Mechanics of Earthquakes and Faulting

Lecture 1

www.geosc.psu.edu/Courses/Geosc508

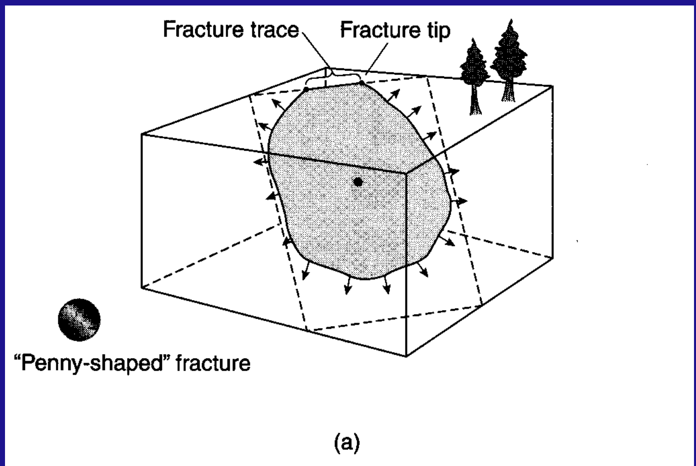
- Surface and body forces
- Tensors, Mohr circles.
- Theoretical strength of materials
- Defects
- Stress concentrations
- Griffith failure criteria

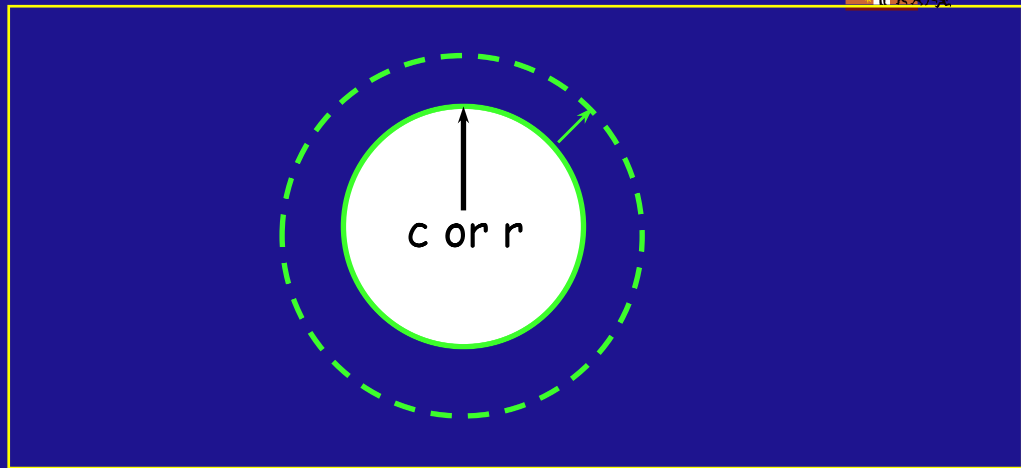
Dislocation model for fracture and earthquake rupture

Dislocation model, circular crack

For an increment of stress ($\Delta\sigma$), how much slip occurs between the crack faces (Δu), and how does that slip vary with position (x, y) and crack radius (c)

$$\Delta u(x,y) = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{c^2 - (x^2 + y^2)}$$





$$\Delta\sigma = \frac{7\pi}{24} G \frac{\Delta u_{max}}{r}$$

$$\Delta\sigma = \frac{7\pi}{16} G \frac{\Delta \bar{u}}{r}$$

Relation between stress drop and slip for a circular dislocation (crack) with radius r

For ν =0.25, Chinnery (1969)

