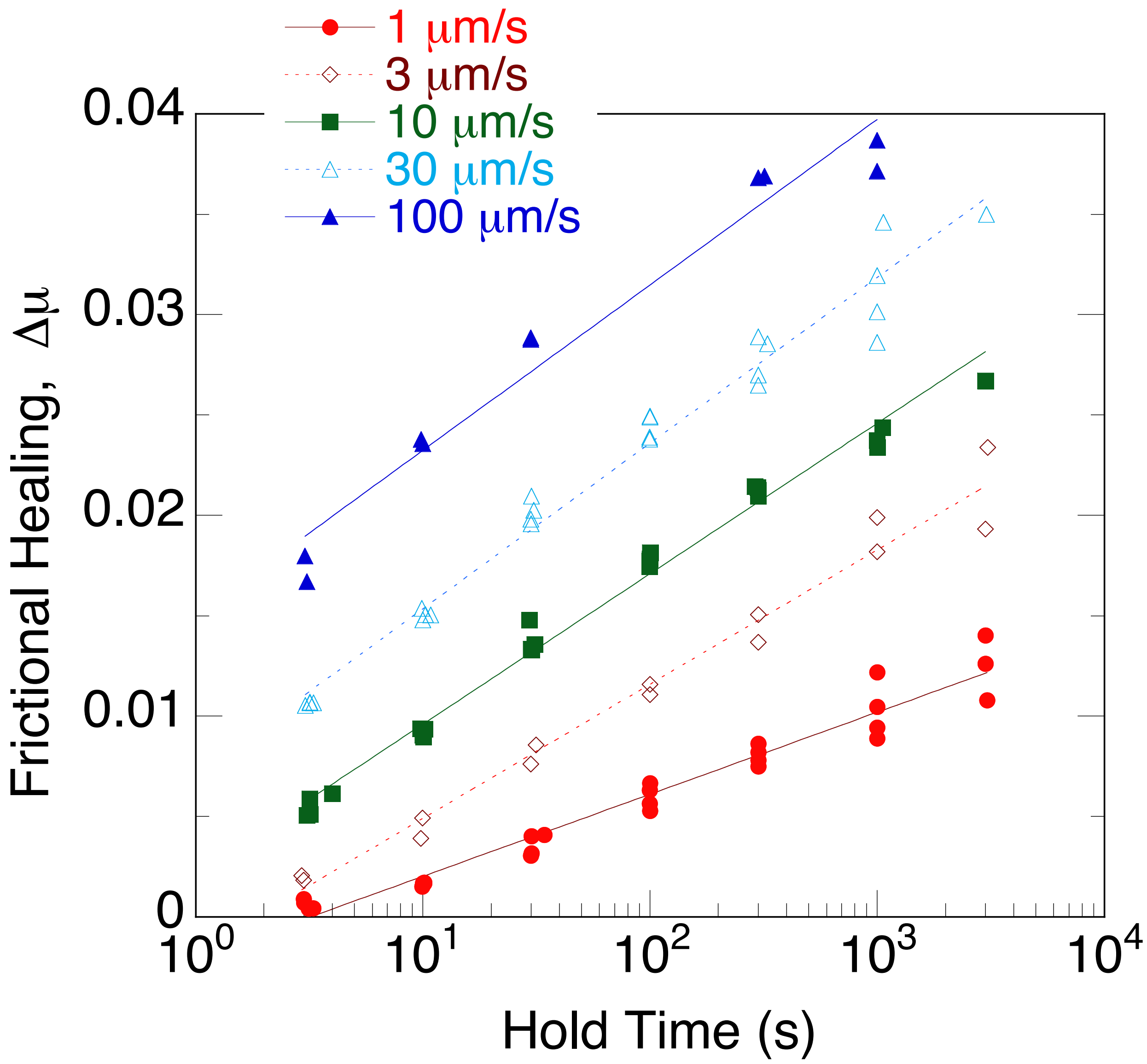


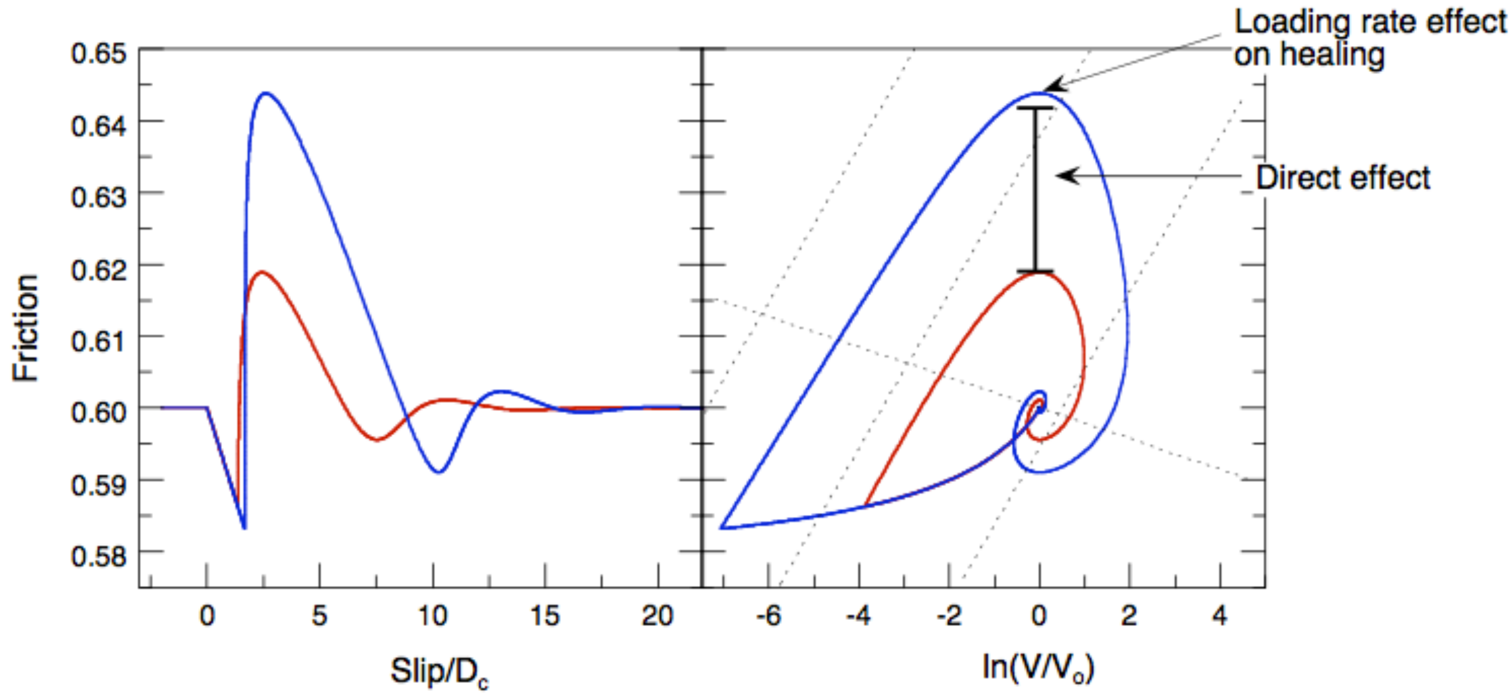
Stressed Aging

Aging rate depends on the rate of shearing



(Marone, 1998, *Nature*)

Phase Plane Plots



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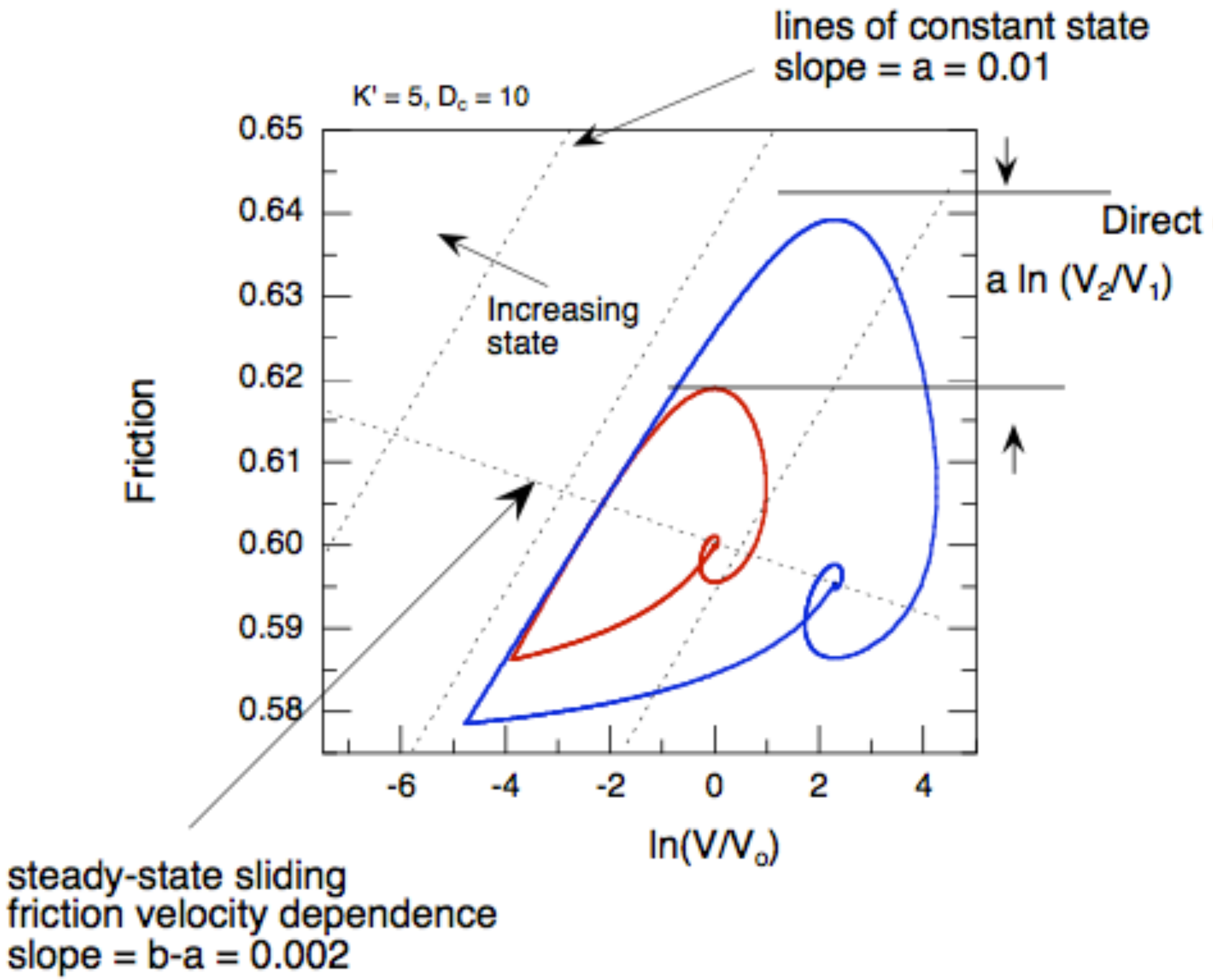
$$\frac{d\mu}{dt} = k(V_{lp} - V)$$

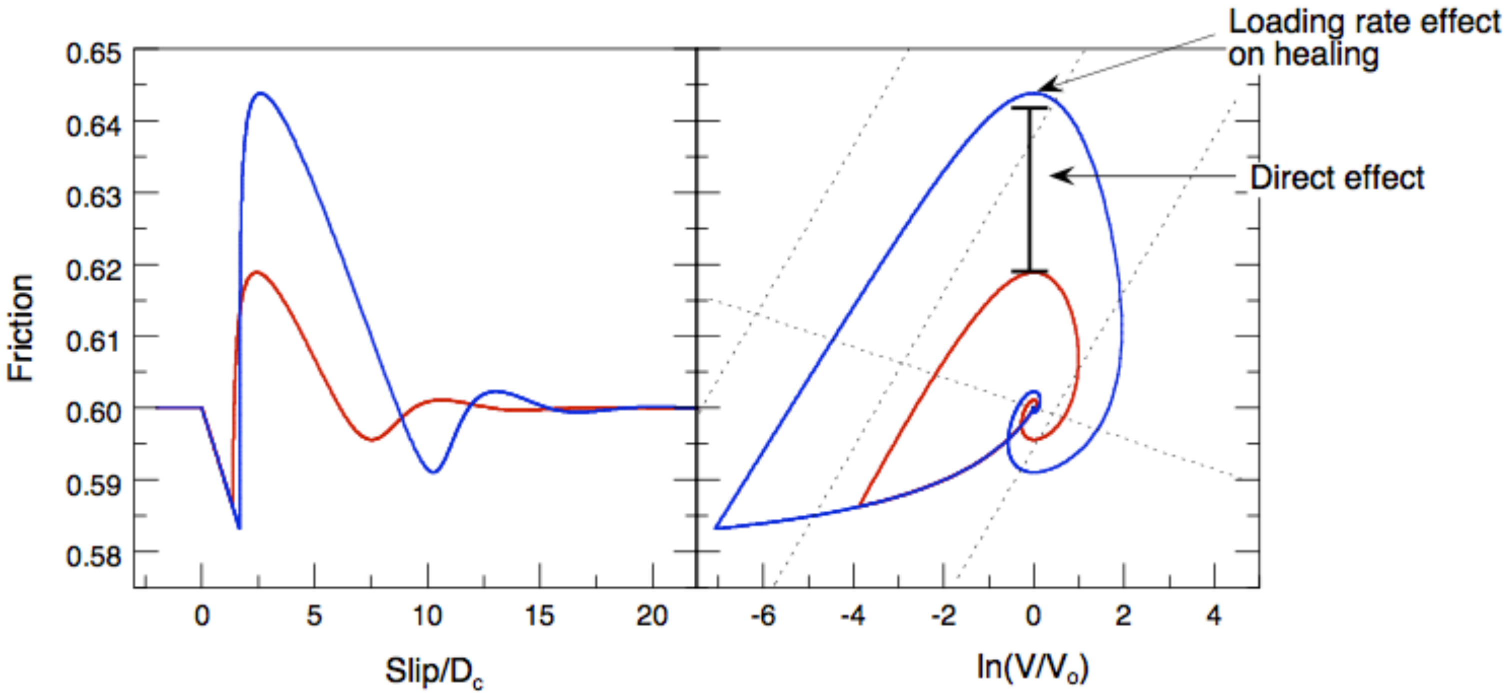
shs test:
1 μm/s
10 μm/s

Phase Plane Plots

shs test:
1 μm/s
10 μm/s



Derivation of the healing rate

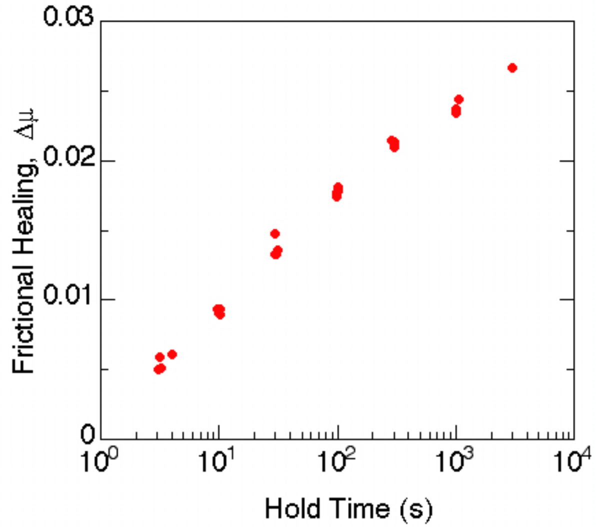


Loading rate effect on frictional healing is due to a combination of the friction direct effect and state evolution

$$\mu(\theta, V) = \mu_o + a\, ln\left(\frac{V}{V_o}\right) + b\, ln\left(\frac{V_o\theta}{D_c}\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

$$\frac{d\mu}{dt} = k(V_{lp} - V)$$

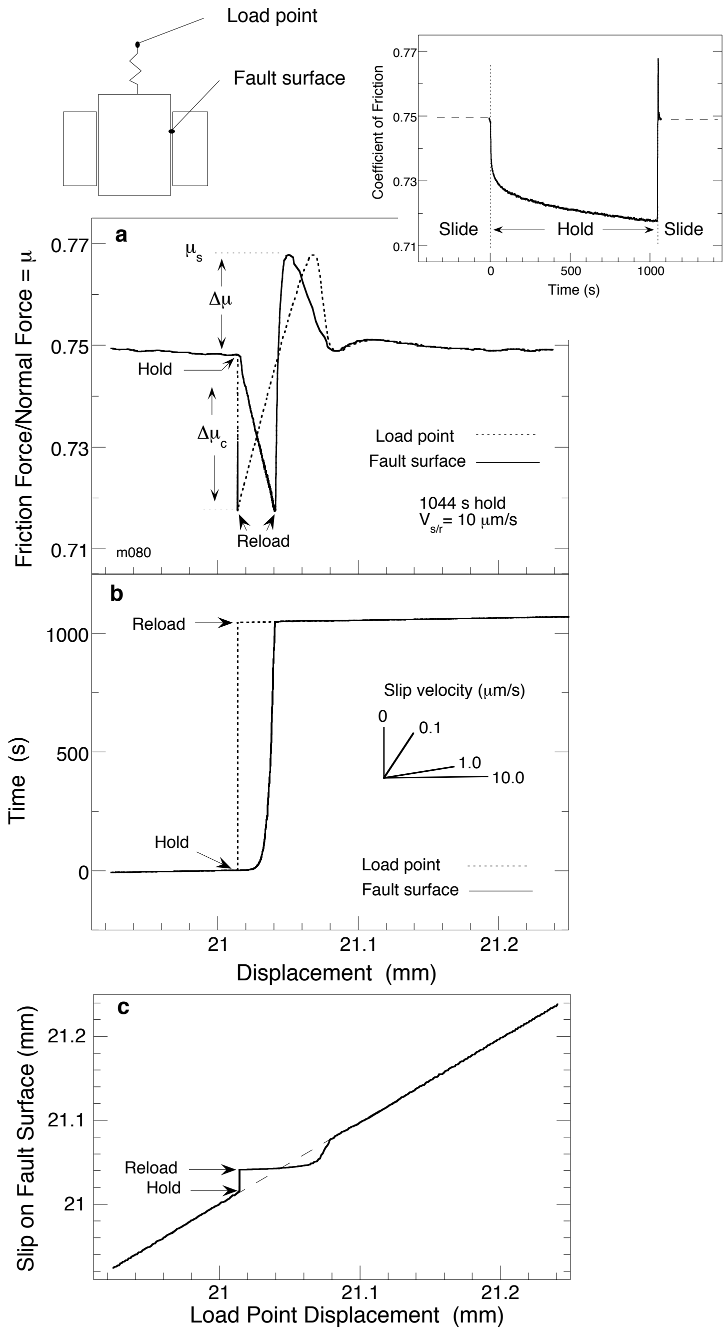


Slide-hold-slide

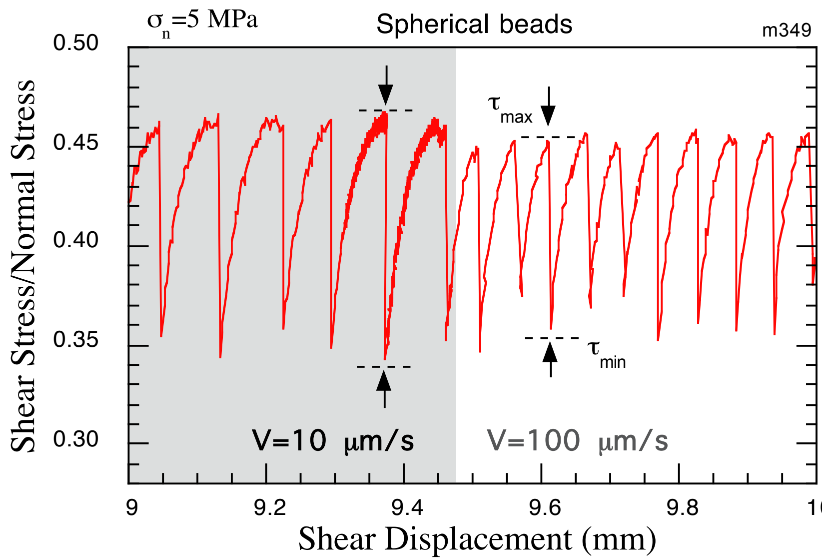
Slip-reload-slip

Earthquake-interseismic healing and reloading-earthquake

The full seismic cycle of stick-slip, frictional restrengthening, and interseismic reloading



