Mechanics of Earthquakes and Faulting

Lecture 1

www.geosc.psu.edu/Courses/Geosc508

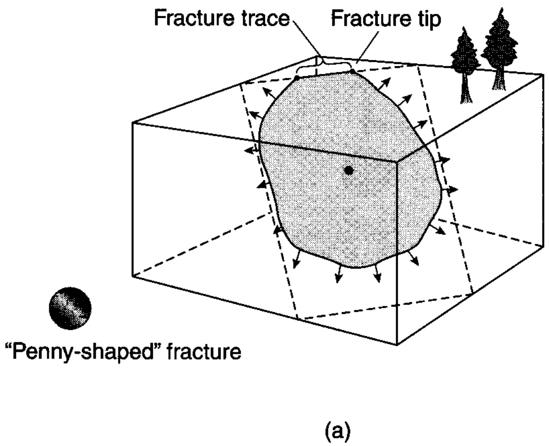
- Surface and body forces
- Tensors, Mohr circles.
- Theoretical strength of materials
- Defects
- Stress concentrations
- Griffith failure criteria

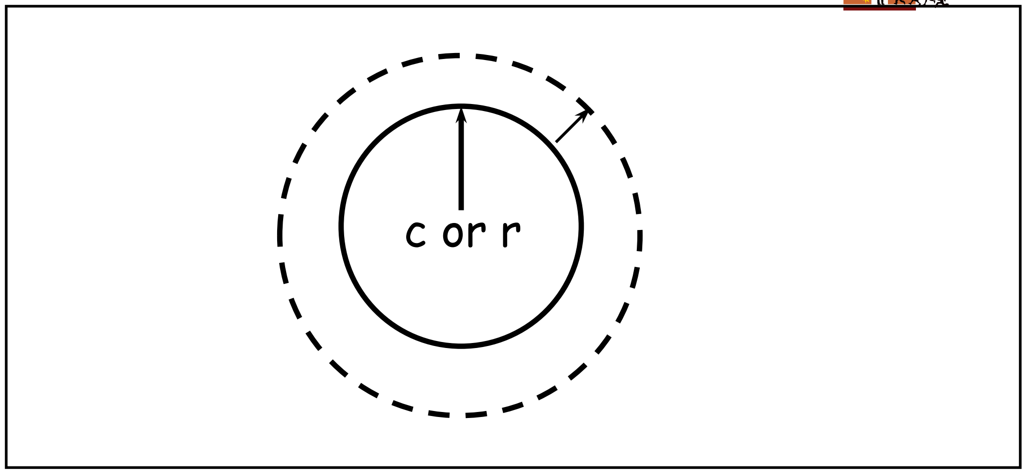
Dislocation model for fracture and earthquake rupture

Dislocation model, circular crack

For an increment of stress ($\Delta\sigma$), how much slip occurs between the crack faces (Δu), and how does that slip vary with position (x, y) and crack radius (c)

$$\Delta u(x,y) = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{c^2-\left(x^2+y^2\right)}$$