- Importance of slip: e.g., $M_o = \mu A u$

Dislocation model for fracture and earthquake rupture

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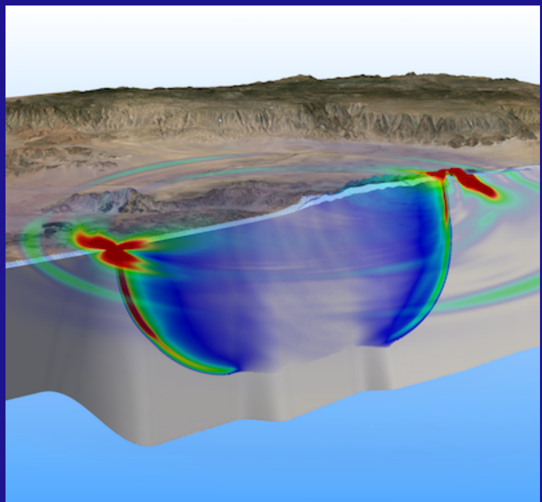
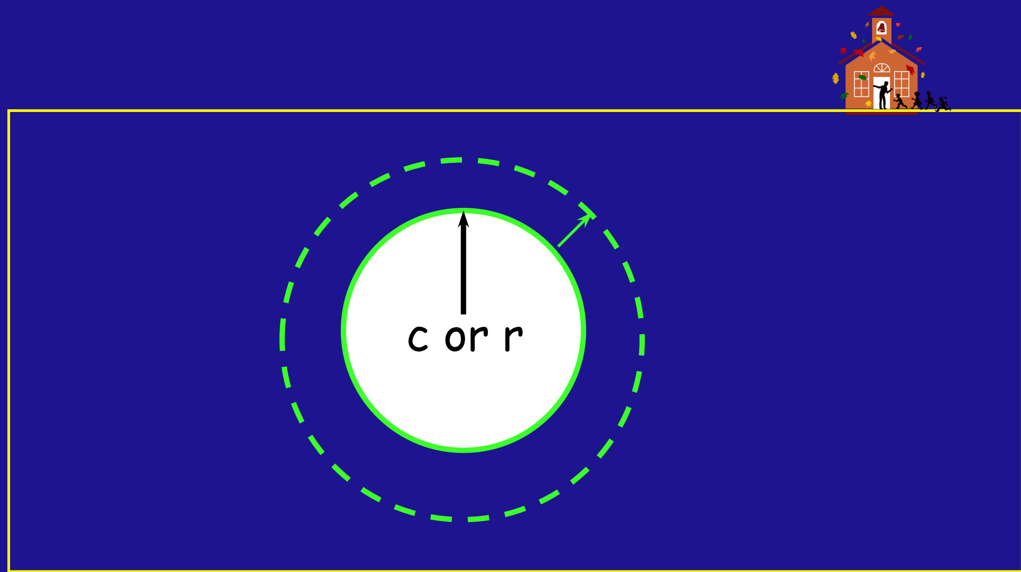




http://schultz.physics.ucdavis.edu/research/virtual_california.html

Stress Transformation

Force Normal to P:

$$A\sigma = \tau_{yx} A \sin\alpha \cos\alpha + \tau_{yy} A \sin\alpha \sin\alpha + \tau_{xy} A \cos\alpha \sin\alpha + \tau_{xx} A \cos\alpha \cos\alpha$$

This can be simplified by eliminating A, and using $\tau_{xy} = \tau_{yx}$ and using the identity $2 \sin\alpha \cos\alpha = \sin 2\alpha$

Normal Stress on Plane P:

$$\sigma = \tau_{xx} \cos^2\alpha + \tau_{xy} \sin 2\alpha + \tau_{yy} \sin^2\alpha$$

Shear Force on P:

$$A\tau = \tau_{yx} A \sin\alpha \sin\alpha - \tau_{yy} A \sin\alpha \cos\alpha - \tau_{xy} A \cos\alpha \cos\alpha + \tau_{xx} A \cos\alpha \sin\alpha$$

This can be simplified to:

Shear Stress on Plane P:

$$\tau = (\tau_{xx} - \tau_{yy}) \cos\alpha \sin\alpha + \tau_{xy} (\sin^2\alpha - \cos^2\alpha)$$

- Stress components are a function of coordinate frame and orientation
- Principal Stresses
  - Shear stresses vanish, only normal stresses
  - By convention, maximum principal stress is σ_1 and $\sigma_1 > \sigma_2 > \sigma_3$, compression is positive

in 2D

τ_{xx}, 0

0, τ_{yy},

Stress Transformation

Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of **Principal Stresses**:

$\sigma = \tau_{xx} \cos^2 \alpha + \tau_{xy} \sin 2\alpha + \tau_{yy} \sin^2 \alpha$

$\tau = (\tau_{xx} - \tau_{yy}) \cos \alpha \sin \alpha + \tau_{xy} (\sin^2 \alpha - \cos^2 \alpha)$