Marone and Saffer, 2013 Figure 4

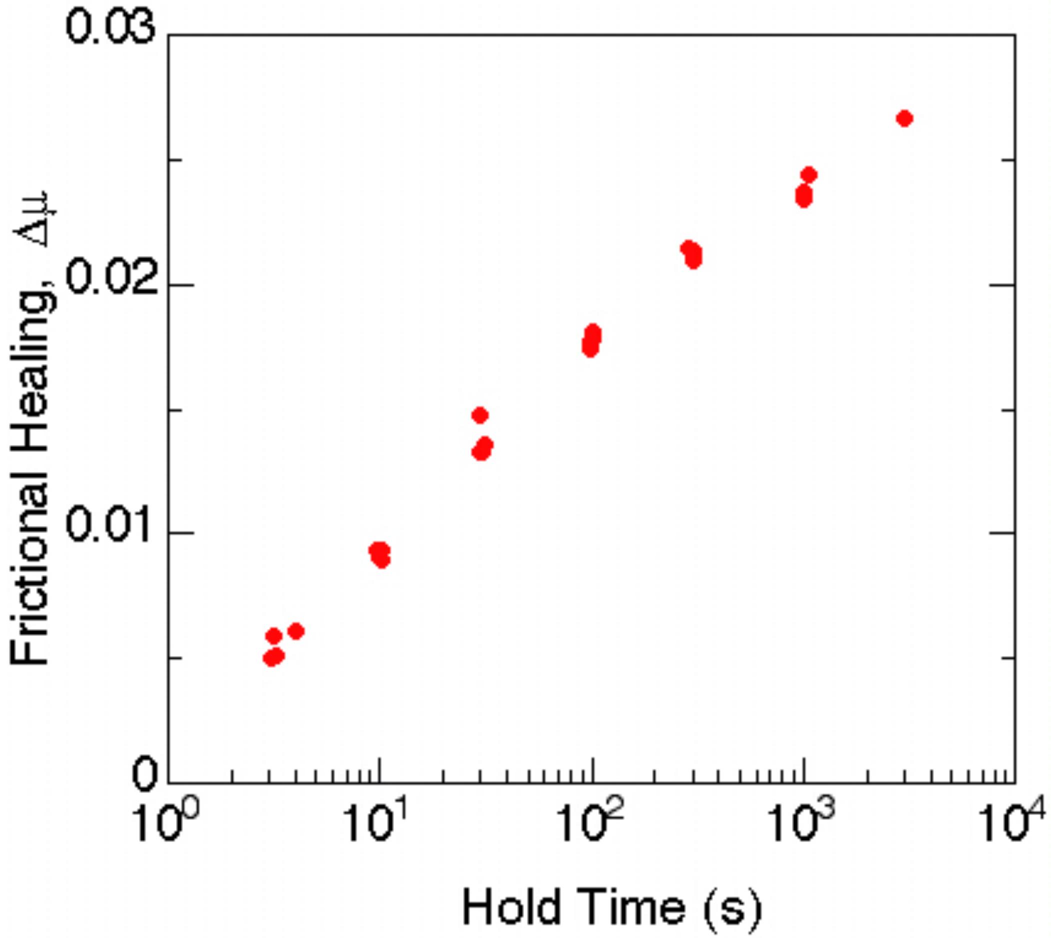


Rate (v) and State (θ) Friction Constitutive Laws

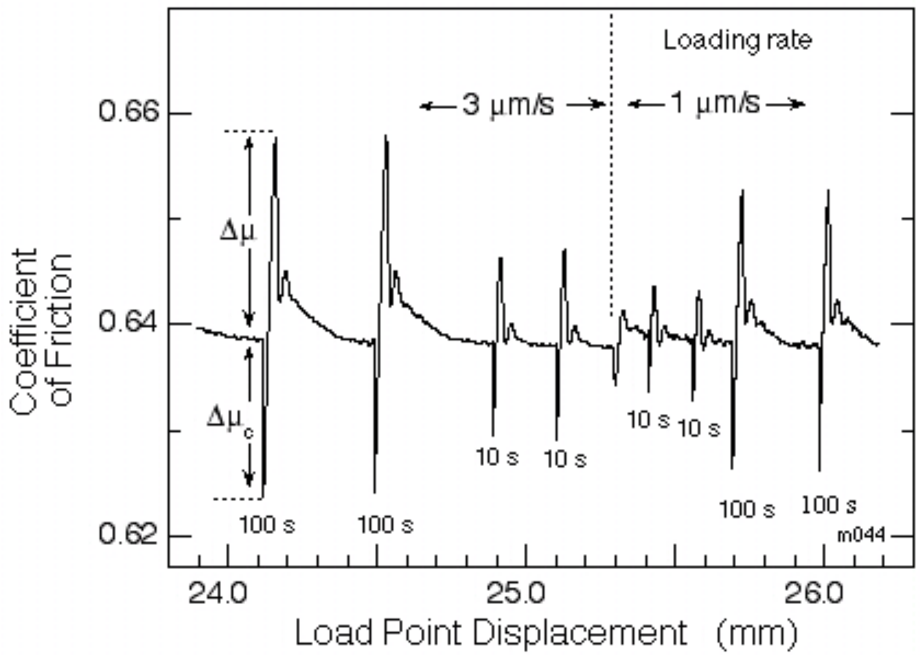
1) $$\mu(\theta, V) = \mu_o + a\ ln\left(\frac{V}{V_o}\right) + b\ ln\left(\frac{V_o\theta}{D_c}\right)$$

2) $$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

3) $$\frac{d\mu}{dt} = k(V_{lp} - V)$$ Elastic Coupling



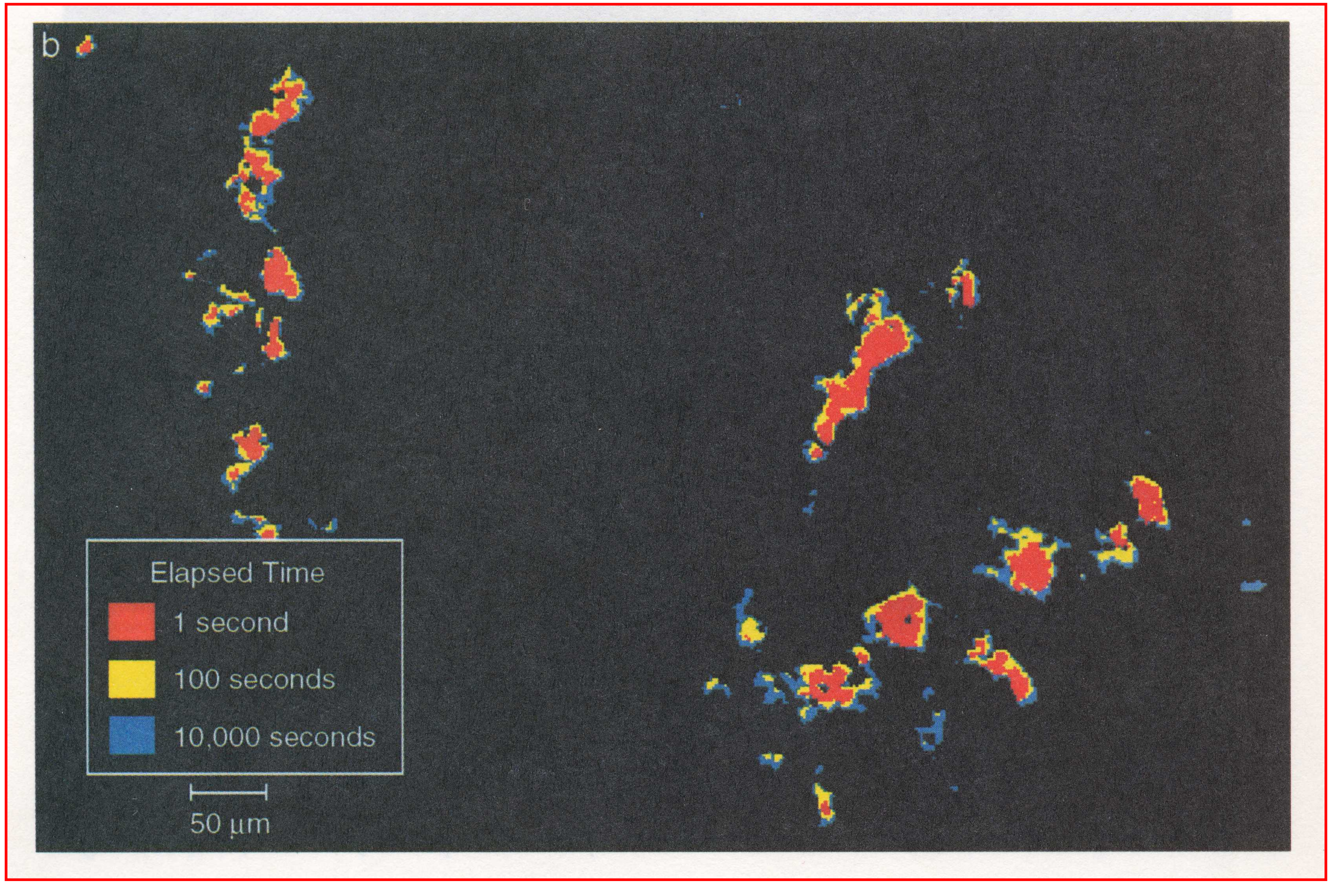
Modeling experimental data



$$\frac{d\mu}{dt} = k(V_{lp} - V)$$

Time dependent yield strength:

$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$



Dieterich and Kilgore [1994]

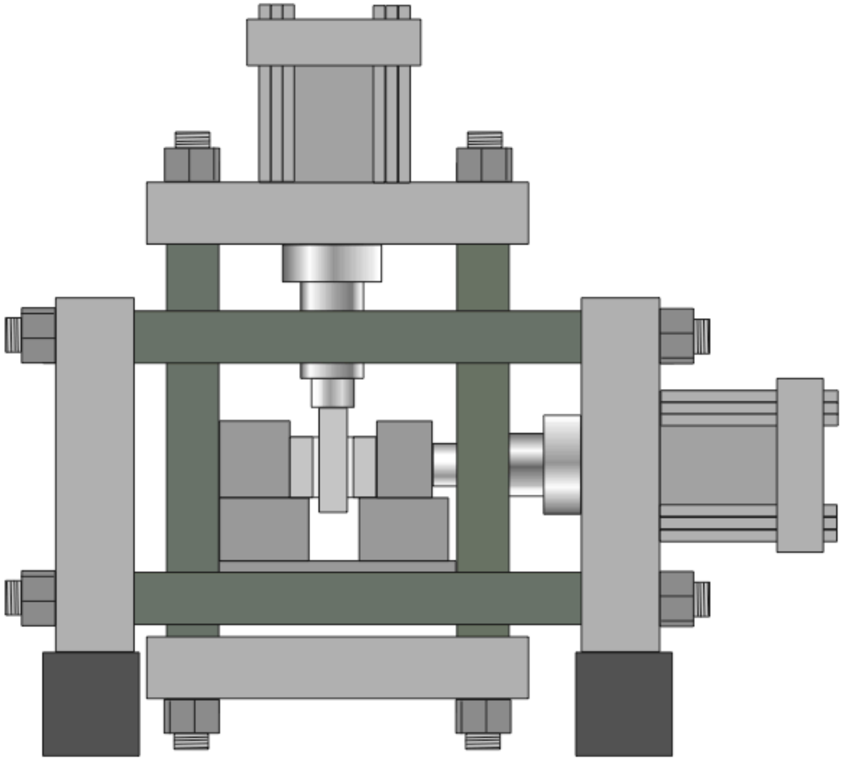
$$\mu = \frac{\tau}{\sigma_n} = \frac{S}{\sigma_y}$$

Time dependent growth of contact (acyrlic plastic)- true static contact

$$\sigma_y = \sigma_o + f(t)$$

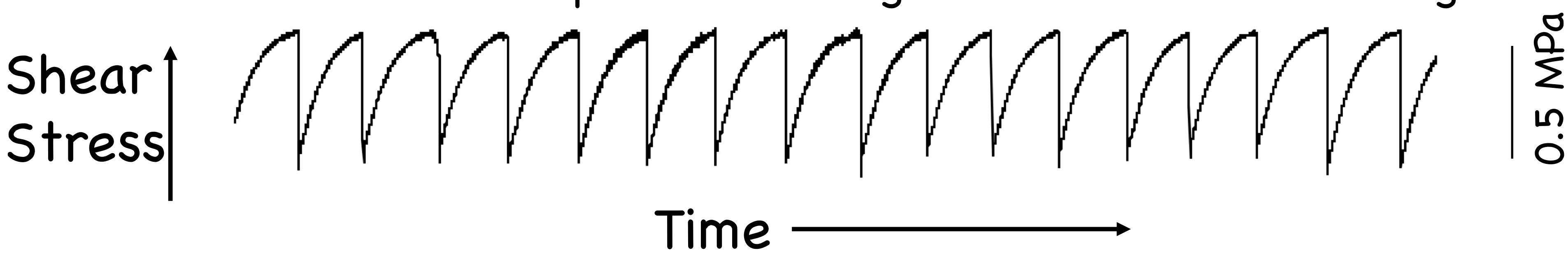
Modified from Beeler, 2003

How do fault/frictional surfaces heal (regain strength) after failure?



Earthquakes & Fault Mechanics: seismic cycle, fault reactivation. *(friction and stick slip: doors, windows, machines, ships in dry dock, dancers…)*

Stick-slip failure during shear at constant loading rate



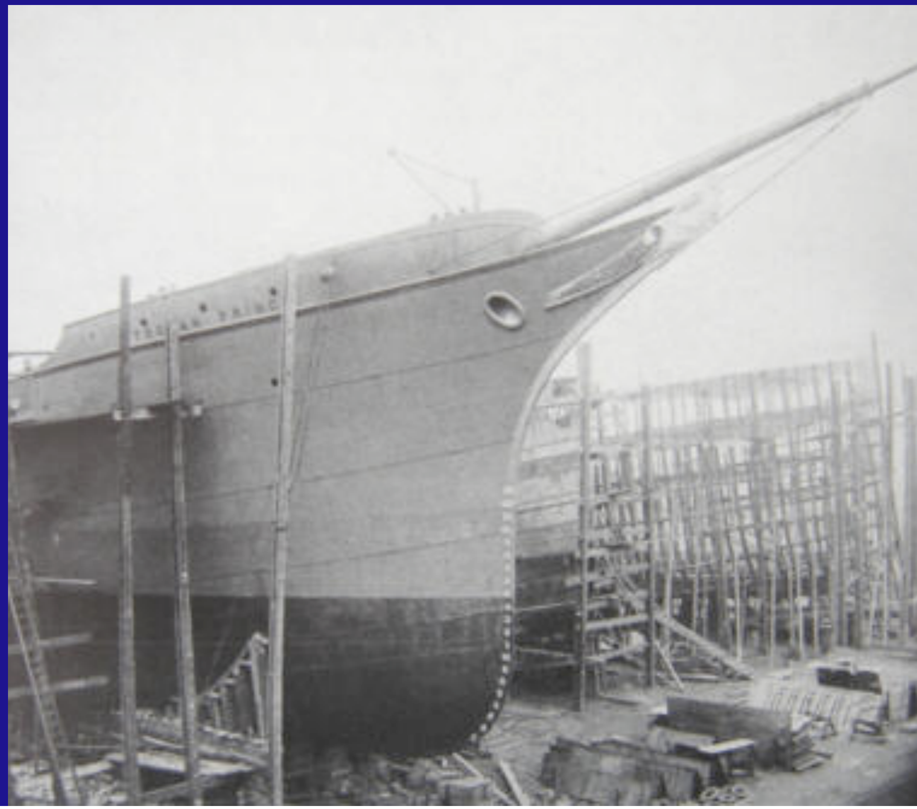
Time dependence of “static” friction
Aging of frictional contacts

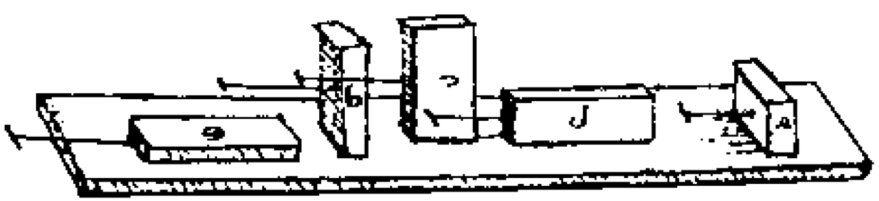
Table 9.1

	T (time of repose, min)	$A+mT^{\mu}$ (static friction force, lbf)
I^{re} observation	0	$A=502$
II^{e}	2	790
III^{e}	4	866
IV^{e}	9	925
V^{e}	26	1,036
VI^{e}	60	1,186
VII^{e}	960	1,535

static friction of two pieces of well-worn oak lubricated with tallow.