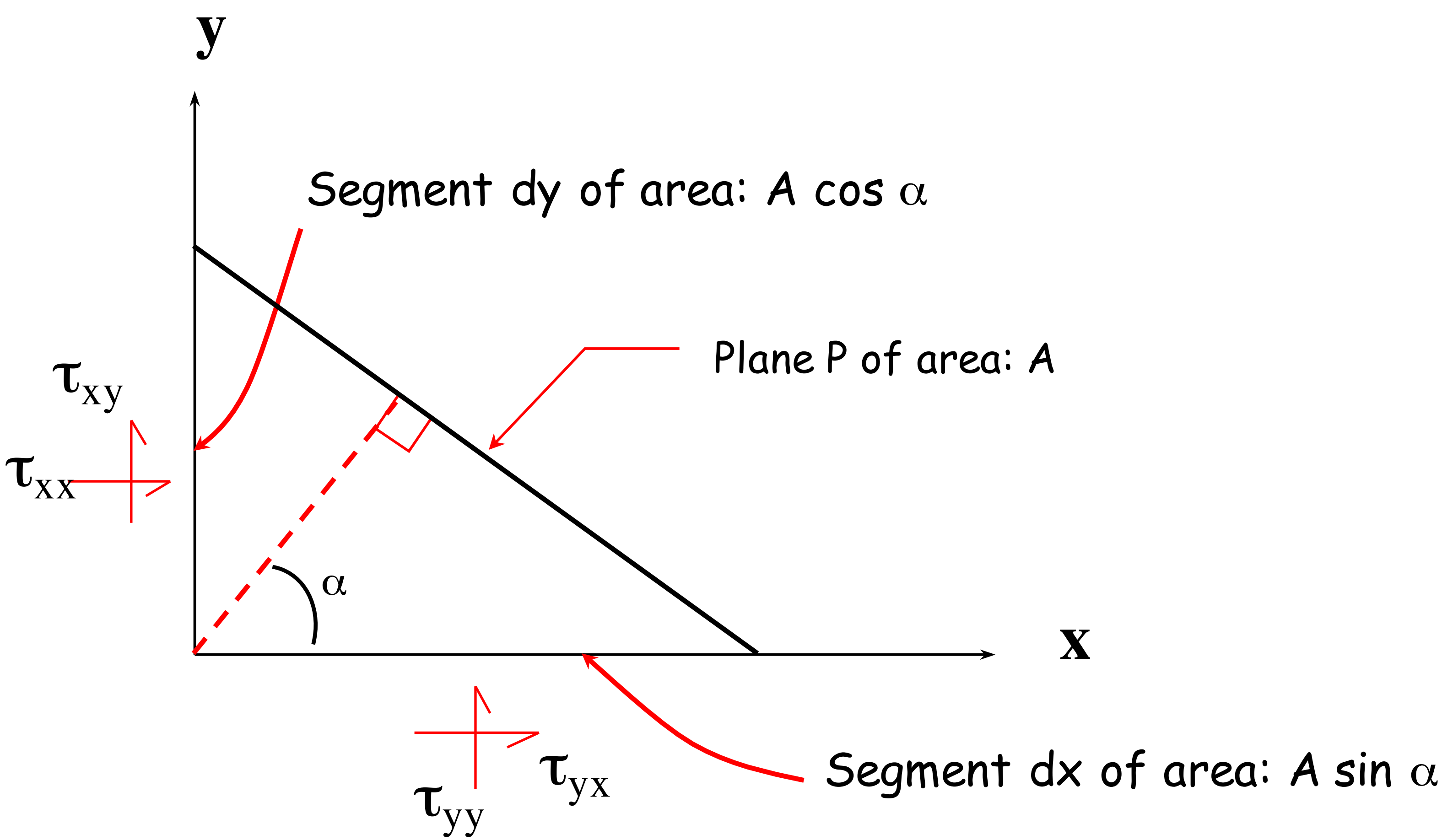


Relation between stress drop and slip for a circular dislocation (crack) with radius r
For ν =0.25, Chinnery (1969)

• Importance of slip: e.g., $M_o = \mu A u$

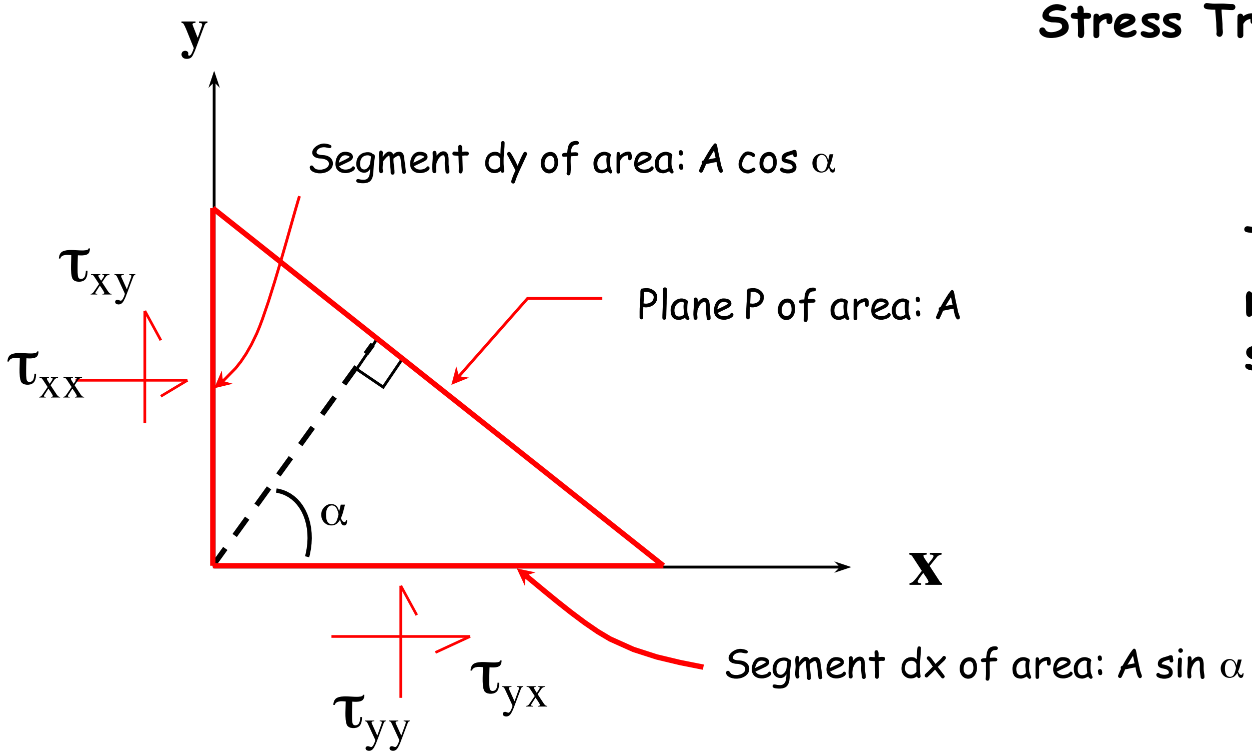
Transformation of Stress From One Coordinate System to Another

- Resolving the applied stress onto a plane, or set of planes, in a different orientation



The forces on plane A must balance those on segments dx and dy

Stress Transformation



The forces on plane A must balance those on segments dx and dy

The force in a direction normal to P (σ A) has contributions from each of the four stress components:

- Force = Stress x Area
- 1) the shear force along dx is and it's component normal to P is τ_{yx} A sin α cos α
- 2) the normal force along dx is and it's component normal to P is τ_{yy} A sin α sin α
- 3) the shear force along dy is and it's component normal to P is τ_{xy} A cos α sin α
- 4) the normal force along dy is and it's component normal to P is τ_{xx} A cos α cos α

Force Normal to P:

$A\sigma = \tau_{yx} \quad + \tau_{yy} \quad + \tau_{xy} \quad + \tau_{xx}$

Stress Transformation

Force Normal to P:

$$A\sigma = \tau_{yx} A \sin\alpha \cos\alpha + \tau_{yy} A \sin\alpha \sin\alpha + \tau_{xy} A \cos\alpha \sin\alpha + \tau_{xx} A \cos\alpha \cos\alpha$$

This can be simplified by eliminating A, using $\tau_{xy} = \tau_{yx}$ and using the identity $2 \sin\alpha \cos\alpha = \sin 2\alpha$

Normal Stress on Plane P:

$$\sigma = \tau_{xx} \cos^2\alpha + \tau_{xy} \sin 2\alpha + \tau_{yy} \sin^2\alpha$$

Shear Force on P:

$$A\tau = \tau_{yx} \qquad - \tau_{yy} \qquad - \tau_{xy} \qquad + \tau_{xx}$$

This can be simplified to:

Shear Stress on Plane P:

$$\tau = (\tau_{xx} - \tau_{yy}) \cos\alpha \sin\alpha + \tau_{xy} (\sin^2\alpha - \cos^2\alpha)$$

- Stress components are a function of coordinate frame and orientation
- Principal Stresses
  - Shear stresses vanish, only normal stresses
  - By convention, maximum principal stress is σ_1 and $\sigma_1 > \sigma_2 > \sigma_3$, compression is positive

in 2D

τ_{xx}, 0

0, τ_{yy},

Stress Transformation

Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of P**rincipal Stresses**:

$\sigma = \tau_{xx} \cos^2 \alpha + \tau_{xy} \sin 2\alpha + \tau_{yy} \sin^2 \alpha$

$\tau = (\tau_{xx} - \tau_{yy}) \cos \alpha \sin \alpha + \tau_{xy} (\sin^2 \alpha - \cos^2 \alpha)$

2D

τ_{xx}, 0

0, τ_{yy},

$\sigma = \tau_{xx} \qquad + \tau_{yy}$

$\tau = (\tau_{xx} - \tau_{yy})$

$\sigma = \sigma_1 \cos^2 \alpha + \sigma_2 \sin^2 \alpha$, Normal Stress

$\tau = (\sigma_1 - \sigma_2) \cos \alpha \sin \alpha$, Shear Stress

Use trig. identities such as $\cos 2\alpha = 1 - 2 \sin^2\alpha$ and $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

Note that these relations make use of the **mean stress** and the **differential stress**

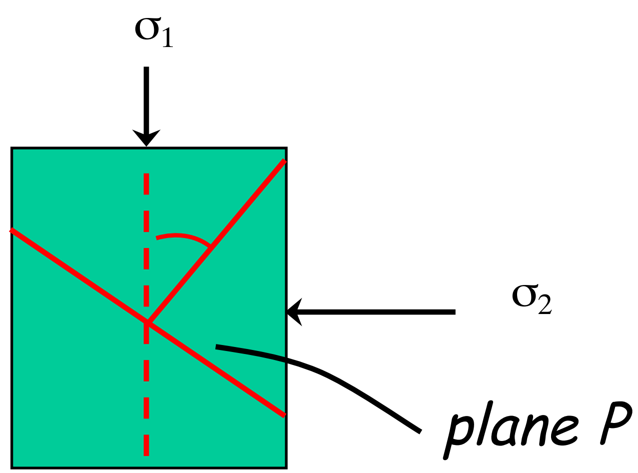
$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

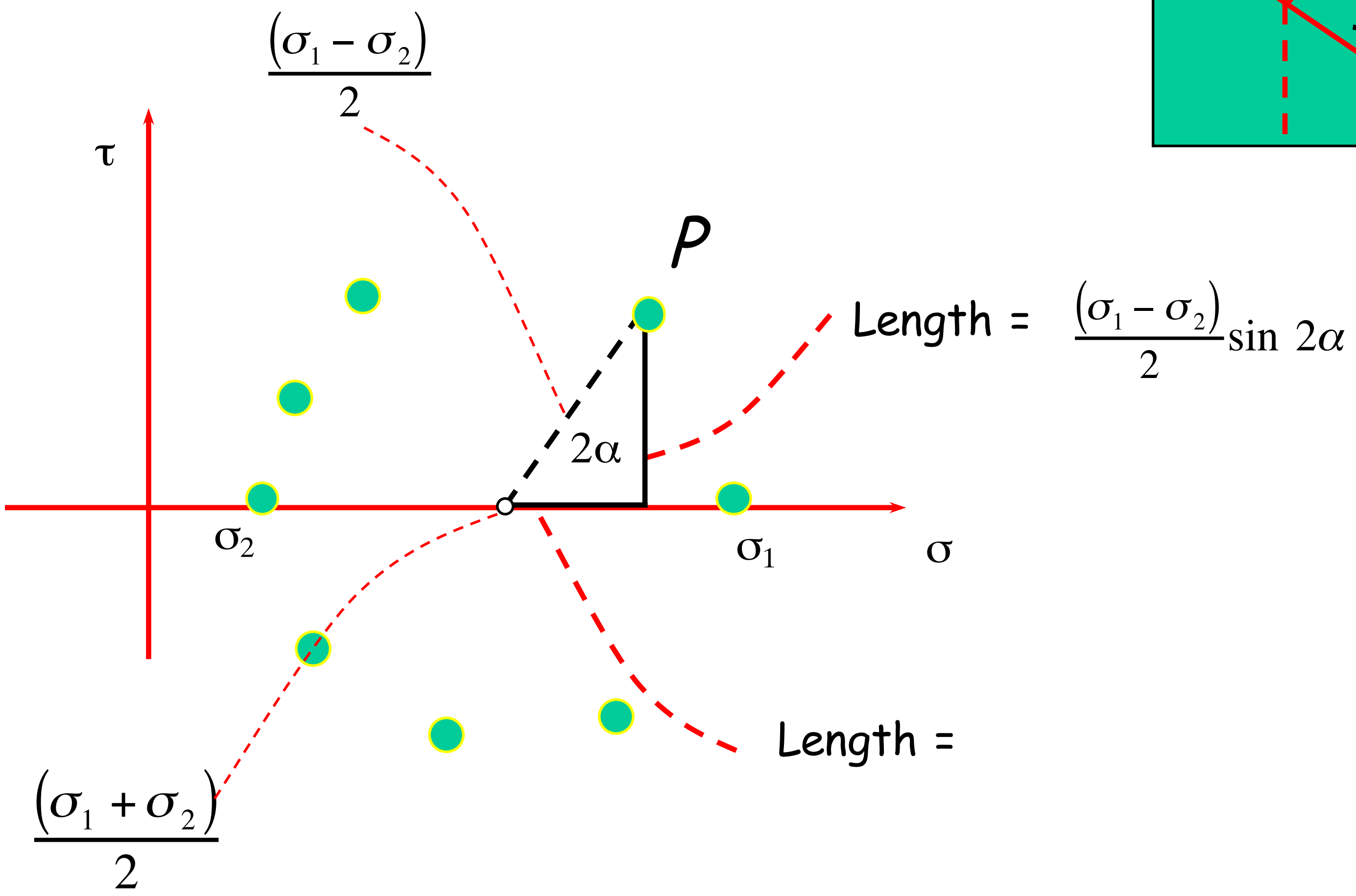
Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of **Principal Stresses**:

Mohr Circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$





Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of **Principal Stresses**:

Mohr Circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

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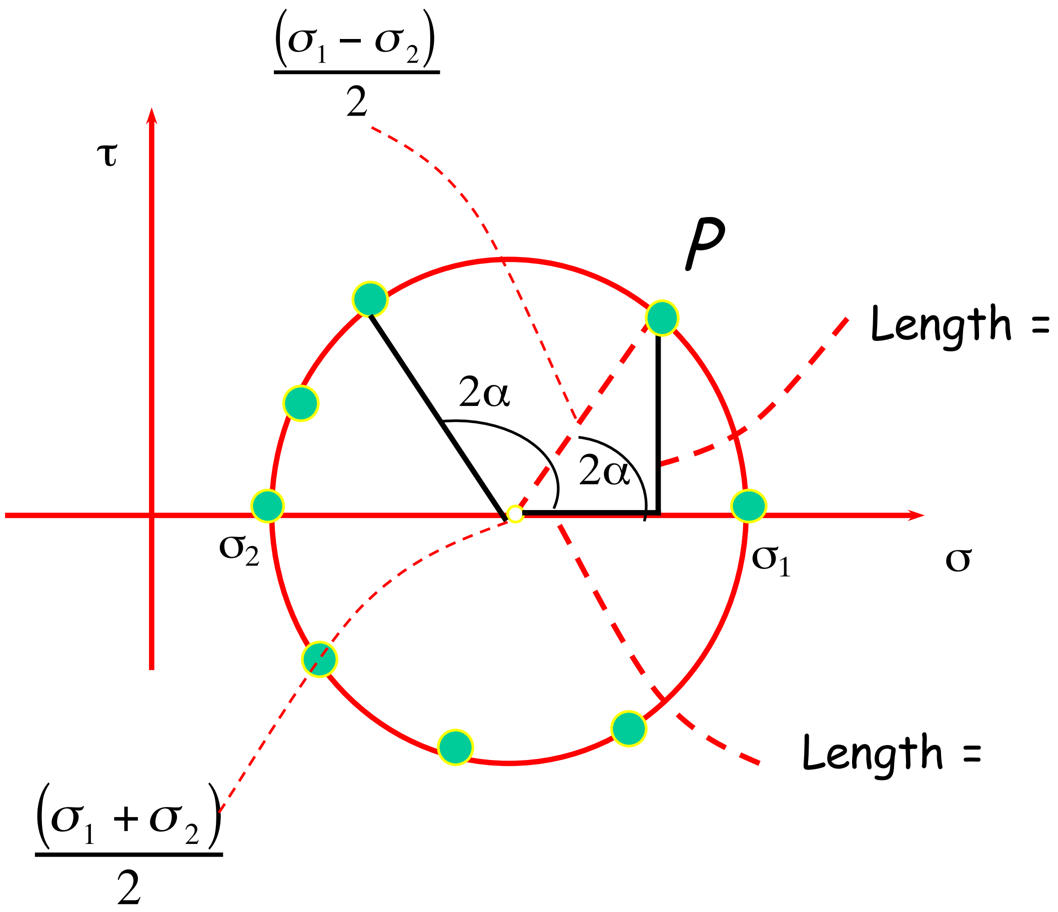
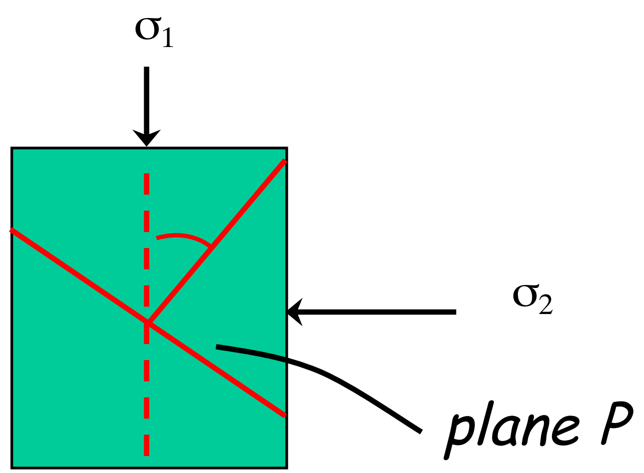
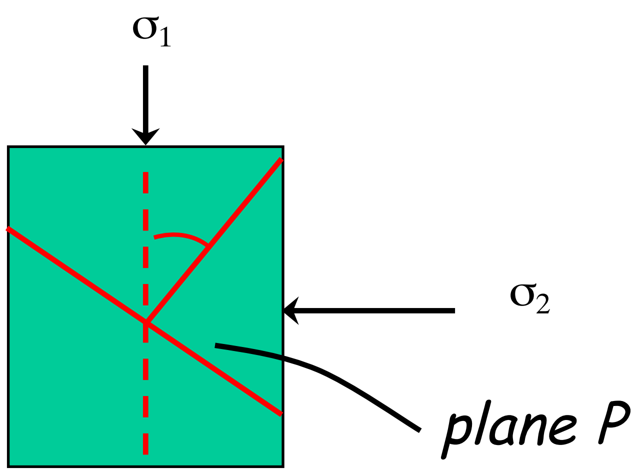
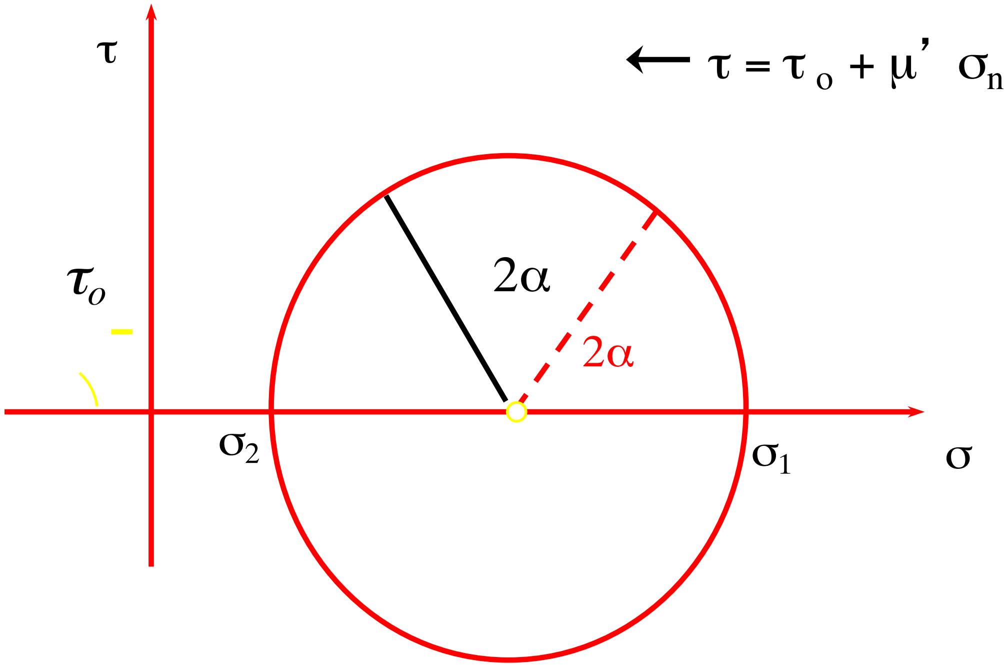