2D

τ_{xx}, 0

0, τ_{yy},

$\sigma = \tau_{xx} \cos^2 \alpha + \tau_{yy} \sin^2 \alpha$

$\tau = (\tau_{xx} - \tau_{yy}) \cos \alpha \sin \alpha$

$\sigma = \sigma_1 \cos^2 \alpha + \sigma_2 \sin^2 \alpha$, Normal Stress

$\tau = (\sigma_1 - \sigma_2) \cos \alpha \sin \alpha$, Shear Stress

Use trig. identities such as $\cos 2\alpha = 1 - 2 \sin^2\alpha$ and $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

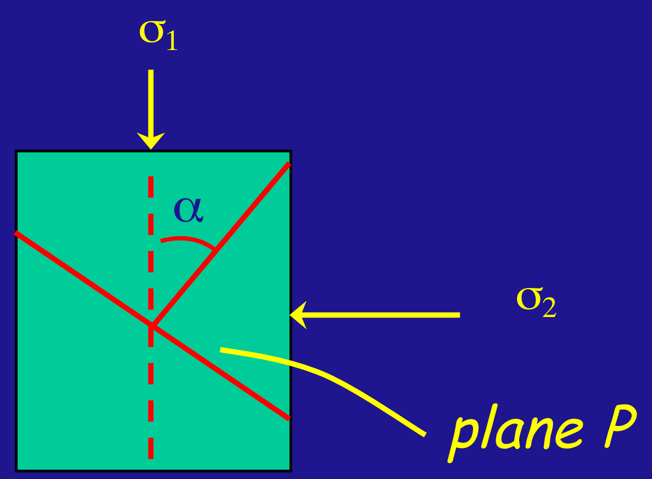
$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

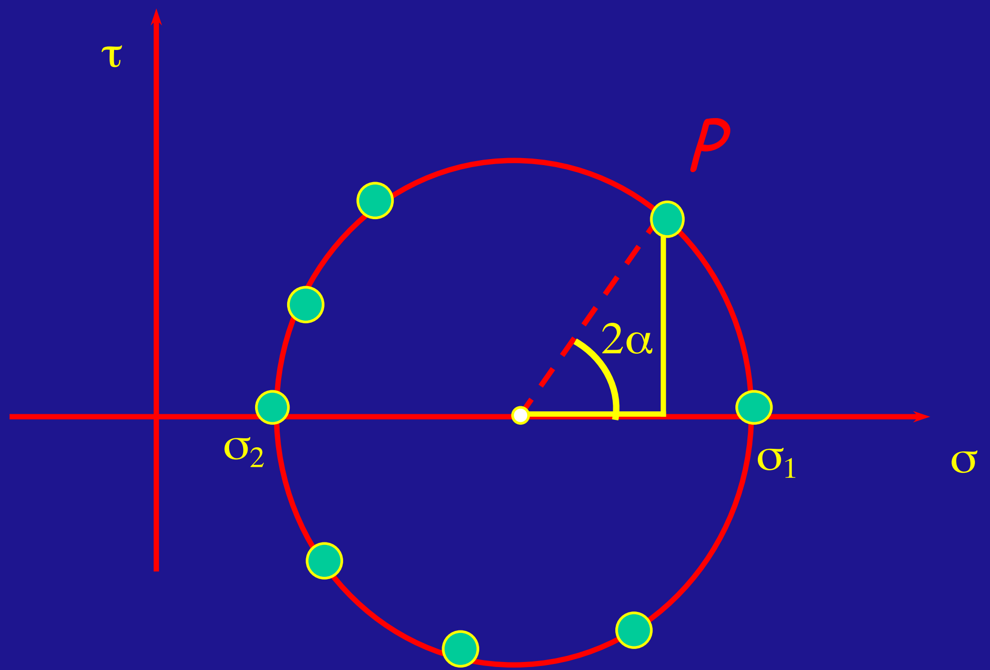
$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

Note that these relations make use of the **mean stress** and the **differential stress**

Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of **Principal Stresses**:

Mohr Circle.