C. A. Coulomb (1736-1806)





Time dependence of “static” friction
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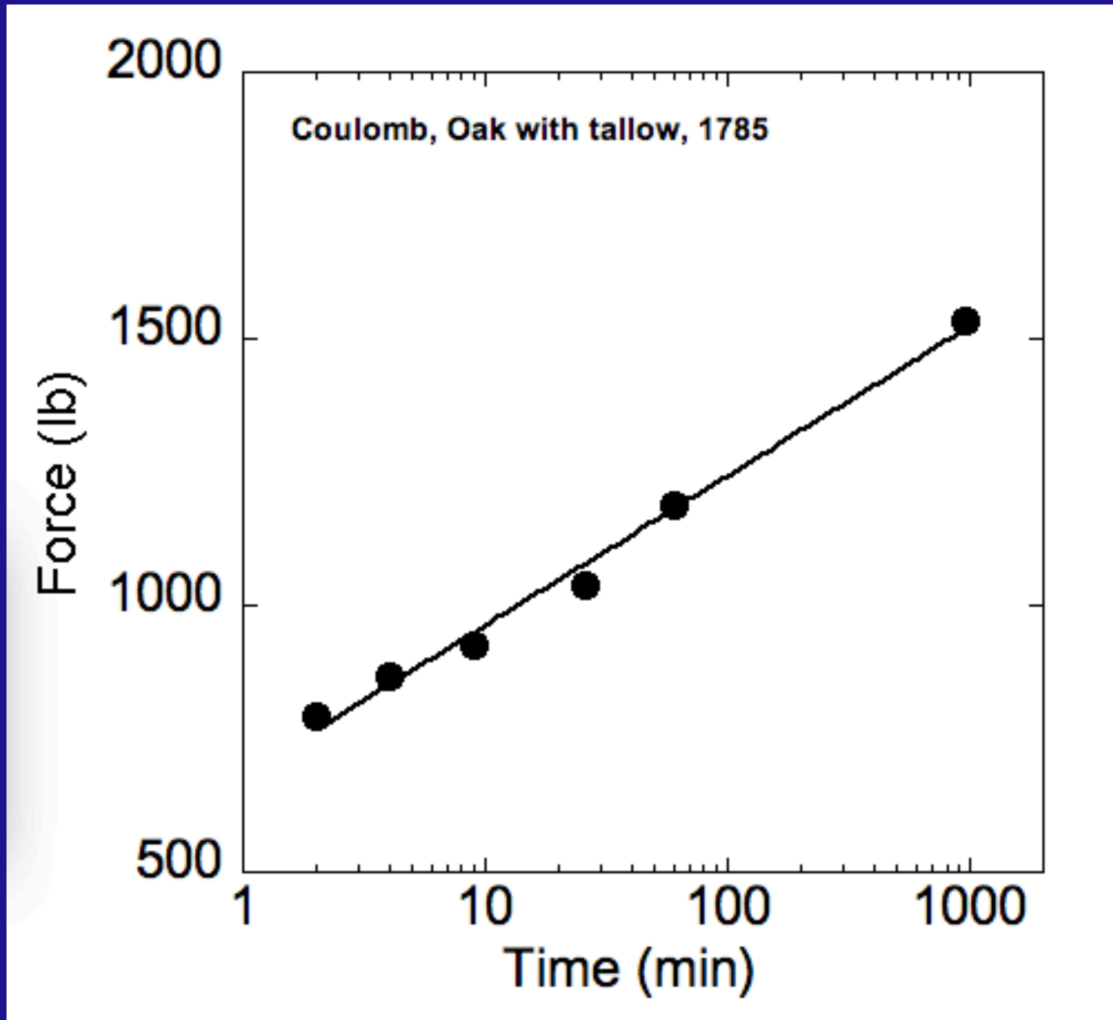
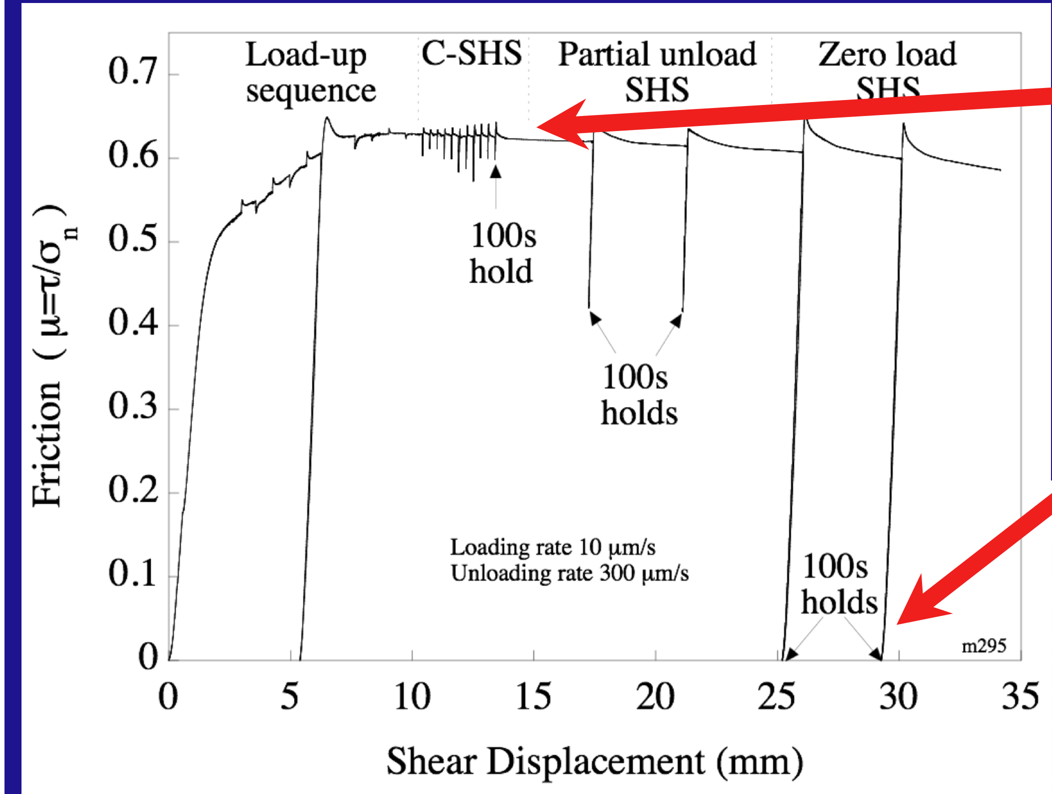
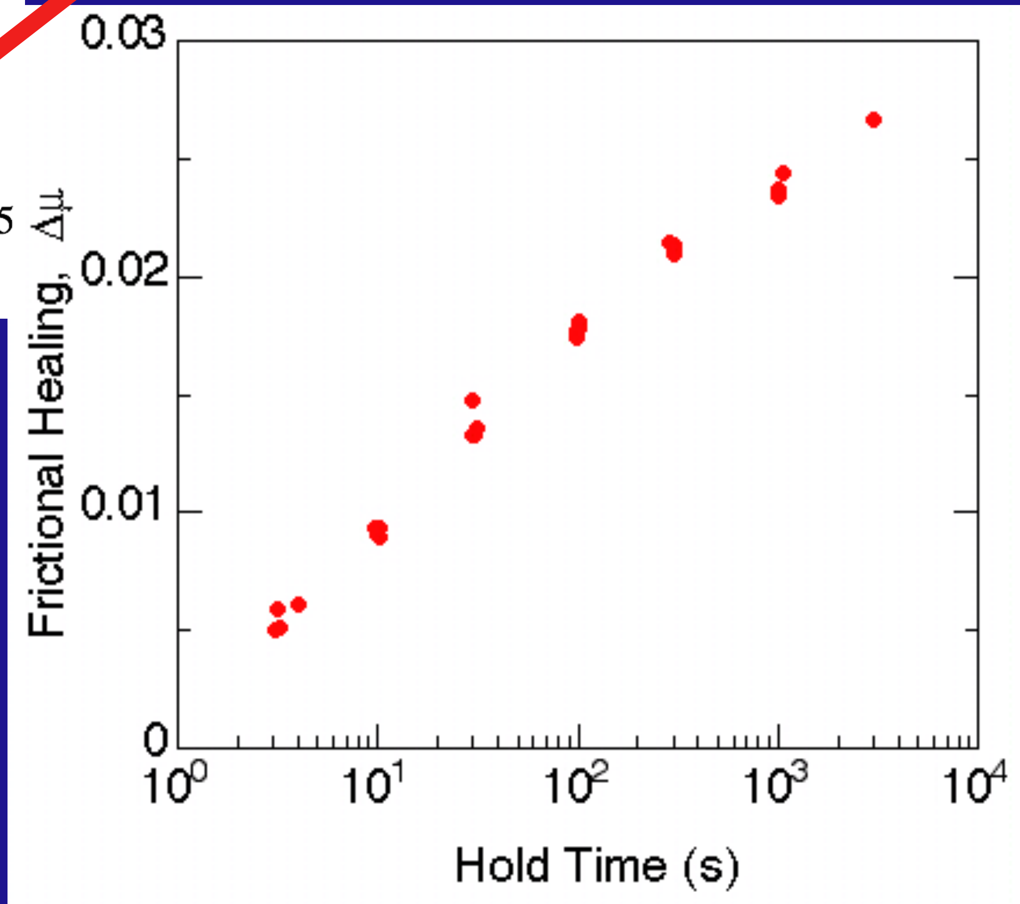


Time dependence of friction in rocks; Macroscopic frictional aging



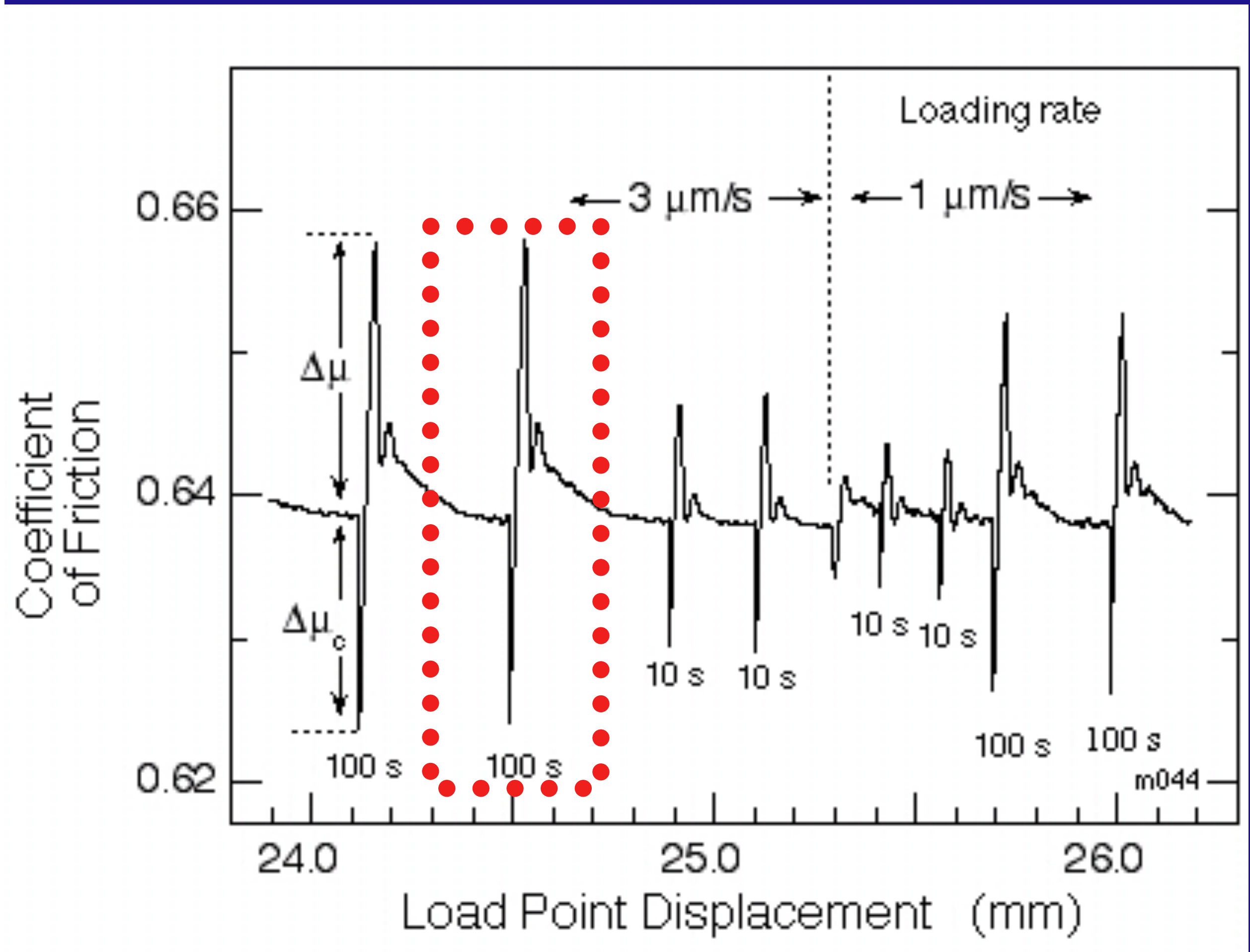
Stressed vs. unstressed aging

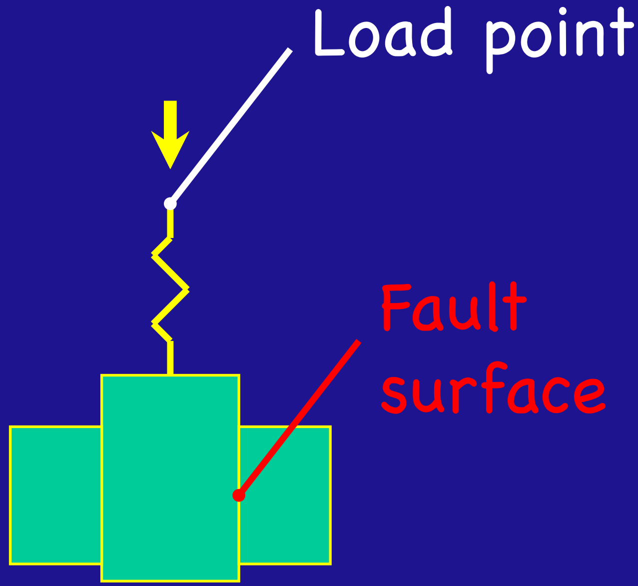
Karner and Marone, J.Geophys. Res, 2001.



Frictional Healing

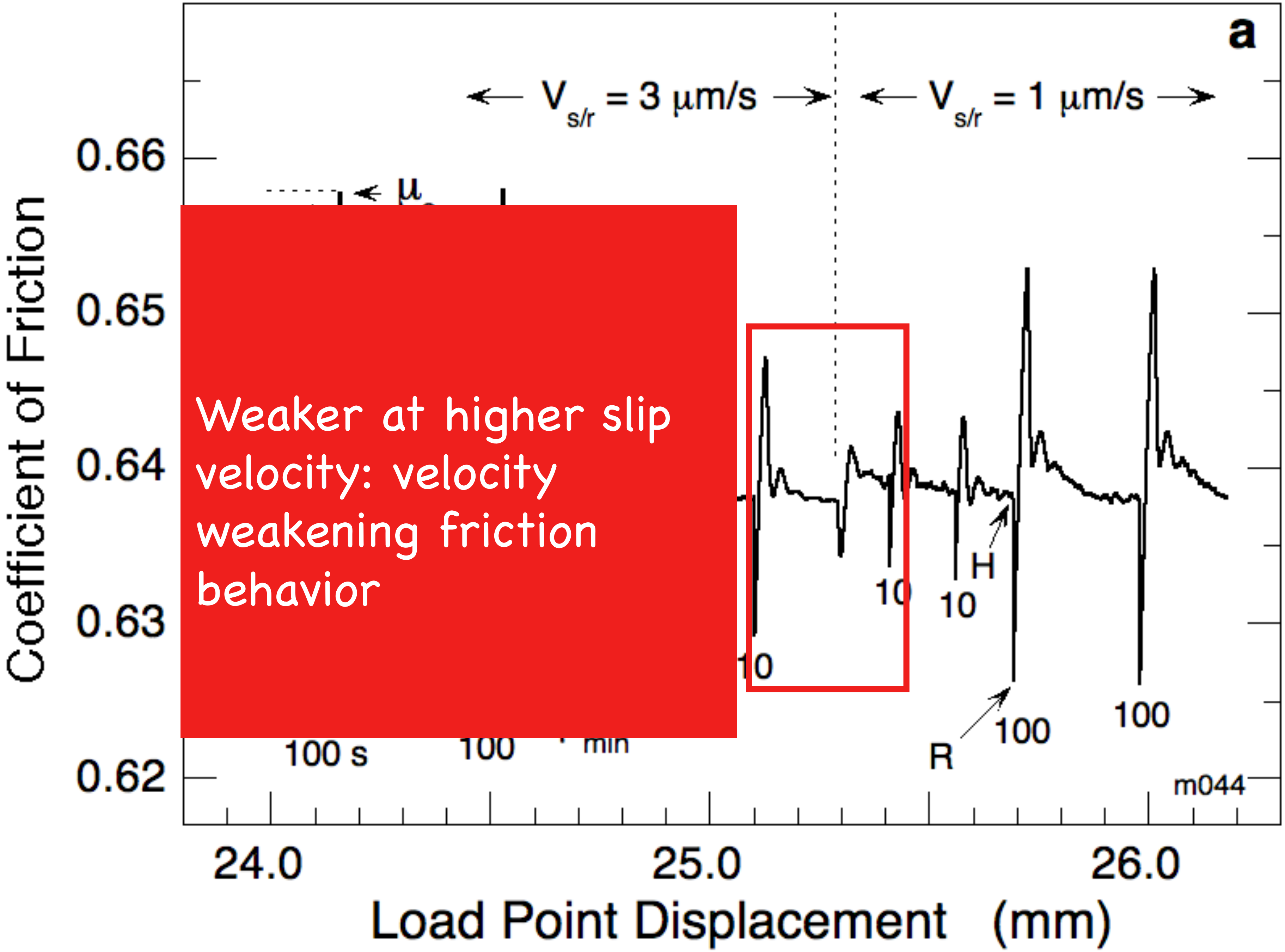
Stressed aging





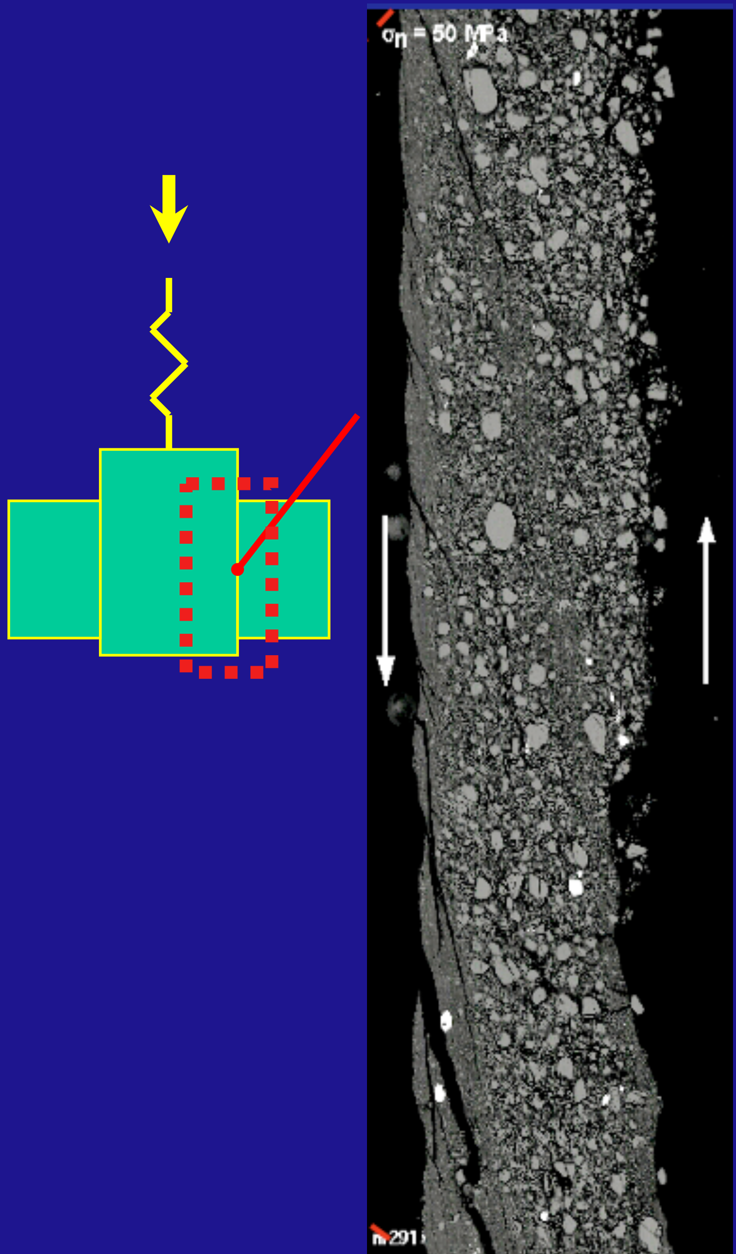
Steady state friction & the rate of healing vary with sliding velocity