Coulomb-Mohr Failure Criterion $\tau = \tau_o + \mu' \sigma_n$

where τ is shear stress τ_o is 'cohesion,' μ' is the coefficient of internal friction and σ_n is normal stress





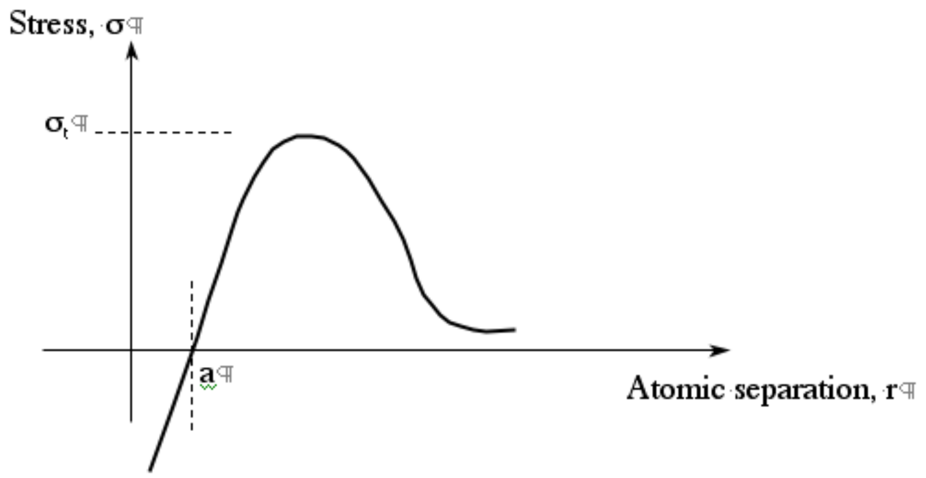
The parameter μ' describes the effect of normal stress on shear strength.

Pressure-dependent brittle failure

Failure stress is higher for things under higher normal stress.

Theoretical strength of materials

- Defects
- Stress concentrations
- Griffith failure criteria
- Energy balance for crack propagation
- Stress intensity factor

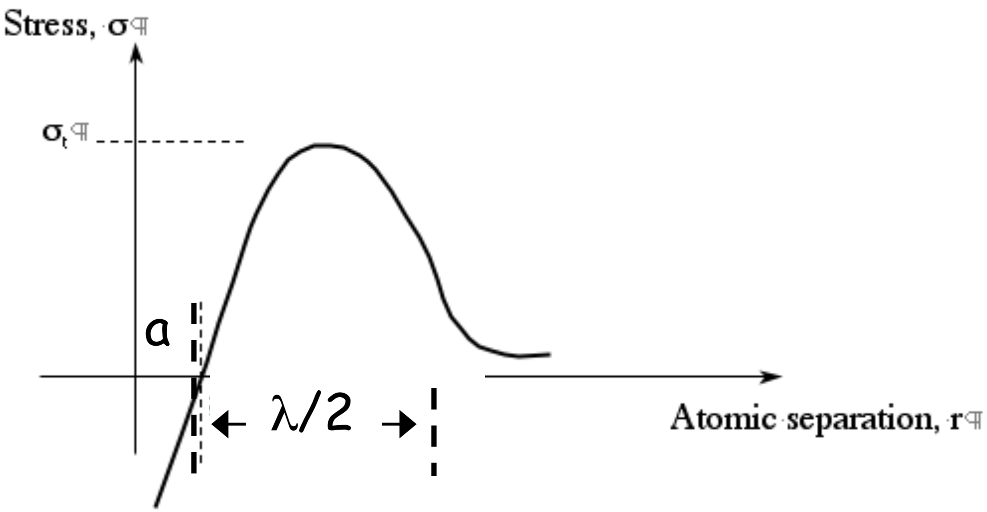


Start by thinking about the theoretical strength of materials –and take crystals as a start. The strength of rocks and other polycrystalline materials will also depend on cementation strength and grain geometry so these will be more complex.

$$\sigma = \sigma_t \sin\left(\frac{2\pi(r-a)}{\lambda}\right),$$

$$\frac{d\sigma}{d(r-a)} = \frac{E}{a} = \frac{2\pi}{\lambda}\sigma_t \cos\left(\frac{2\pi(r-a)}{\lambda}\right),$$