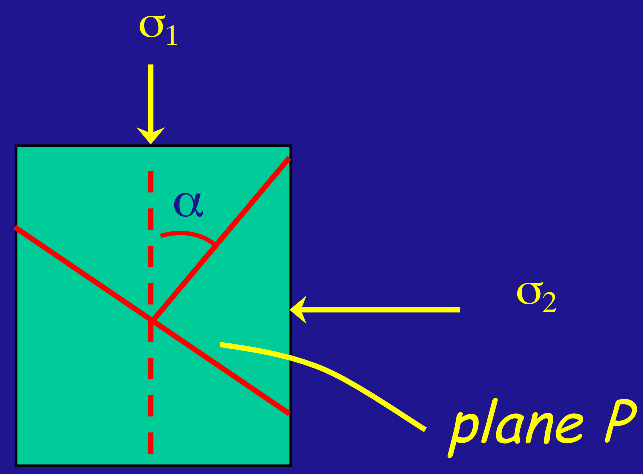


Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of **Principal Stresses**:

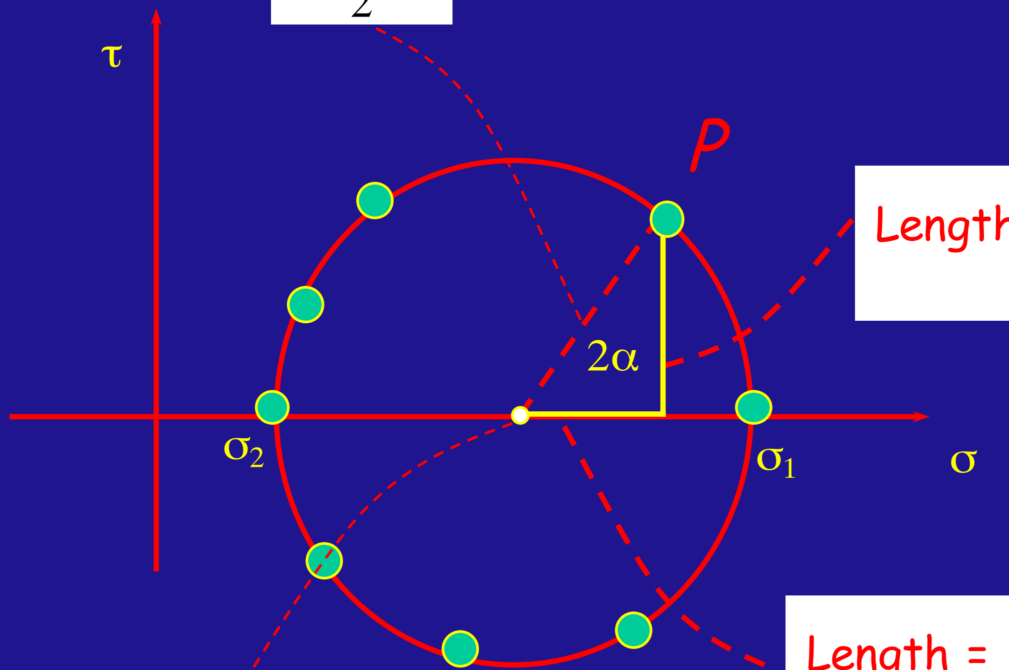
Mohr Circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2}\cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$