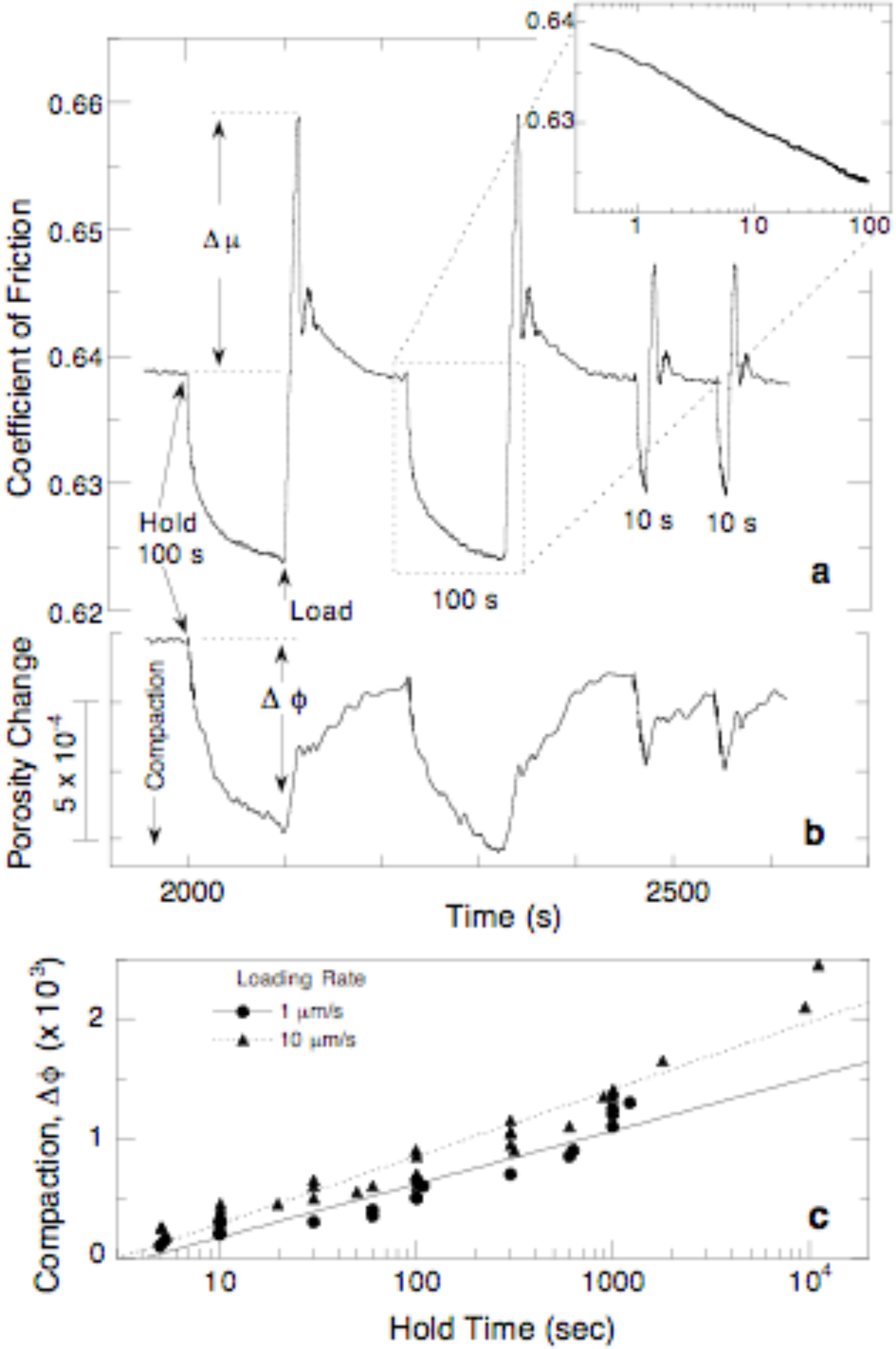
Angular quartz particles (100-150 μm), 3 mm thick, 25 MPa normal stress. Marone, Nature, 1998

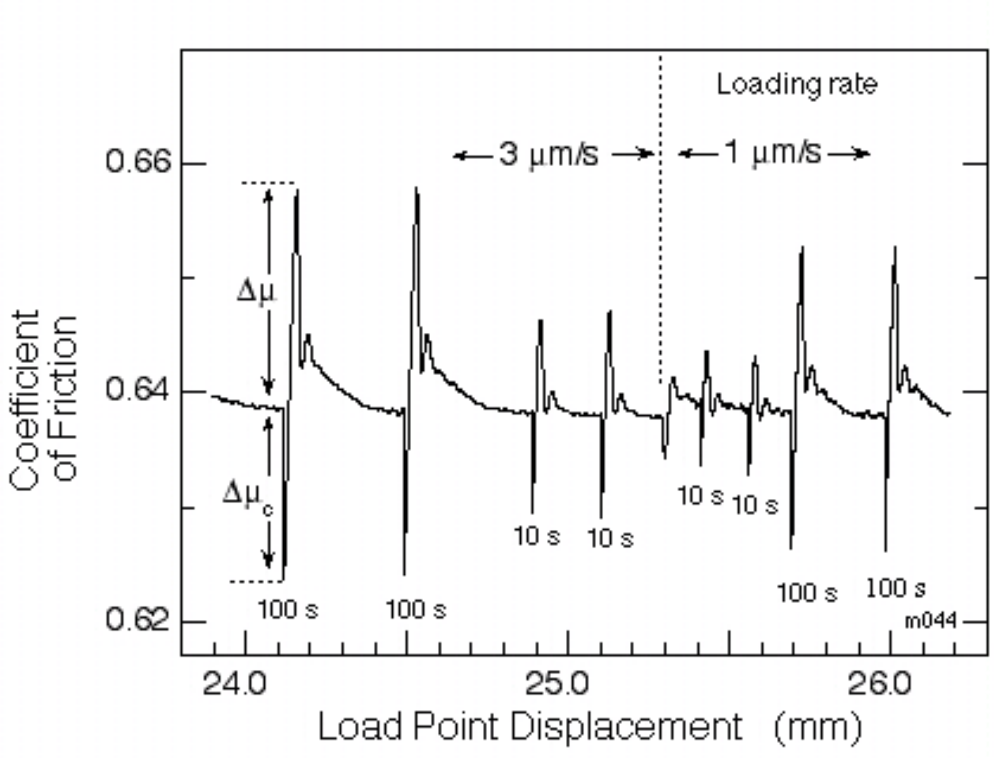


Marone, Nature, 1998

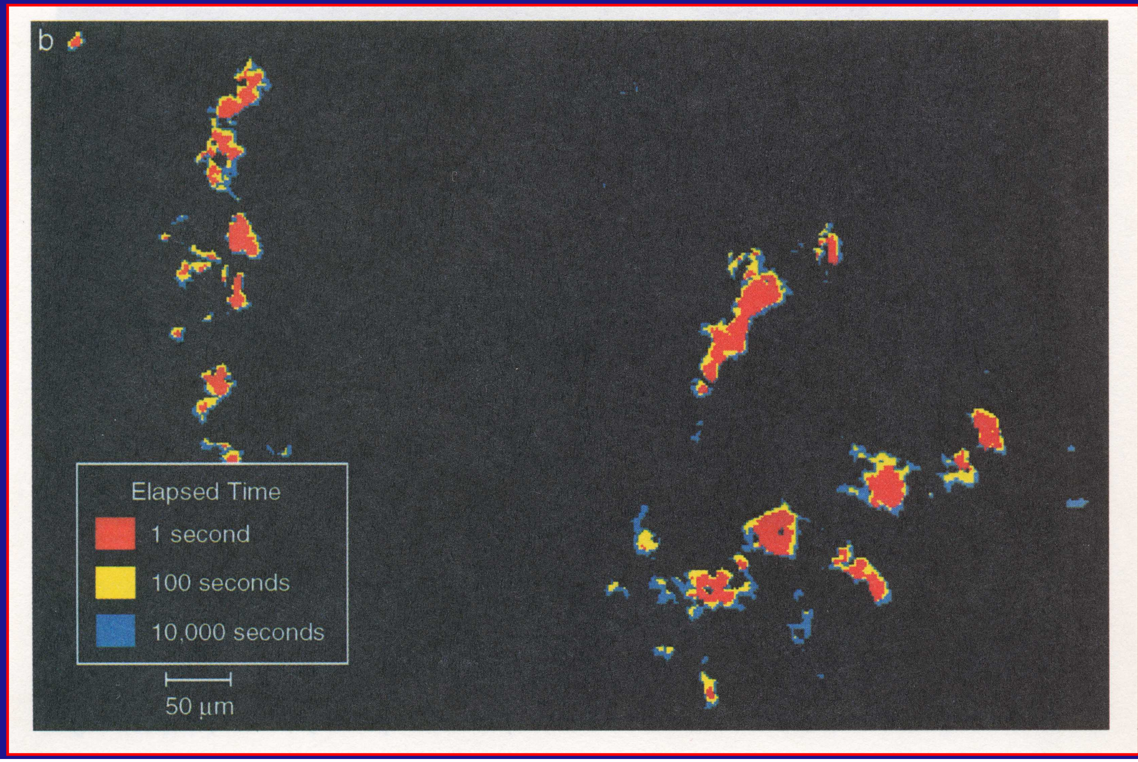
Consider the role of compaction and densification



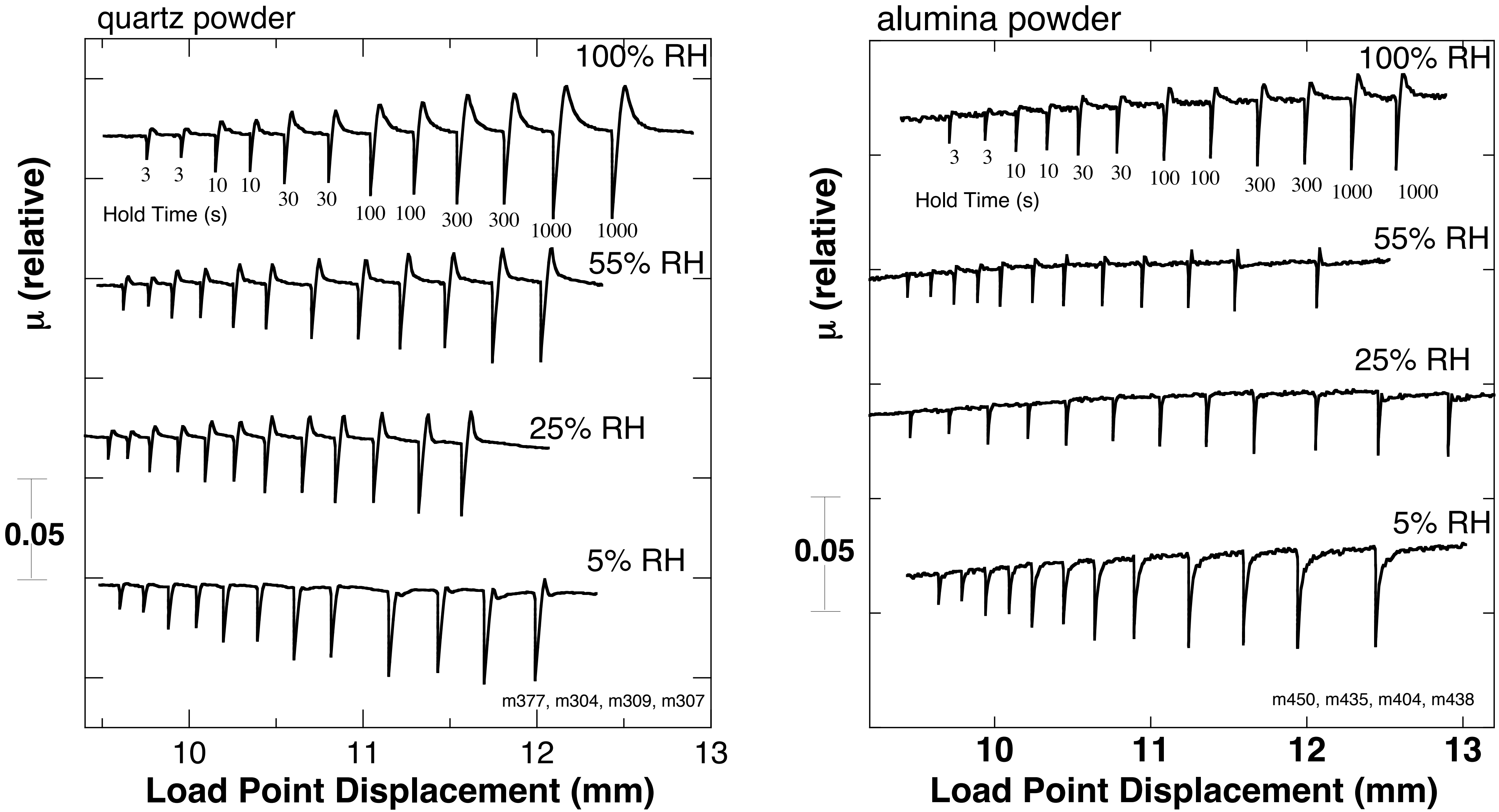




Frictional aging.
How does it work?



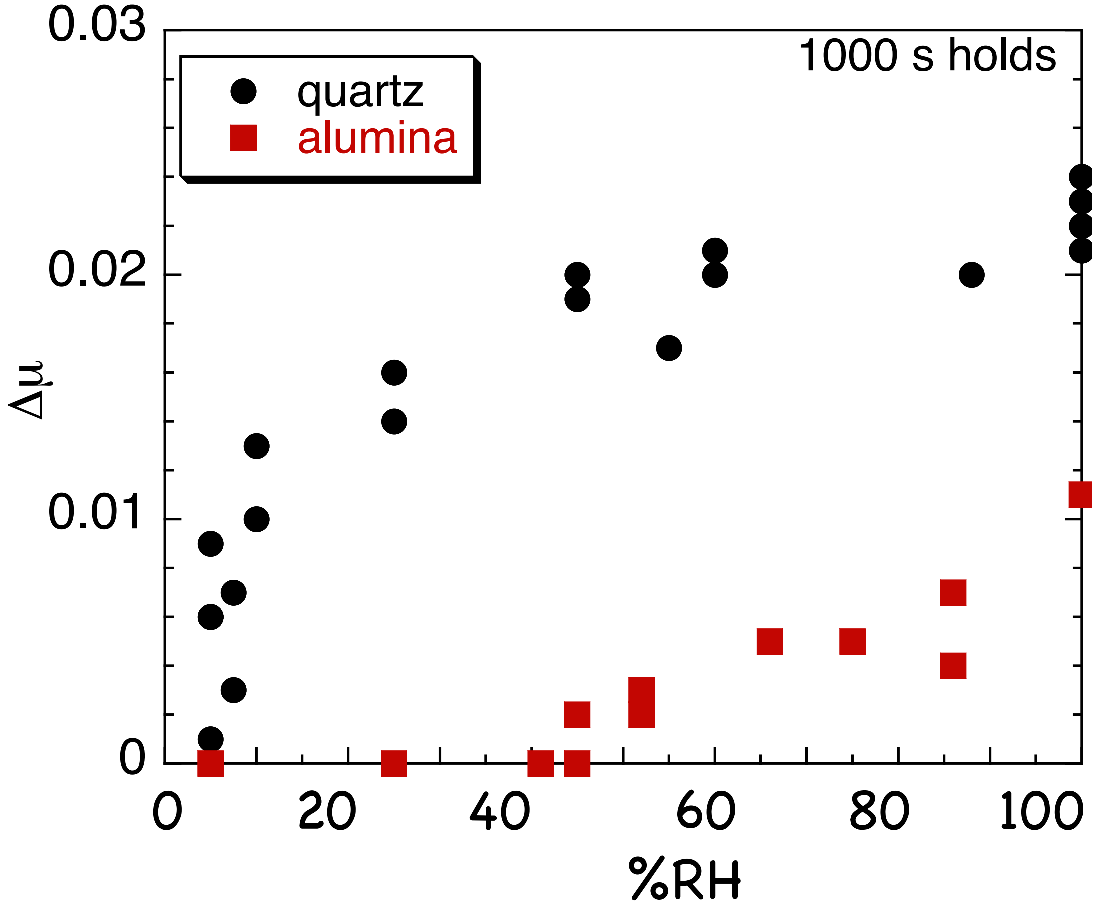
Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions



In-situ Particle Comminution; Production of Fresh Surface Area

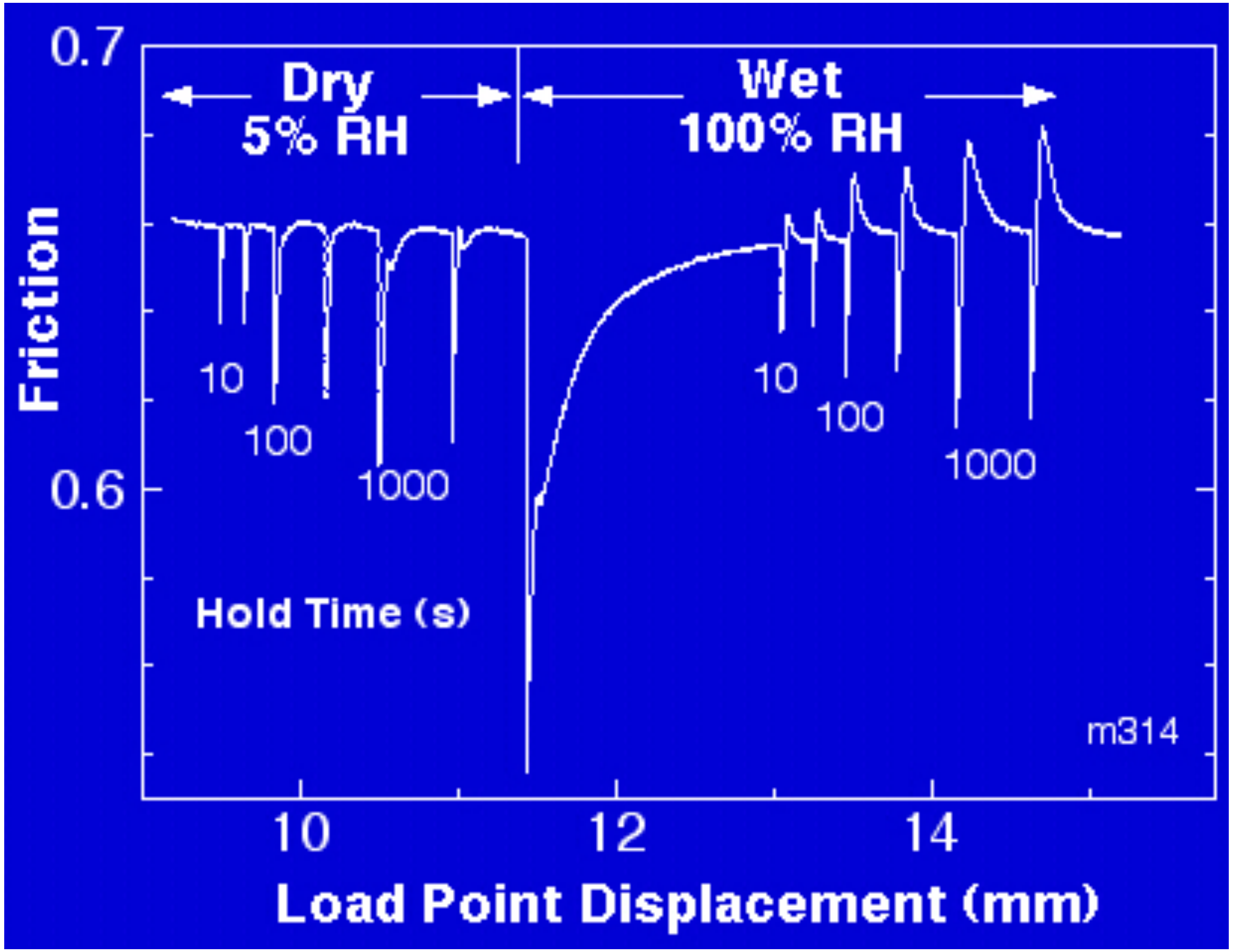
Frye and Marone, JGR 2002

Granular quartz



Hydrolytic Weakening causes enhanced rate of strengthening

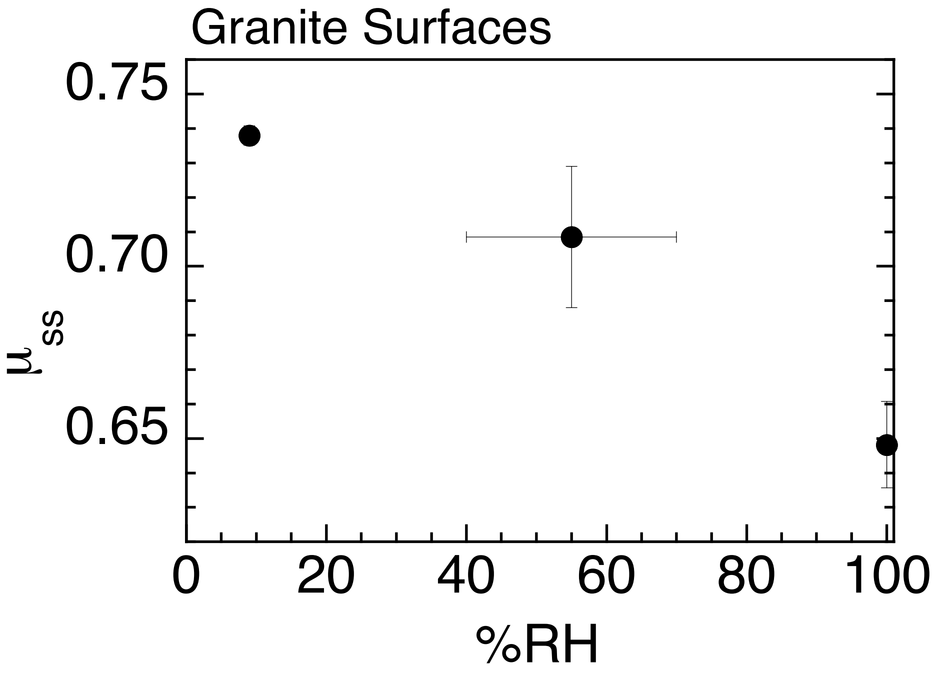
Chemically-Assisted Frictional Aging; Creep at Adhesive Contact Junctions



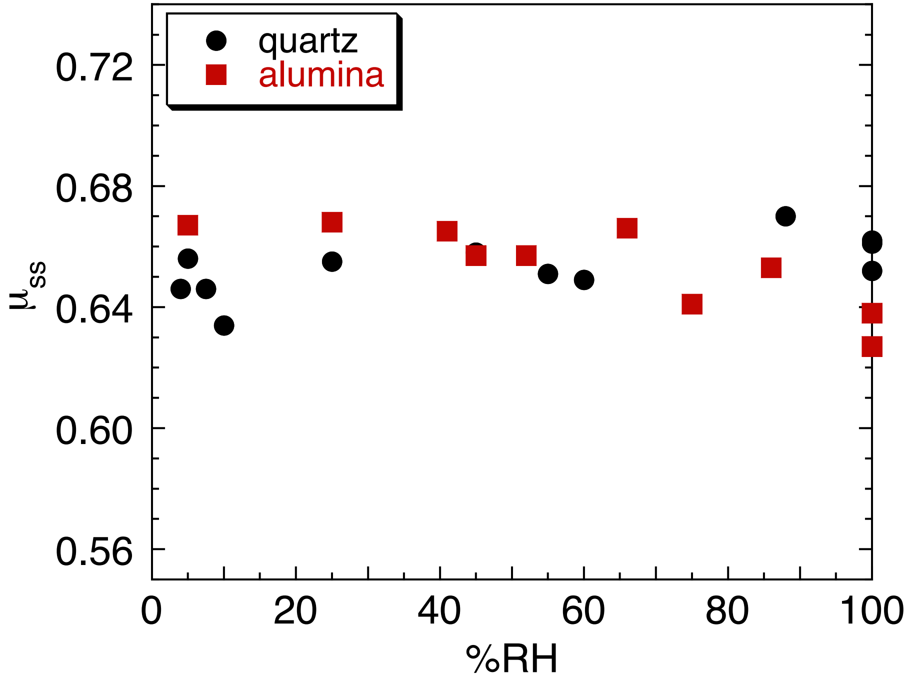
Frye and Marone, JGR 2002

Hydrolytic Weakening causes enhanced rate of strengthening, but base level frictional strength is unchanged

Granite Surfaces



Granular Materials



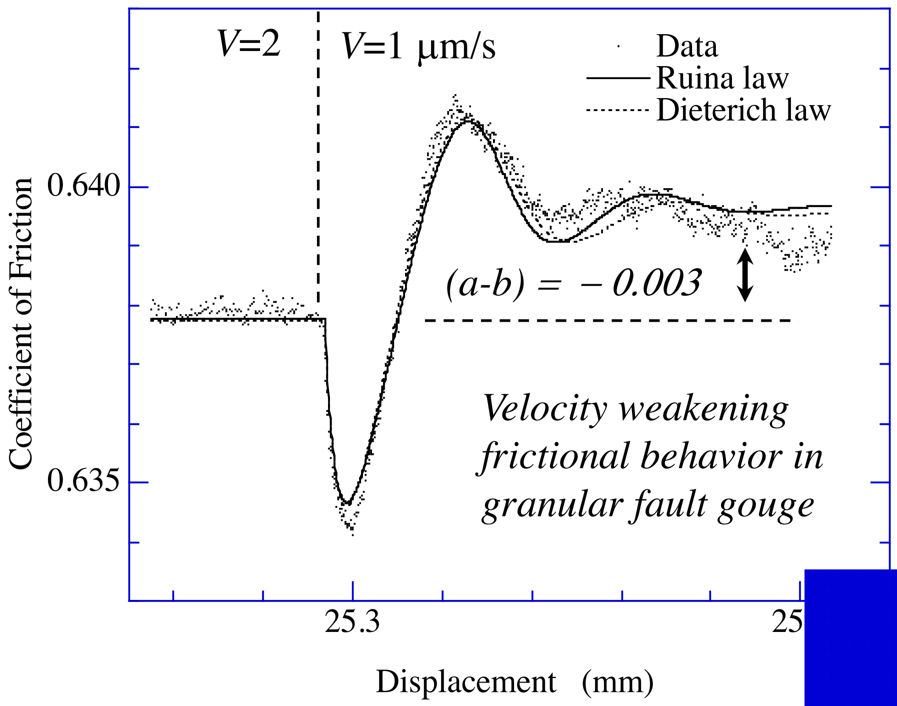
Solid Surfaces: Base level of frictional strength decreases with increasing water content (cf. Dieterich & Conrad, 1984)

Granular Materials: Frictional strength is independent of water content

Interpretation: Contact junctions subject to time dependent strengthening or growth, which inhibits sliding, but particle rolling is not affected by these factors.

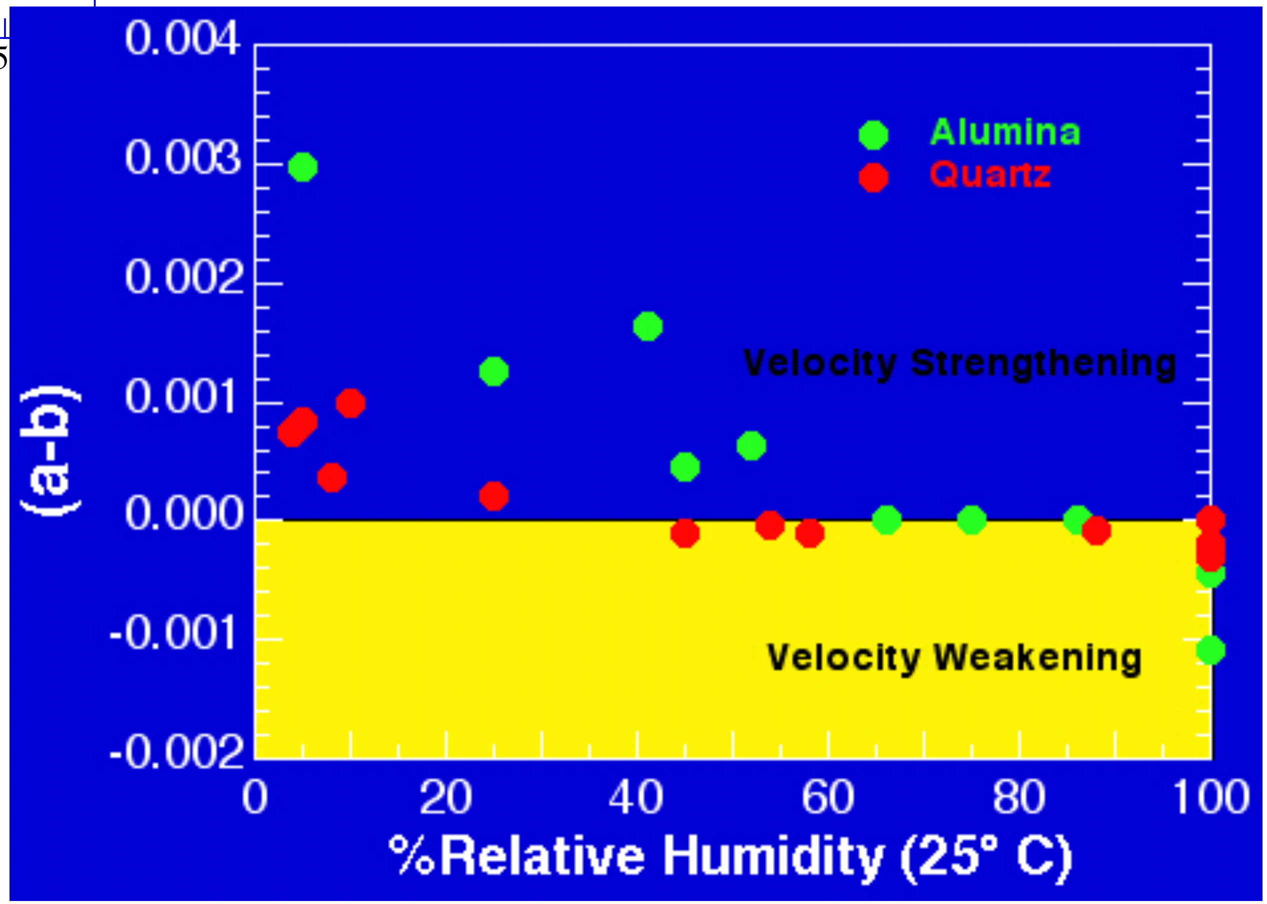
Frye and Marone, JGR 2002

Empirical laws, based on laboratory friction data



Velocity dependence of steady state friction varies changes from positive to negative. (cf. Tullis and co-workers)

Chemically-assisted creep at adhesive contact junctions

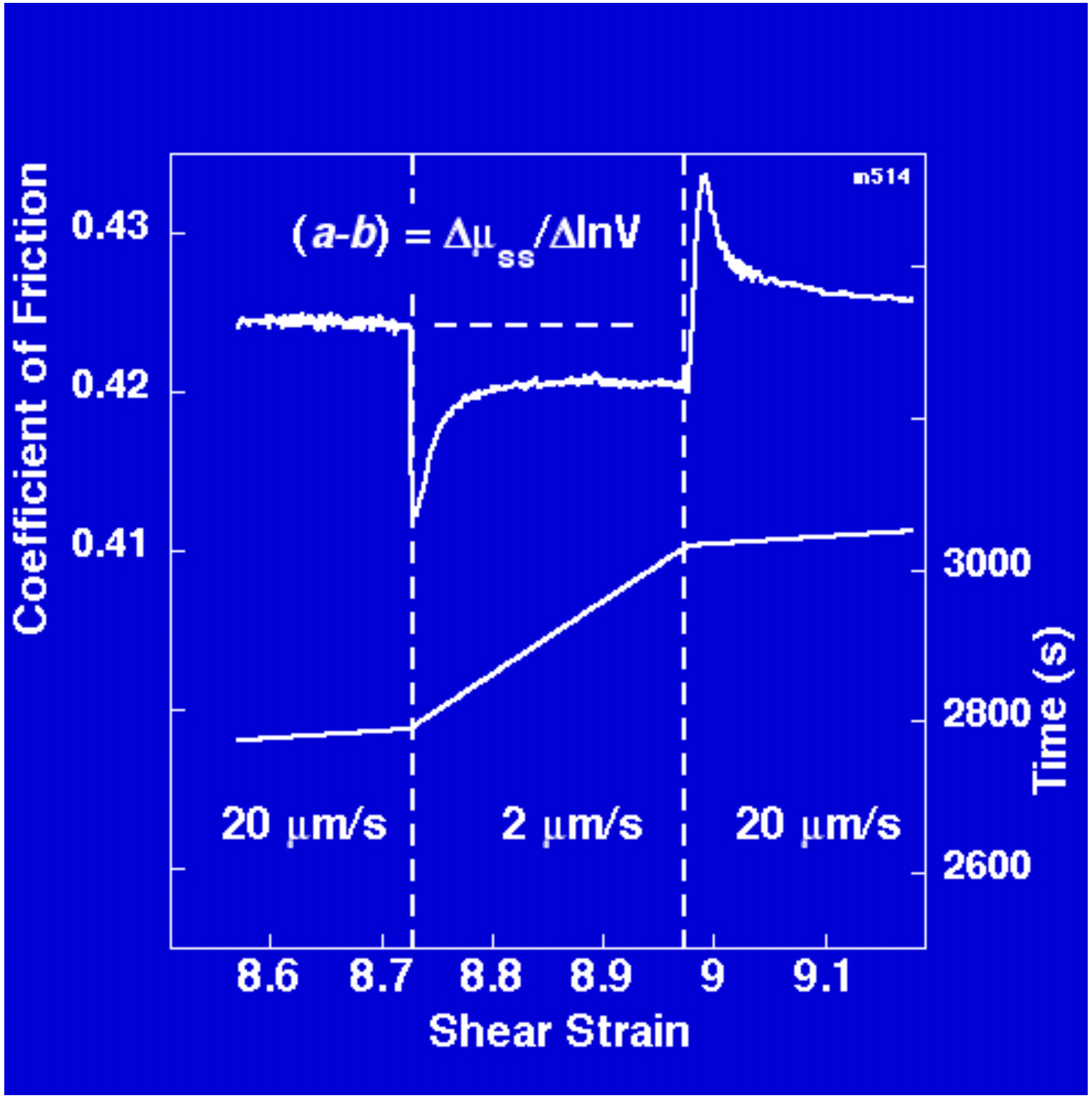


Frye and Marone, JGR 2002

Measuring the velocity dependence of friction

Frictional Instability

Requires (a-b) < 0



Constitutive Modelling

Rate and State Friction Law

Elastic Interaction, Testing Apparatus

$$\mu(\theta,v) = \mu_0 + a\ln\left(\frac{v}{v_o}\right) + b\ln\left(\frac{v_o\theta}{D_c}\right)$$