Since $\frac{(r-a)}{\lambda} << 1$, we can re-write this as: $\sigma_t = \frac{E\lambda}{2\pi a}$



Theoretical strength, σ_t, of simple crystals:
Bonds must break along a lattice plane

Consider a tensional stress field, and take *a* as the equilibrium lattice spacing. Approximate the region around the peak strength as a sinusoid, wavelength λ
Then, for small changes in lattice spacing: the rate of stress change is related to E.

$$\sigma = \sigma_t \sin\left(\frac{2\pi(r-a)}{\lambda}\right),$$

$$\frac{d\sigma}{d(r-a)} = \frac{E}{a} = \frac{2\pi}{\lambda}\sigma_t \cos\left(\frac{2\pi(r-a)}{\lambda}\right),$$

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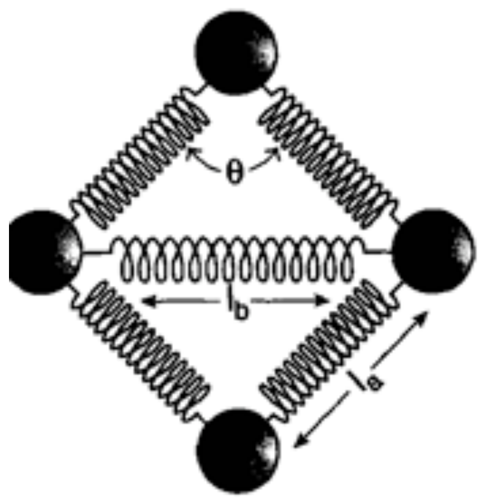
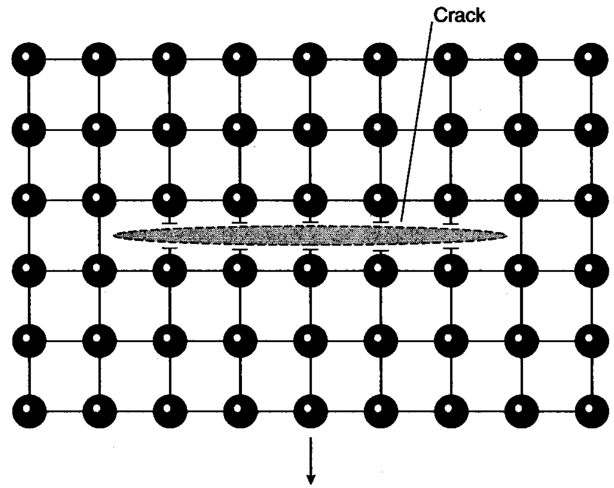


Tensile Strength of single x' l, by our approximation: $$\sigma_t = \frac{E\lambda}{2\pi a}$$

The strain energy and stress is zero at thermodynamic equilibrium, which occurs at r= 3a/2 and since a ≈ λ, the theoretical strength is about E/2π. (See Scholz, Ch. 1.1 for additional details).

$$\sigma_t \approx \frac{E}{2\pi}$$



•This type of calculation was carried out in the early 1900's and people immediately realized that there was a problem.

•Experiments showed that E was on the order of 10's of GPa, whereas the tensile strength of most materials is closer to 10's of MPa.

•Griffith proposed a solution in two classic papers in the early 1920's –but the proof of his ideas had to wait until the invention of the electron microscope.

Bottom line: Defects.

Defects severely reduce the strength of brittle materials relative to the theoretical estimate. Flaws exist at all scales from atomic to the specimen size (laboratory sample size or continent scale, in the case of plate tectonics)

Stress concentrations around defects cause the local stress to reach the theoretical strength.

Two types of defects cause two types of deformation:

- cracks and crack propagation lead to brittle deformation;
- dislocations and other types of atomic misregistration lead to plastic flow and 'ductile' deformation.

Brittle deformation generally leads to catastrophic failure and separation of lattice elements.

Plastic flow produces permanent deformation without loss of lattice integrity.

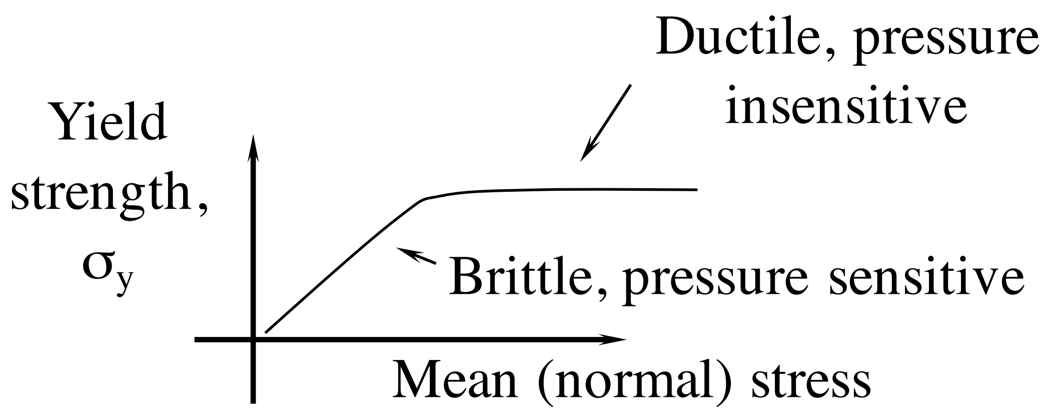
Scholz generalizes these modes of deformation to make a connection with lithospheric deformation.

The upper lithosphere deforms primarily by brittle mechanisms and can be referred to as the **schizosphere** (lit. the broken part), whereas

the lower lithosphere deforms by ductile mechanisms and can be classified as the **plastosphere**.

Rheology and Deformation. Definitions.