$$\frac{(\sigma_1 - \sigma_2)}{2}$$



Length = $\frac{(\sigma_1 - \sigma_2)}{2}\sin\ 2\alpha$

Length = $\frac{(\sigma_1 - \sigma_2)}{2}\cos 2\alpha$

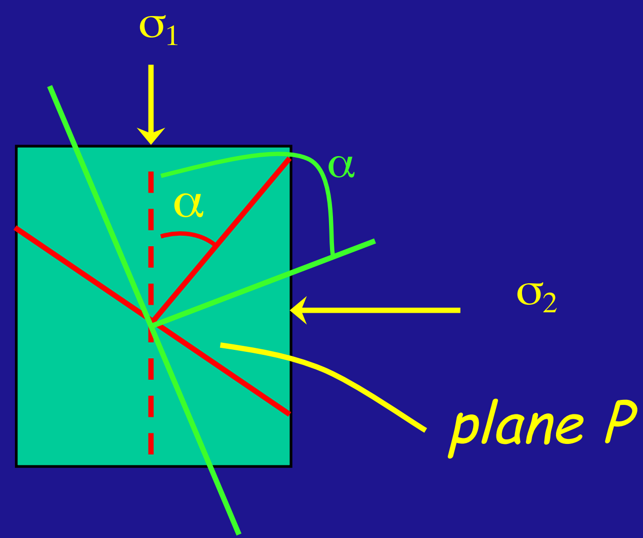
$$\frac{(\sigma_1 + \sigma_2)}{2}$$

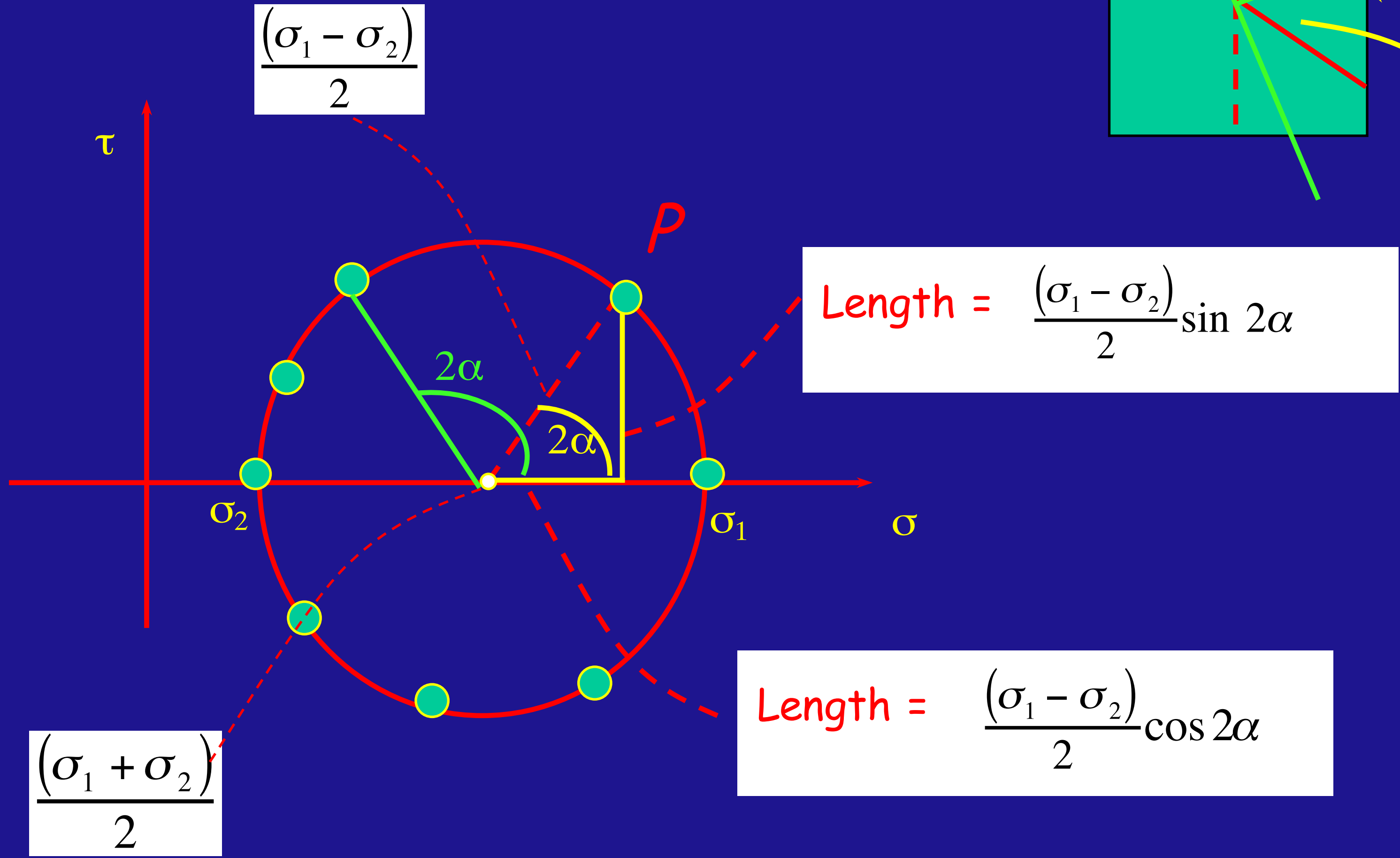
Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of **Principal Stresses**:

Mohr Circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2}\cos 2\alpha$$

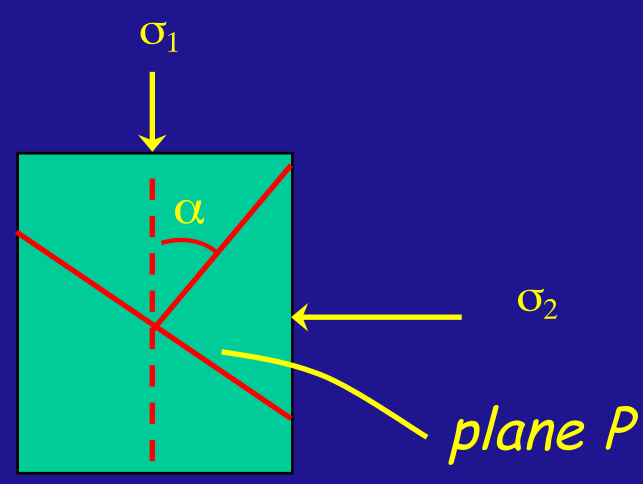
$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