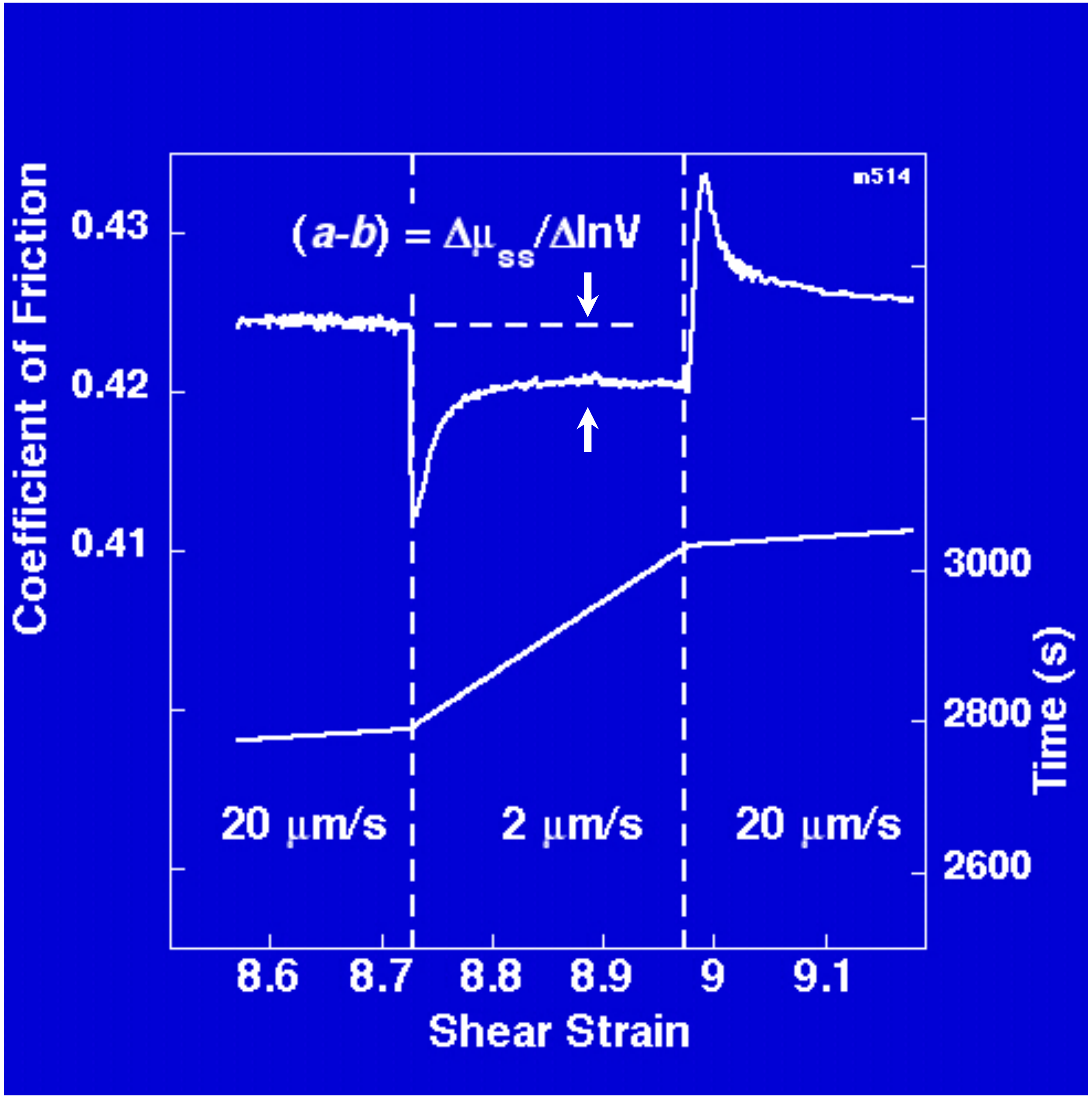
$$\frac{d\theta}{dt} = 1 - \frac{v\,\theta}{D_c}$$

$$\theta_{ss} = \frac{D_c}{v}$$

$$\Delta\mu_{ss} = (a-b)\ln\left(\frac{v}{v_o}\right)$$

$$\frac{d\mu}{dt} = k'\,(v_{lp} - v)$$

Results: Velocity stepping
Measuring the velocity dependence of friction



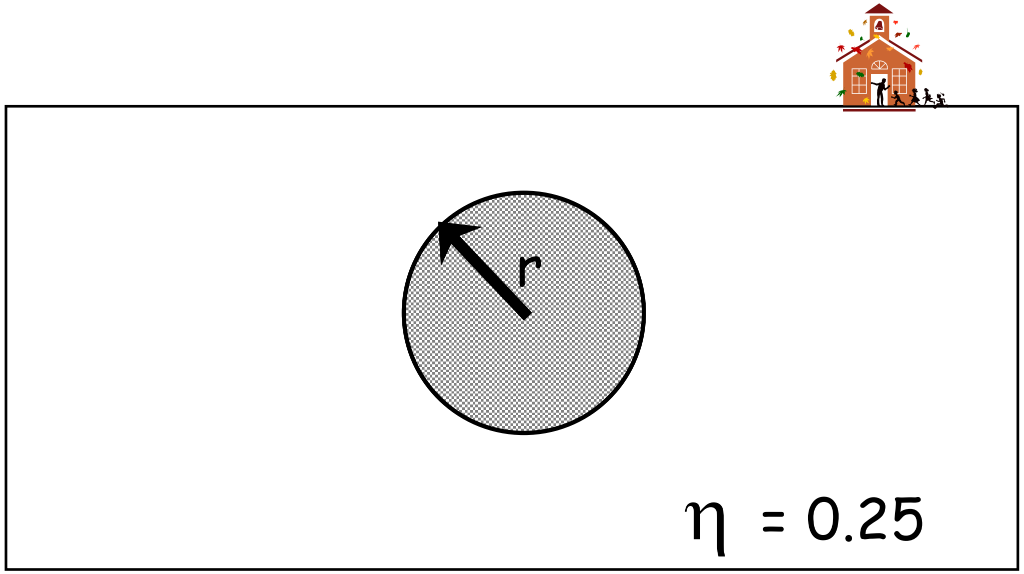
Frictional Instability

Requires K < K_c

$$K_c = \frac{\sigma_n (b - a)}{D_c}$$

This example shows steady-state velocity strengthening: (a-b) > 0

Rupture Patch Size for Earthquake Nucleation



$$K = \frac{\Delta\sigma}{\bar{u}} = \frac{7\pi}{16}\frac{G}{r}$$

$$K_c \approx \frac{\sigma_n(b-a)}{D_c}$$

earthquake nucleation when $\frac{K}{K_c} \approx 1.0$

$$h^* = r_c \approx C\frac{GD_c}{\sigma_n(b-a)}$$

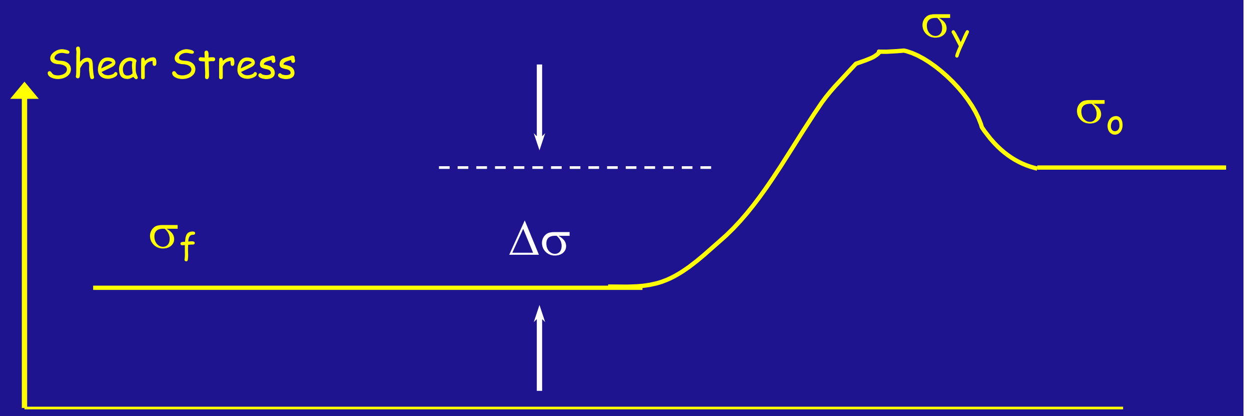
See also: Earthquake nucleation on rate and state faults – Aging and slip laws, Ampuero & Rubin, JGR 2008

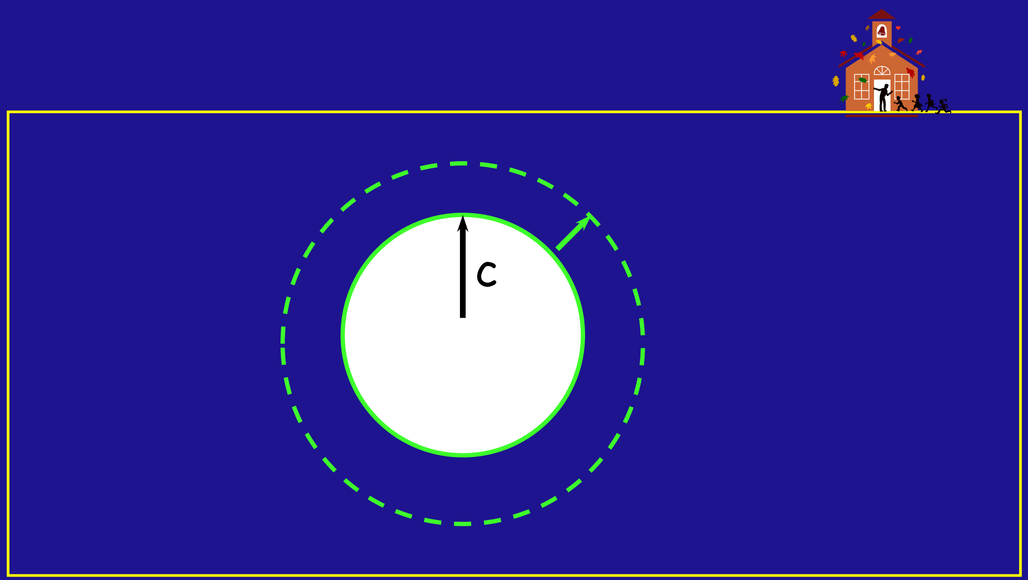
Dislocation model for fracture and earthquake rupture

Dislocation model, circular crack

$\Delta\sigma = (\sigma_o - \sigma_f)$

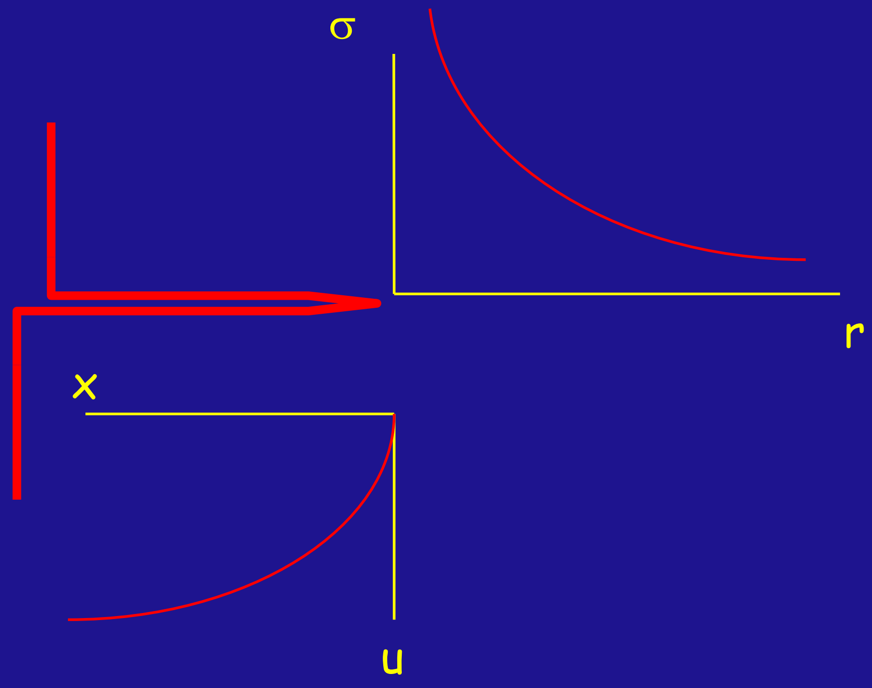
$$\Delta u(x,y) = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{c^2 - \left(x^2 + y^2\right)}$$





Crack tip stress field, real materials

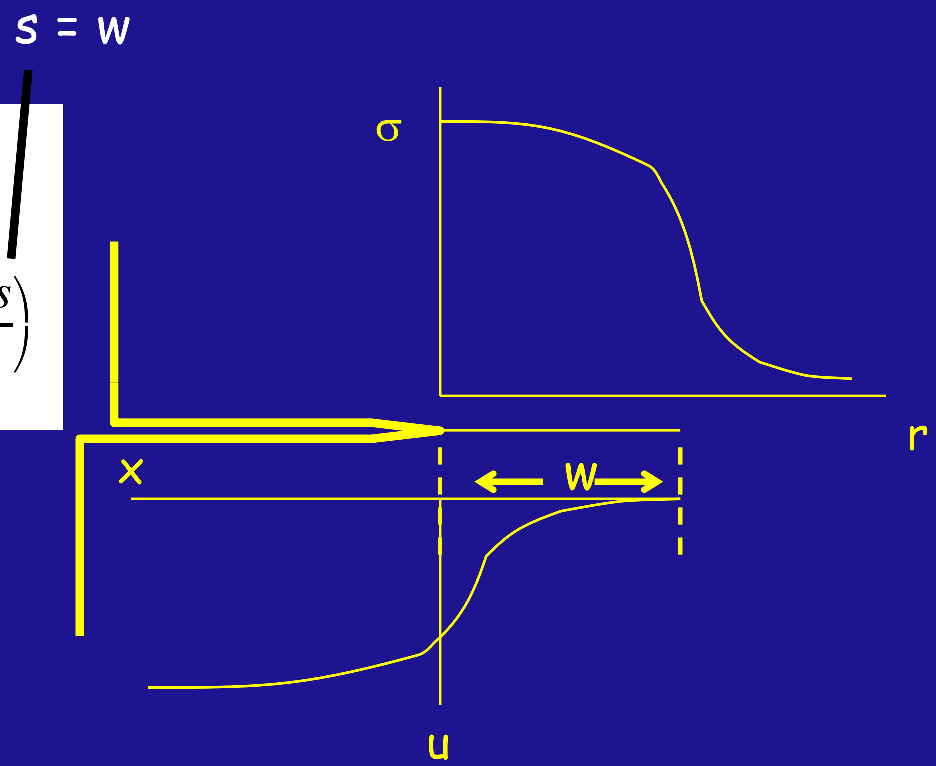
- Singular crack (Eshelby)



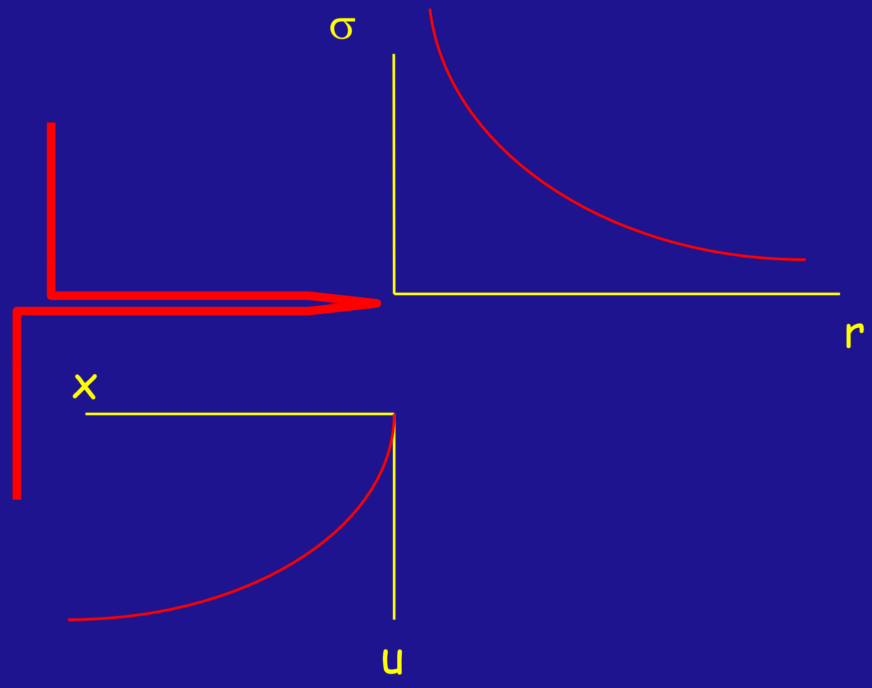
- Dugdale (Barenblatt)
  - Assume a yield criterion, σ_y, within a crack tip region

$$\Delta u = \frac{(1-\eta)(\sigma_o - \sigma_f)\; c}{2\pi\mu}\; f(\theta,\ \theta_2),\ \ see\ \ 1.30$$

$$\theta = \cos^{-1}\left(\frac{2x}{c}\right) \quad for\ \ |x| < \frac{c}{2} \quad and \quad \theta_2 = \cos^{-1}\left(\frac{c-2s}{c}\right)$$



Crack tip stress field, real materials



- Singular crack (Eshelby)

- Dugdale (Barenblatt)
  - Assume a yield criterion, σ_y, within a crack tip region

s = w

$$\Delta u = \frac{(1-\eta)(\sigma_o - \sigma_f)\ c}{2\pi\mu} f(\theta,\ \theta_2),\ \ see\ \ 1.30$$

$$\theta = \cos^{-1}\left(\frac{2x}{c}\right)\ \ for\ \ |x| < \frac{c}{2}\ \ and\ \ \theta_2 = \cos^{-1}\left(\frac{c-2s}{c}\right)$$

- e.g., can we 'read' the state of stress in the crust from earthquake (fault) data