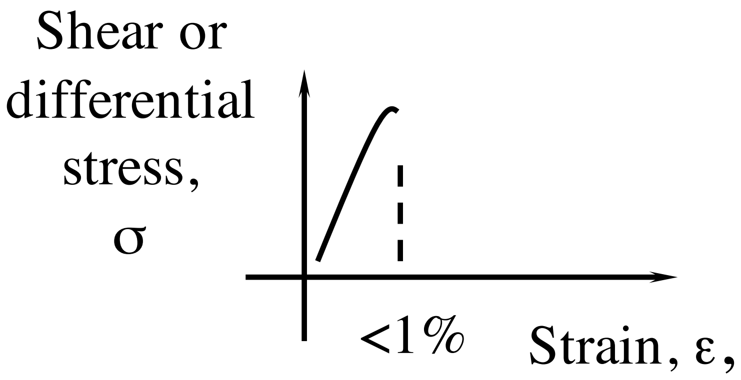
The terms brittle and ductile can be defined in a number of ways. One def. is given above. Another important operational definition involves the stress-strain characteristics and the dependence of strength on mean (or normal stress).



Brittle and Ductile (or plastic) deformation can be distinguished on the basis of whether the yield strength depends on pressure (mean stress or normal stress).

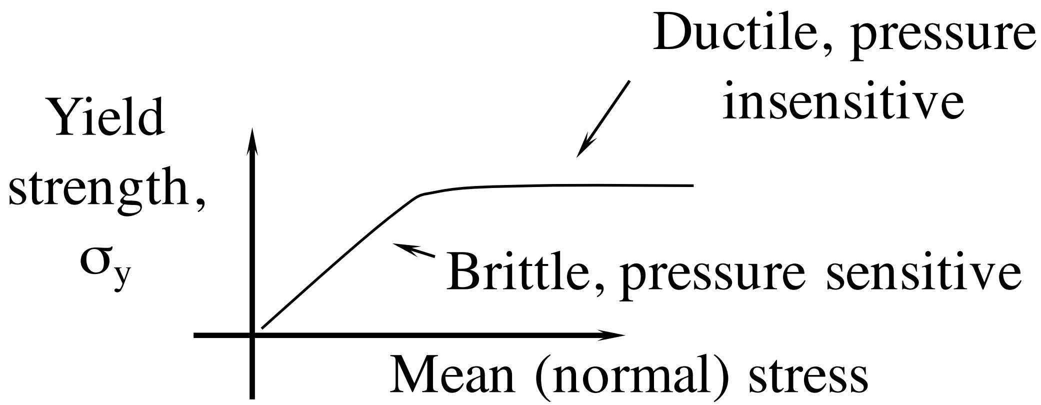
Rheology and Deformation. Definitions.

The term 'brittle' is also used to describe materials that break after very little strain.



Fracture toughness describes a material's ability to deform without breaking.
- Brittle materials (like glass or ceramics) have low toughness.
- Plastics have high toughness

What causes the pressure sensitivity of brittle deformation?



- Volume change. Brittle deformation involves volume change –dilatancy or compaction.
- 'Dilation' means volume increase. Dilatancy describes a shear induced volume increase. The term was introduced to describe deformation of granular materials – but dilation also occurs in solid brittle materials via the propagation of cracks.

- Work is done to increase volume against the mean stress during brittle deformation, thus the pressure sensitivity of brittle deformation.

- Ductile deformation occurs without macroscopic volume change, due to the action of dislocations. Dislocation motion allows strain accommodation.

Stress concentrations around defects.

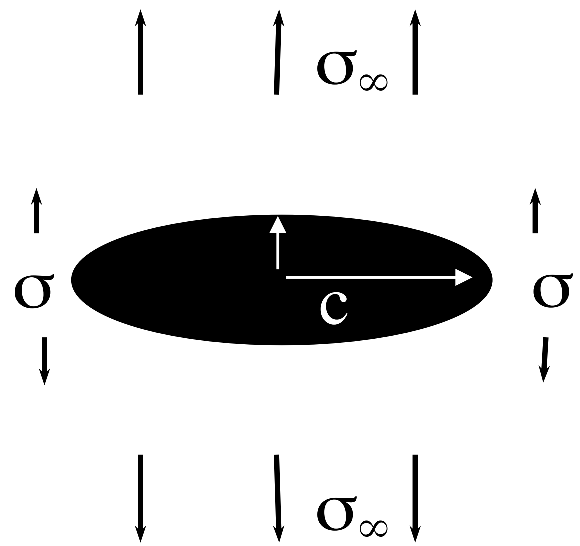
In general, the stress field around cracks and other defects is quite complex, but there are solutions for many special cases and simple geometries

Scholz gives a partial solution for an elliptical hole in a plate subject to remote uniform tensile loading (ρ is the local curvature)

$$\sigma = \sigma_\infty \left(1 + 2\frac{c}{b}\right)$$

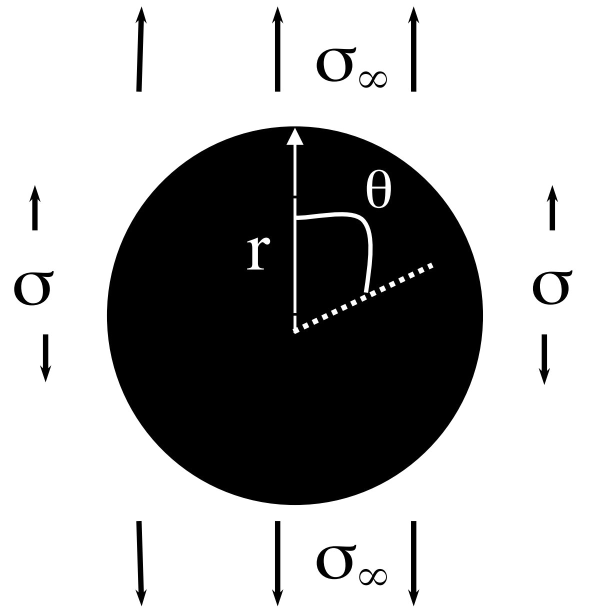
$$\sigma = 2\sigma_\infty \left(1 + 2\sqrt{\frac{c}{\rho}}\right)$$