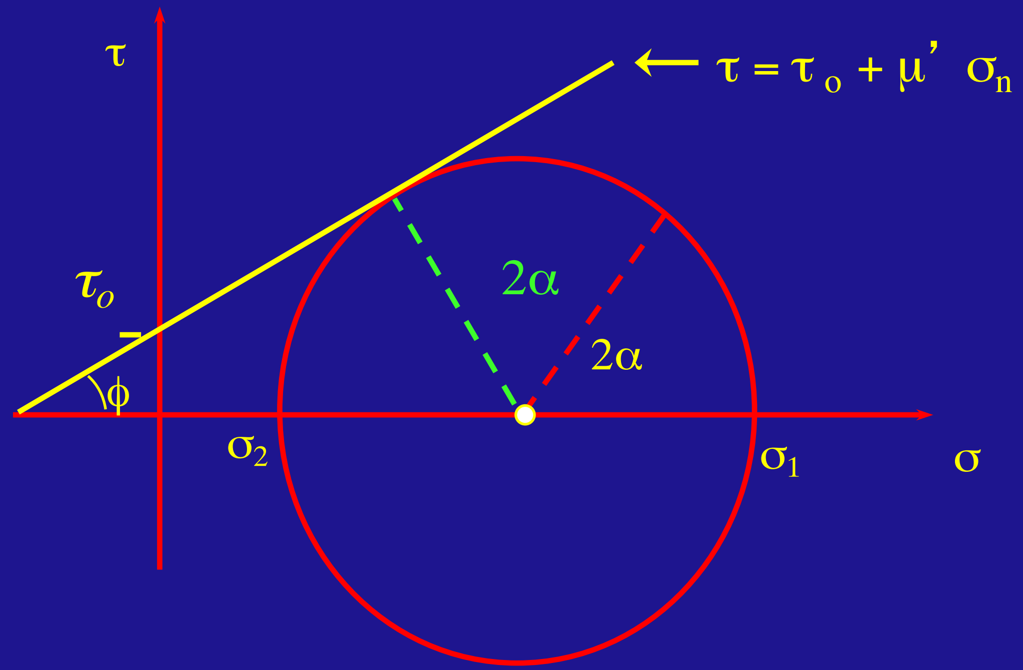




Coulomb-Mohr Failure Criterion $\tau = \tau_o + \mu' \sigma_n$

where τ is shear stress τ_o is 'cohesion,' μ' is the coefficient of internal friction and σ_n is normal stress





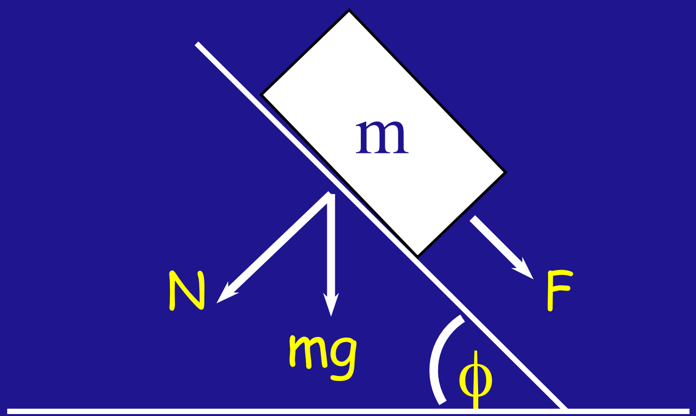
The parameter μ' describes the effect of normal stress on shear strength.

Pressure-dependent brittle failure

Failure stress is higher for things under higher mean stress.

Friction and Faulting.

Write down the forces and stresses for a simple experiment.
Take a mass on an inclined plane, as in a simple friction experiment.