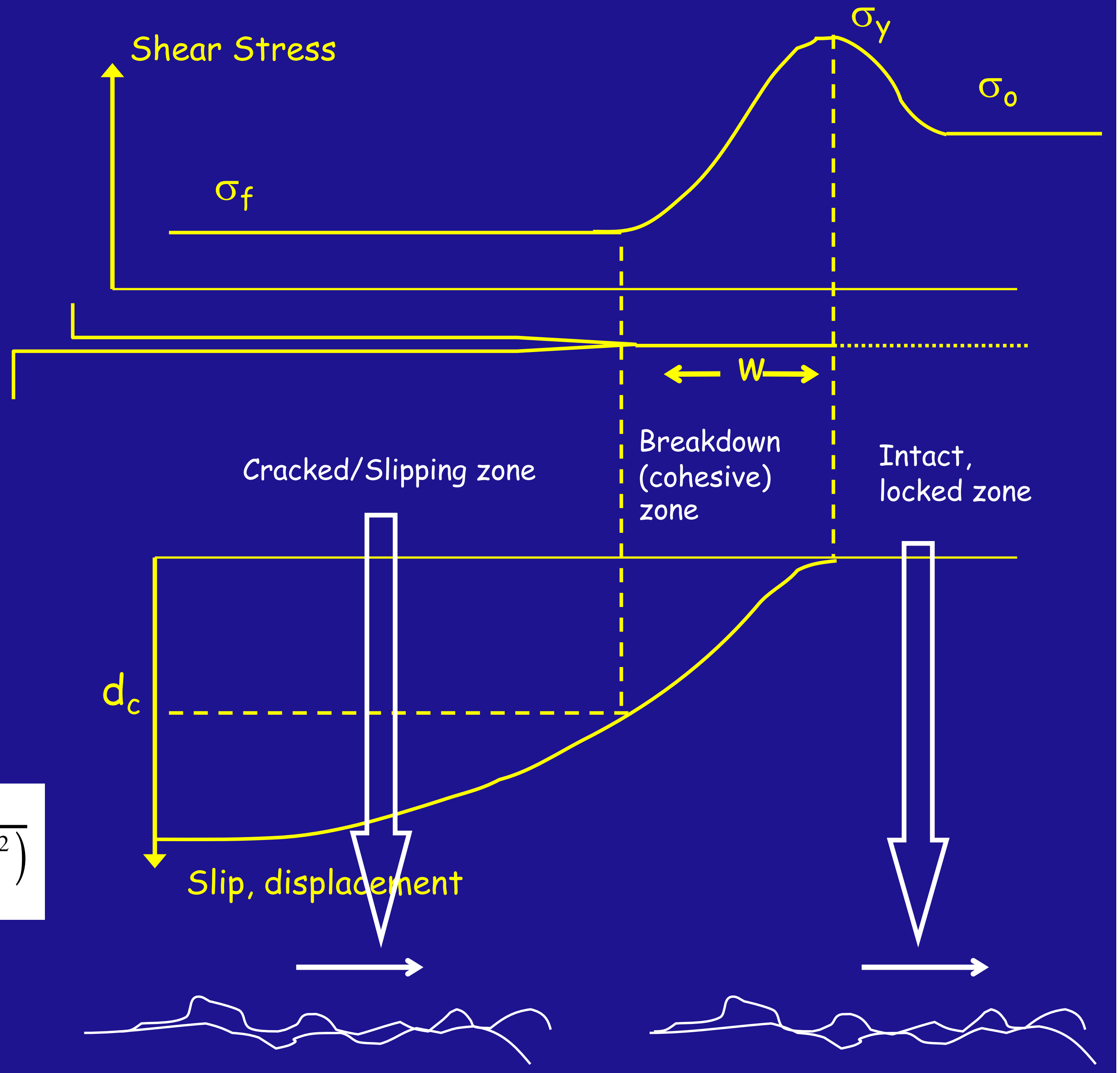
Barenblatt, G. I., 1959, The formation of brittle cracks during brittle fracture. General ideas and hypotheses. Axially-symmetric cracks. Appl. Math. Mech. 23, 1273 – 1282.
Barenblatt, G. (1962). The mathematical theory of equilibrium cracks in brittle fracture. Advances in Applied Mechanics, 7, 55–129.
Dugdale, D. (1960). Yielding of steel sheets containing slits. Journal of the Mechanics and Physics of Solids, 8, 100–104.

Cohesive zone crack model, applies to fracture and/or friction

- Dugdale (Barenblatt)

$$\Delta u = \frac{(1-\eta)(\sigma_o - \sigma_f)\ c}{2\pi\mu}\ f(\theta,\ \theta_2),\ see\ 1.30$$

$$\theta = \cos^{-1}\left(\frac{2x}{c}\right)\ \ for\ |x| < \frac{c}{2}\ \ and\ \ \theta_2 = \cos^{-1}\left(\frac{c-2s}{c}\right)$$

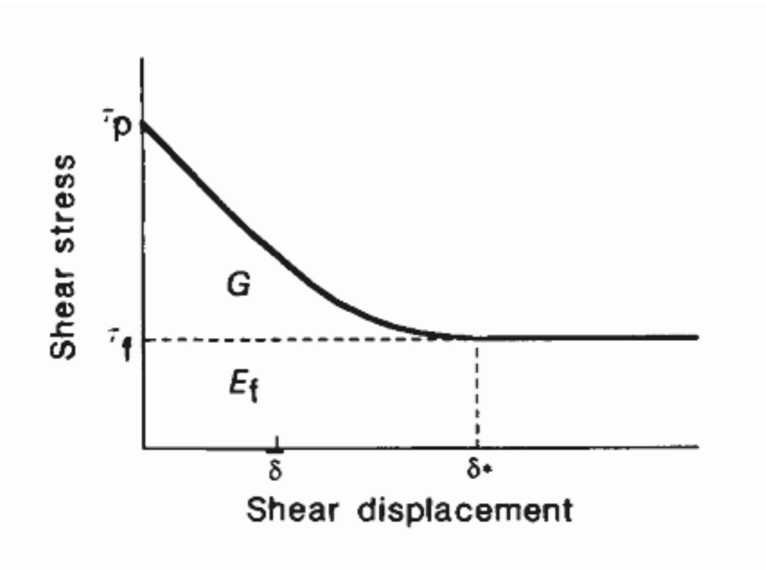


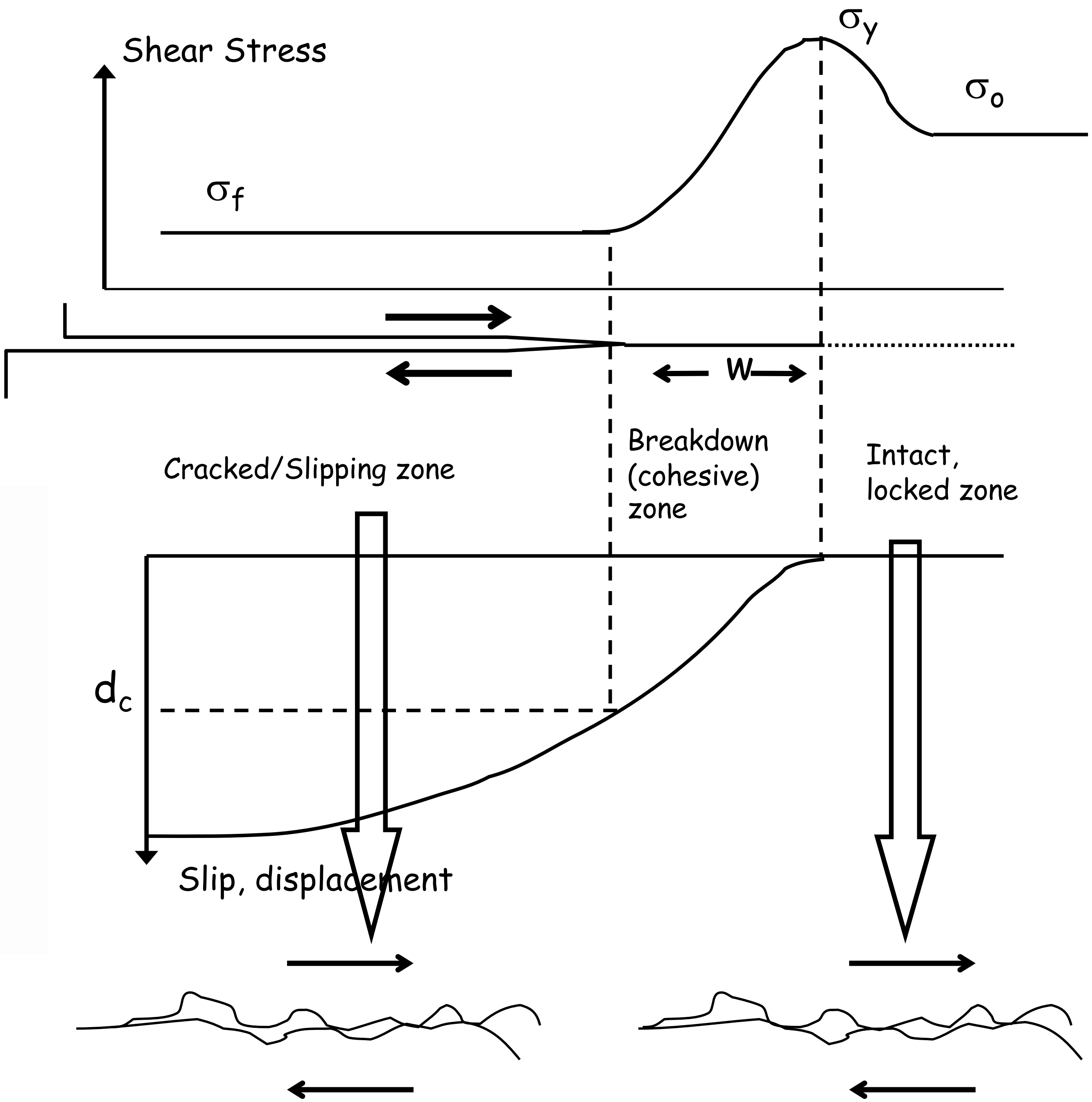
Dislocation model, circular crack

$\Delta\sigma = (\sigma_o - \sigma_f)$

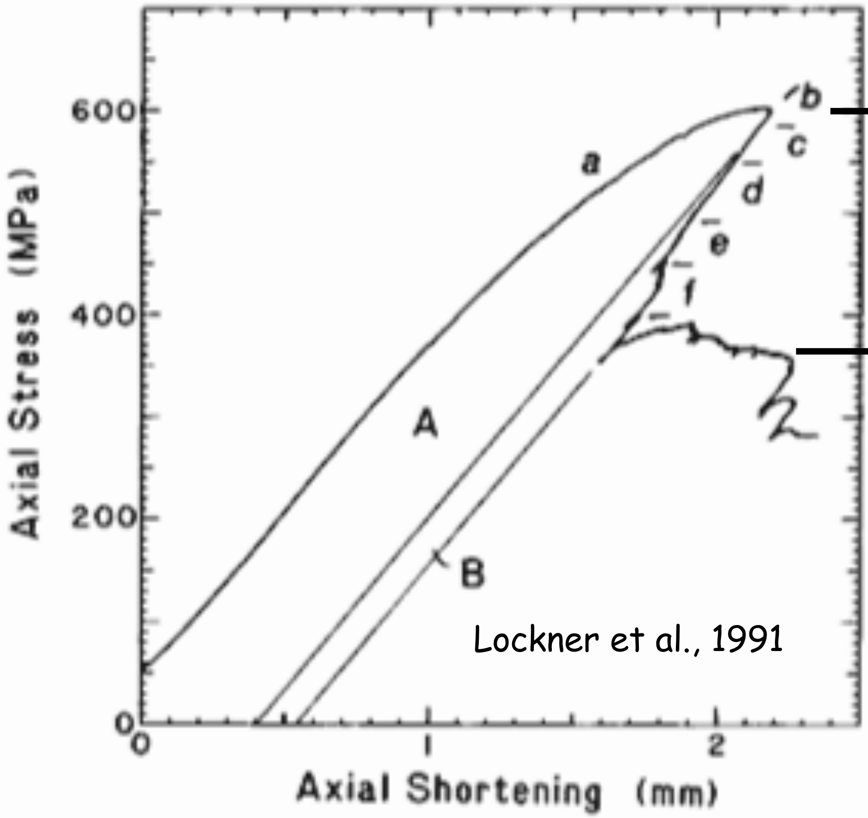
$$\Delta u(x,y) = \frac{24}{7\pi}\ \frac{\Delta\sigma}{\mu}\sqrt{c^2 - (x^2 + y^2)}$$

Cohesive zone, slip weakening crack model for friction

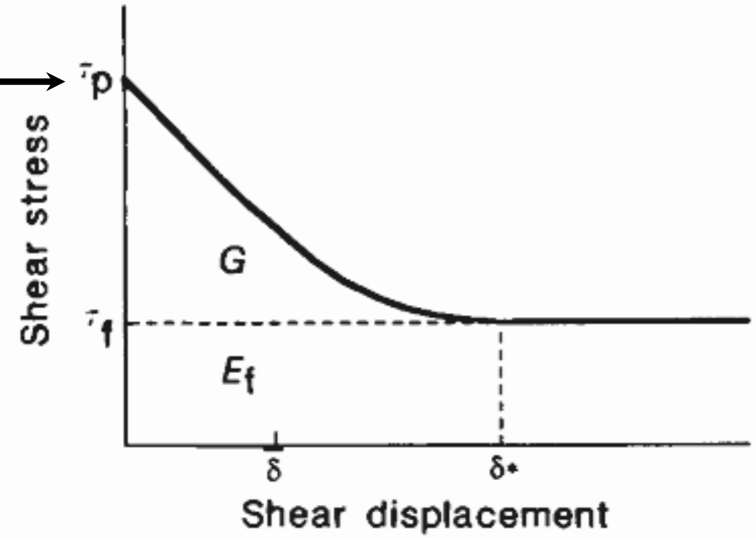




Shear Fracture Energy from Postfailure Behavior



Lockner et al., 1991

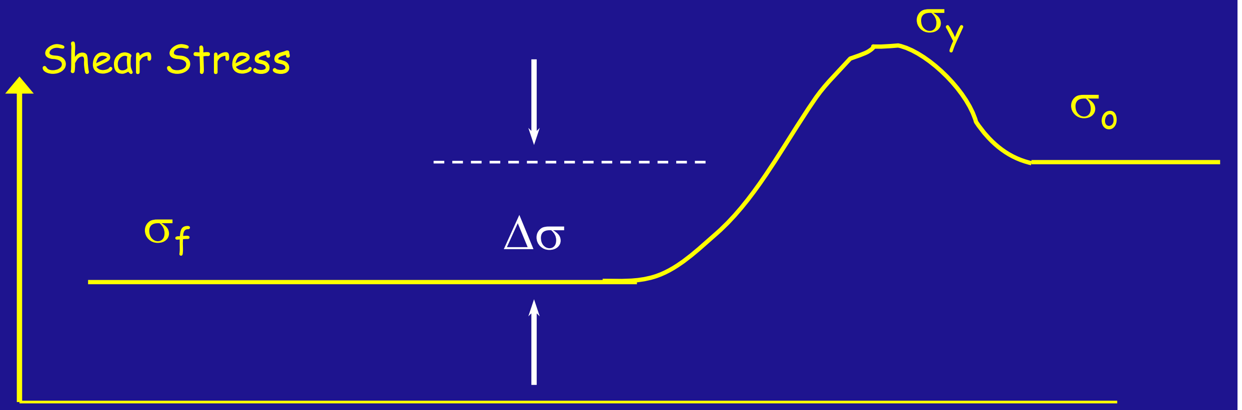


Inferred shear stress vs. slip relation for slip-weakening model. (based on Wong, 1982)

Wong, 1982, found that shear stress dropped ~ 0.2 GPa over a slip distance of ~50 microns.

Exercise: Estimate G from this data and compare it to the values reported in Scholz (Table 1.1) and Wong, 1982.

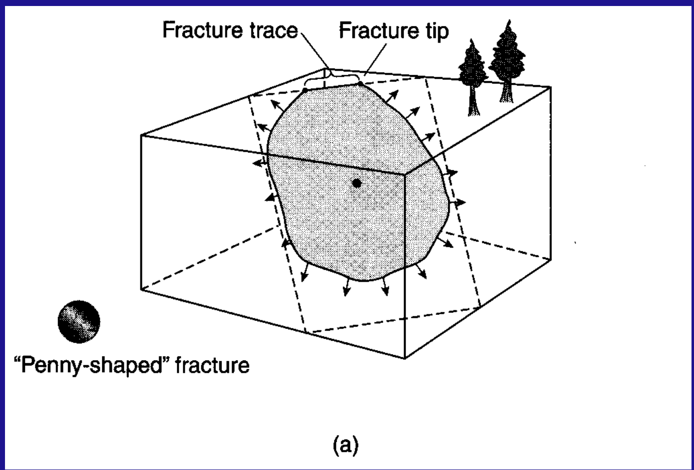
Dislocation model for fracture and earthquake rupture



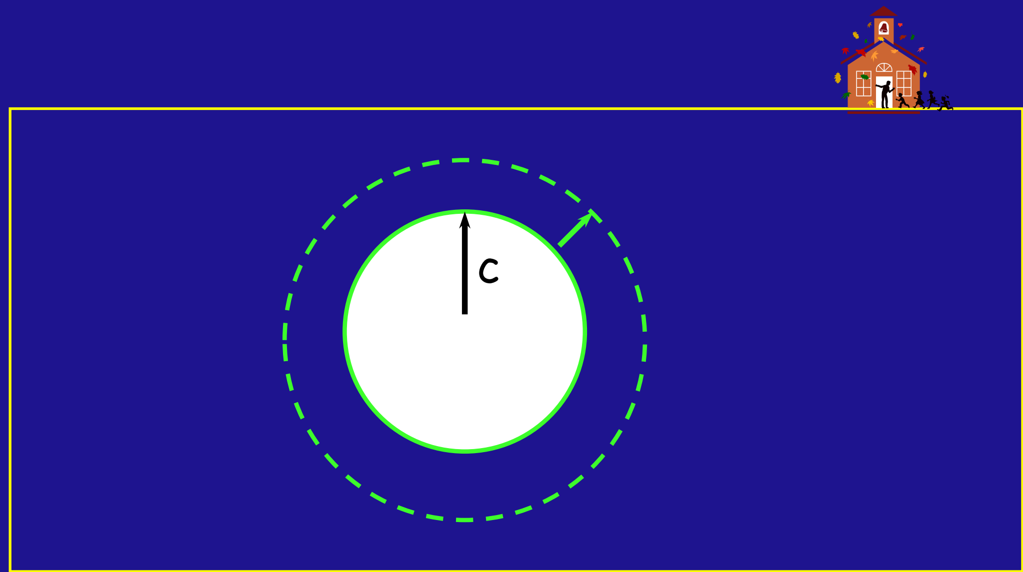
Dislocation model, circular crack

$\Delta\sigma = (\sigma_o - \sigma_f)$

$$\Delta u(x,y) = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{c^2 - \left(x^2 + y^2\right)}$$



(a)



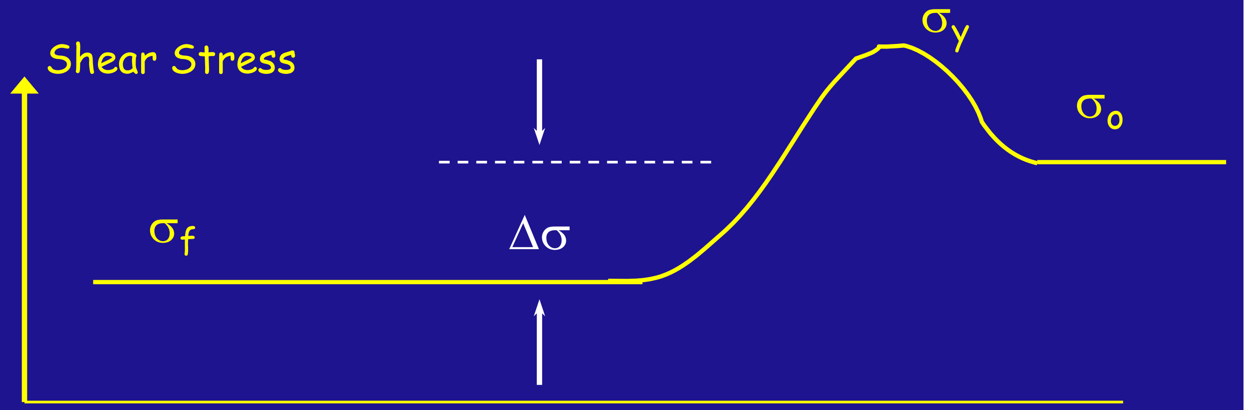
$$\Delta\sigma = \frac{7\pi}{16} G \frac{\Delta\bar{u}}{r}$$

Relation between stress drop and slip for a circular dislocation (crack) with radius r

For η=0.25, Chinnery (1969)

- Importance of slip: e.g., $M_o = \mu A u$

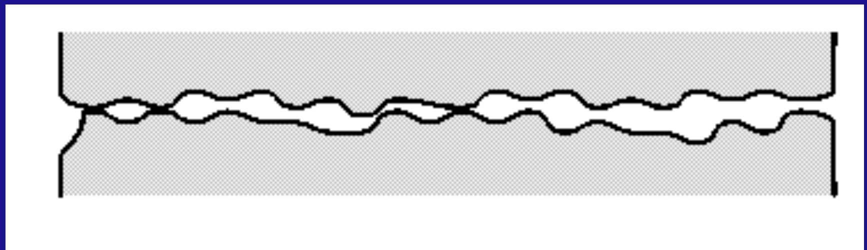
Dislocation model for fracture and earthquake rupture

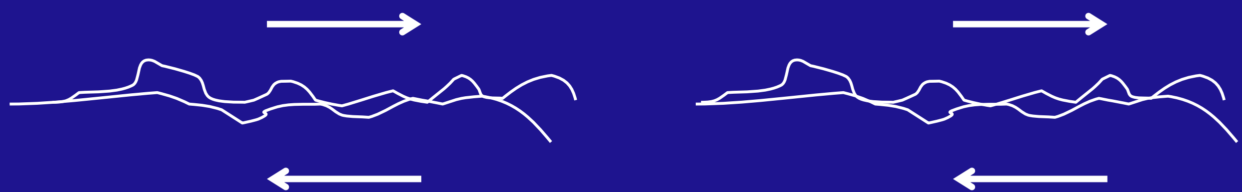


Dislocation model, circular crack

$\Delta\sigma = (\sigma_o - \sigma_f)$

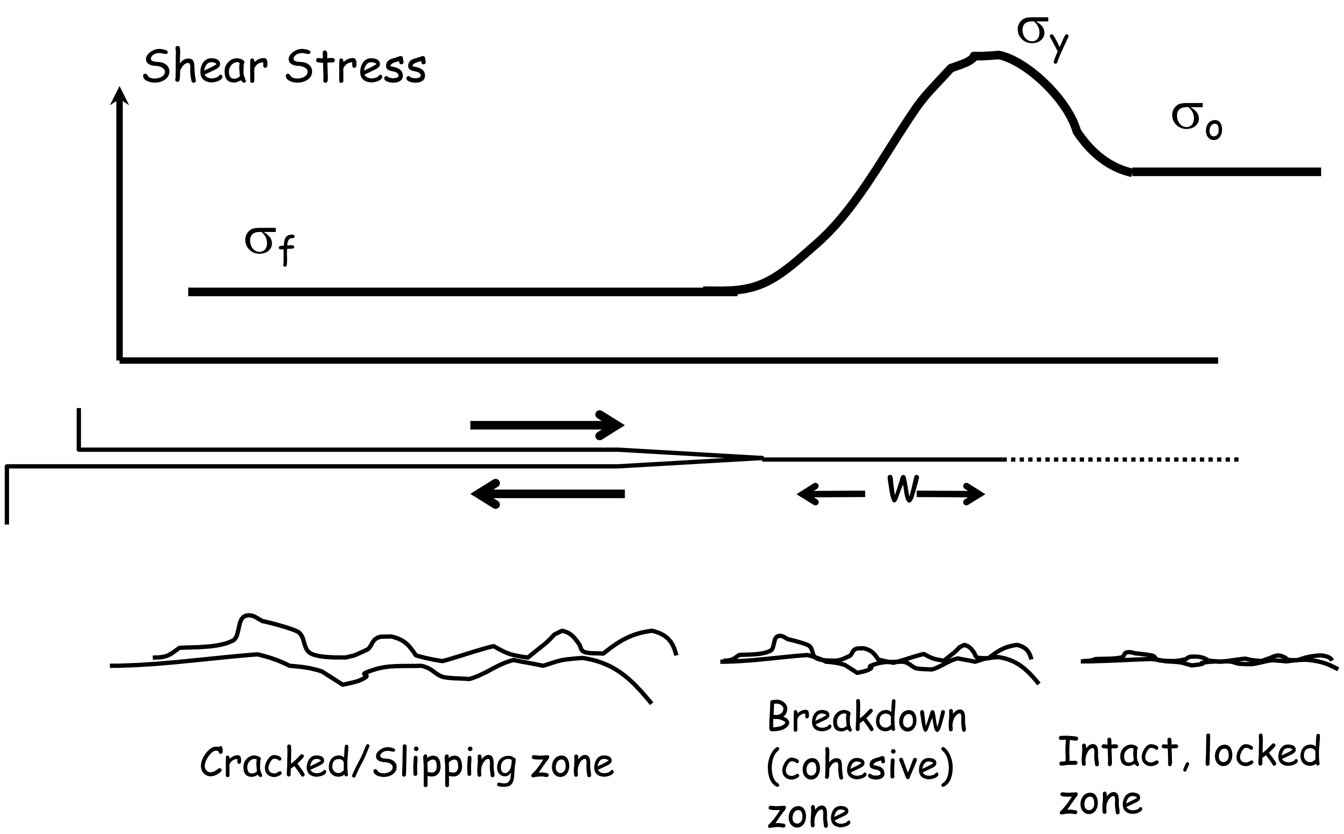
$$\Delta u(x,y) = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{c^2 - \left(x^2 - y^2\right)}$$

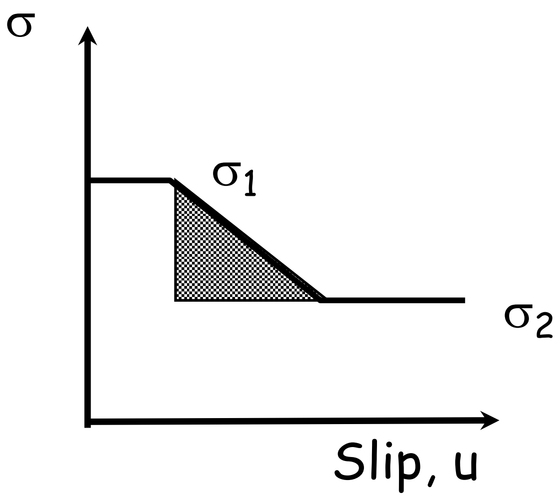


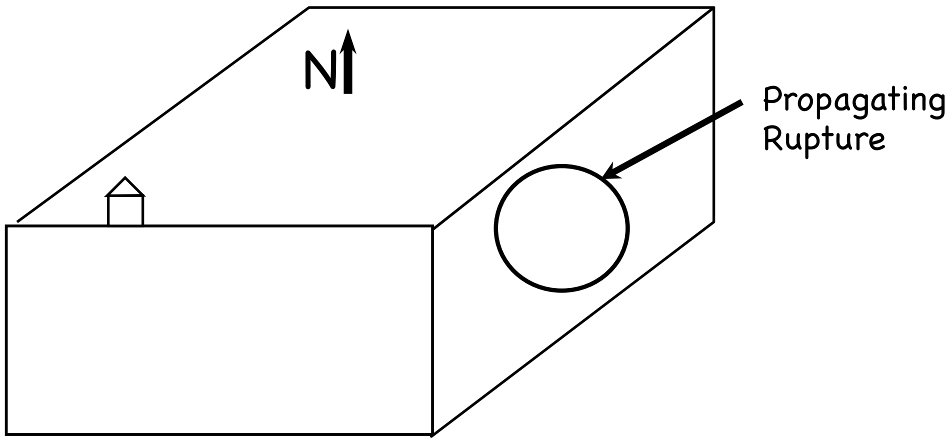


Mode II crack propagation at speed Vr

cohesive zone/slip weakening crack model for friction







cohesive zone/slip weakening crack model for friction

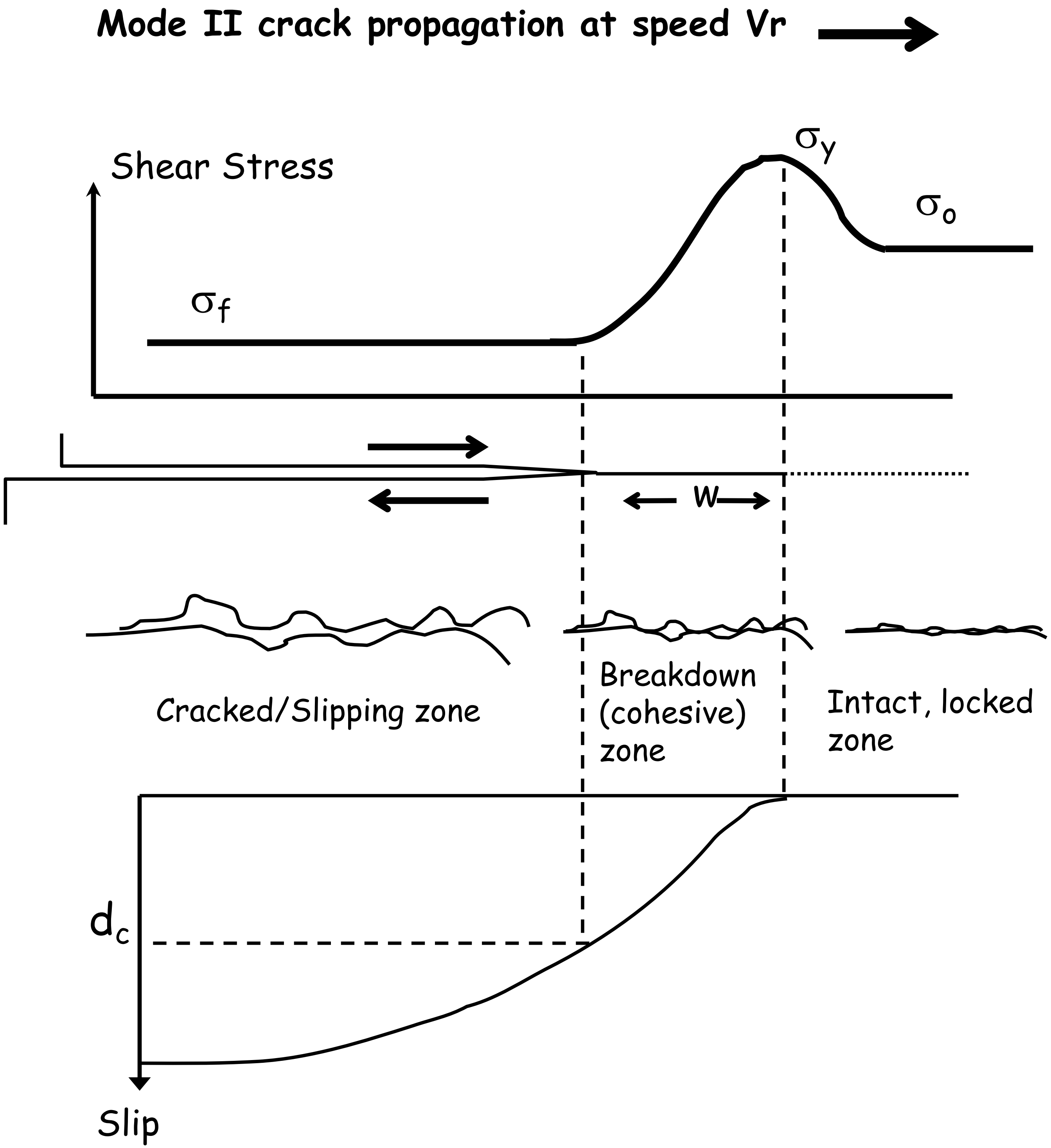
f_{max} scales as:

f_{max} = w/Vr

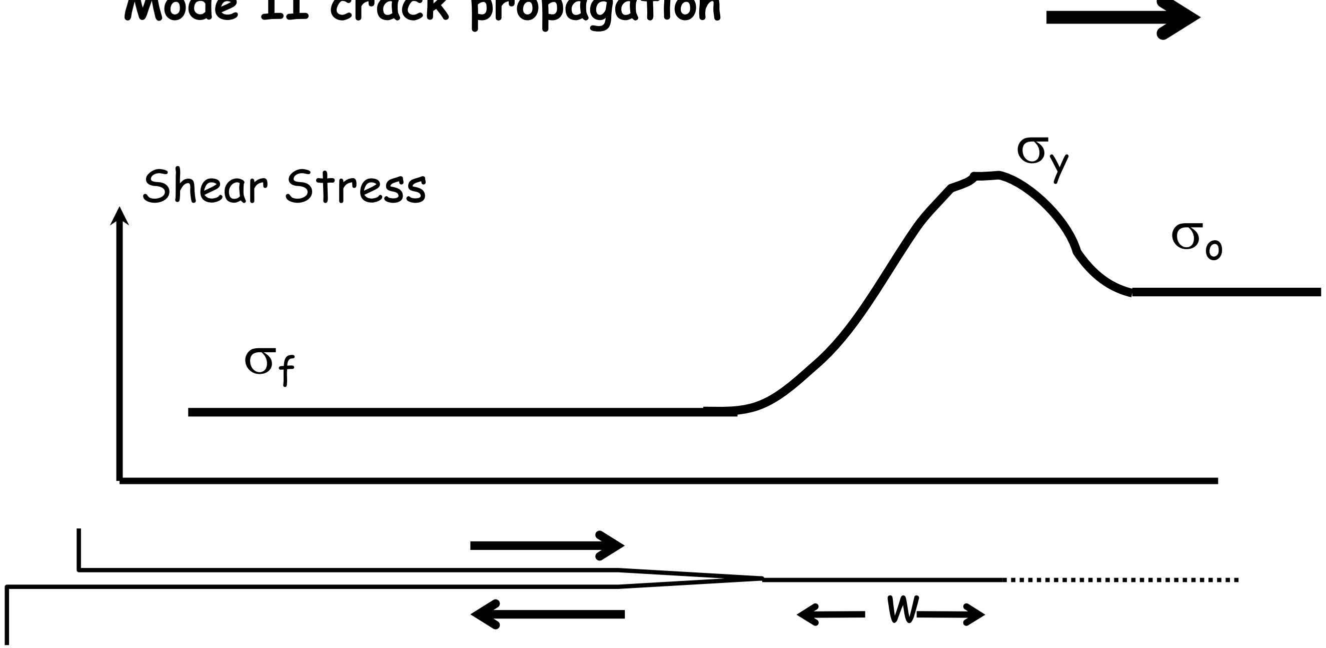
$$\Delta\sigma = \frac{7\pi}{24} G \frac{u_{max}}{r}$$

$$\Delta\sigma = \frac{7\pi}{24} G \frac{D_c}{w}$$

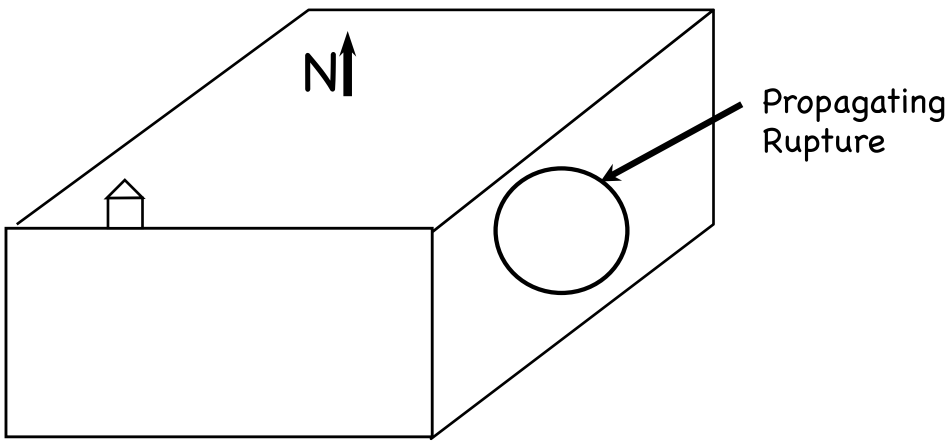
$$D_c = \frac{24}{7\pi} \frac{\Delta\sigma}{G} w$$



Mode II crack propagation



Cohesive zone length w scales as:



$$w \approx C \frac{D_c G}{\Delta\sigma}$$

$$\frac{w}{D_c} \approx C \frac{G}{\Delta\sigma}$$

Slip weakening model (Ida, 1972, 1973)

$$R_c = \frac{CGD_c}{\sigma_n (b-a)} \text{ or } L_c = \frac{E \; D_c}{2(1-\nu^2)\sigma_n \Delta\mu}$$

Recall the derivation of this result:

Frictional Instability

Requires K < K_c

$$K_c = \frac{\sigma_n(b-a)}{D_c}$$

$$\Delta\sigma = \frac{7\pi}{24} G \frac{u_{max}}{r}$$

Relation between stress drop and slip for a circular dislocation (crack) with radius r

For ν =0.25, Chinnery (1969)

$$\Delta\sigma = \frac{7\pi}{24} G \frac{D_c}{w}$$

$$D_c = \frac{24}{7\pi} \frac{\Delta\sigma}{G} w$$