Normal Force: $N = m\,g\cos\phi$

Shear Force: $F = m\,g\sin\phi$

Coefficient of friction $\mu = F/N$

$$\mu = \frac{F = m\,g\sin\phi}{N = m\,g\cos\phi} = \tan\phi$$

Amonton's Law:

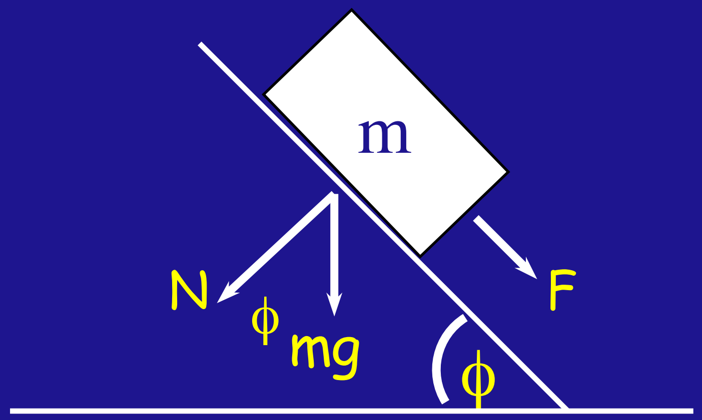
$F/N = \mu = \tan\phi$

This can also be written in terms of stresses: $\tau = \mu\,\sigma_n$

But, note that in general: $\mu \neq \mu'$ and $\phi \neq \phi'$

That is, the coefficient of sliding friction is not equal to the coefficient of internal friction.

Mass on an inclined plane, as in a simple friction experiment.



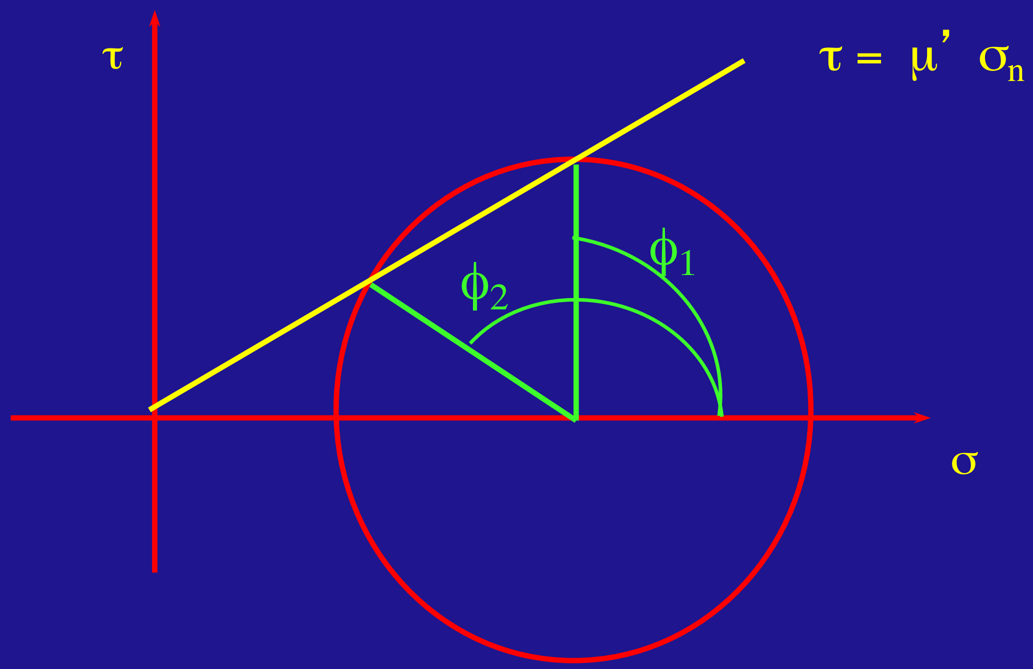
Coefficient of friction $\mu = F/N$

Amonton's Law:

$F/N = \mu = \tan \phi$

In terms of stresses: $\tau = \mu \sigma_n$

This failure criterion can be plotted on a Mohr diagram



For this stress state, the Mohr plot shows that frictional failure would occur on any plane of orientation between ϕ_1 and ϕ_2

- Discussion of Handin, JGR, 1969 and Chapter 1 Scholz

- Rankine's condition: what is it?
- What is the coefficient of internal friction?
- How are material properties a function of the state of stress? What did Handin mean by this statement on p. 5344 (top left)?
- Rocks pass from brittle to ductile deformation mechanism with what changes in strain rate, temperature and pressure? What does this mean for the linearity of the Mohr envelope?