Crack tip stresses



Malvern (1969) gives a full solution for a circular hole or radius r = a

$$\sigma_{\theta\, Max} = \sigma_{\infty}\left(a,\ \frac{\pi}{2}\right) = 3\sigma_{\infty}$$



Full solution for a circular hole of radius r=a

$$\sigma_r = \frac{\sigma_\infty}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_\infty}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma_\infty}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_\infty}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = -\frac{\sigma_\infty}{2}\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$

Full solution for a circular hole of radius r=a

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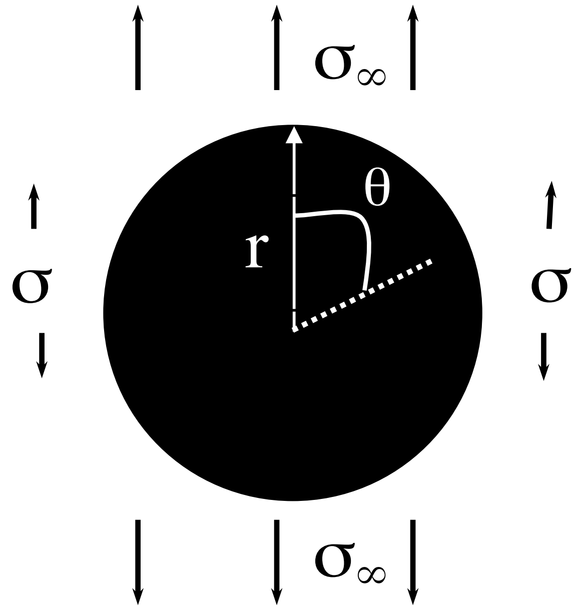
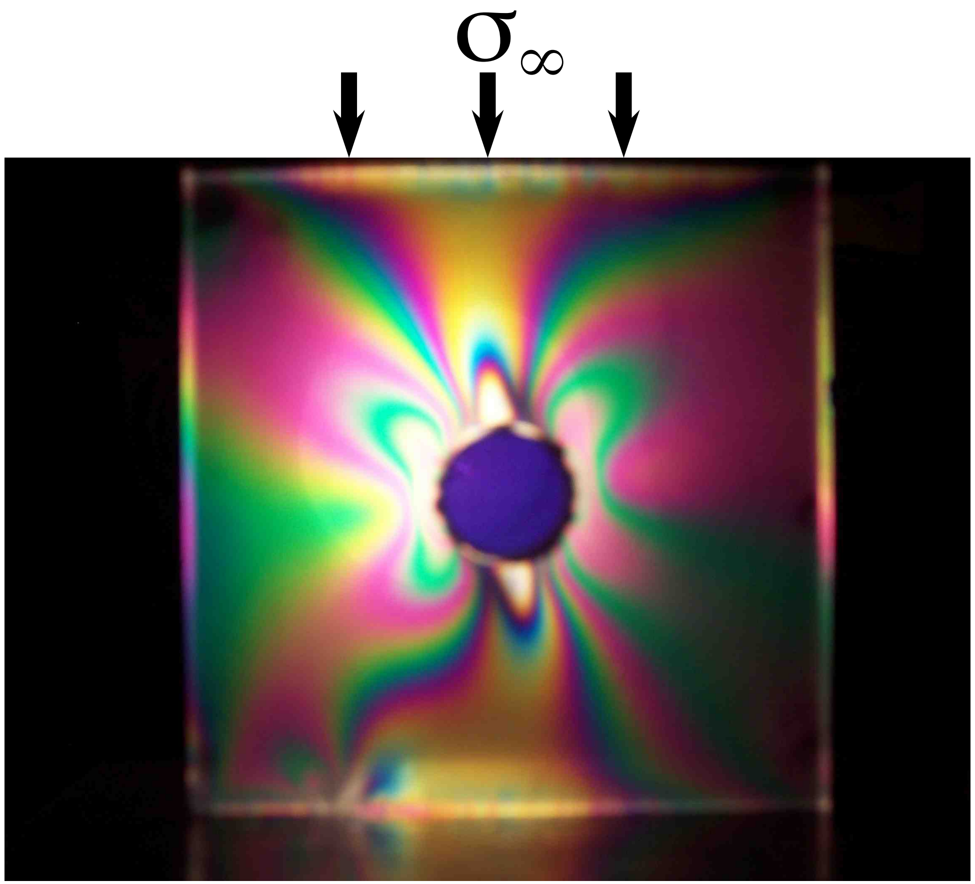




Bond separation and specific surface energy.

- Fracture involves creation of new surface area.
- The specific surface energy is the energy per unit area required to break bonds.

Two surfaces are created by separating the material by a distance $\lambda/2$ and the work per area is given by stress times displacement.

$$2\gamma = \int_0^{\frac{\lambda}{2}} \sigma_t \sin\left(\frac{2\pi(r-a)}{\lambda}\right) \; d(r-a)$$

$$= \frac{\lambda\sigma_t}{\pi}$$

This yields the estimate: .

$$\gamma = \frac{Ea}{4\pi^2}$$

The surface energy is a fundamental quantity and we will return to it when we talk about the energy balance for crack propagation and the comparison of laboratory and seismic estimates of G, the fracture energy.

Can crack mechanics help to solve, quantitatively, the huge discrepancy between the theoretical (~10 GPa) and observed (~10 MPa) values of tensile strength?

For a far field applied stress of σ_∞, we have crack tip stresses of

$$\sigma \approx 2\sigma_\infty \sqrt{\frac{c}{\rho}}$$

Taking σ as σ_t, we can combine the relations for

theoretical strength $\sigma_t = \frac{E\lambda}{2\pi a}$ and surface energy $\gamma = \frac{Ea}{4\pi^2}$

to get:

$$\sigma_t = \sqrt{\frac{E\gamma}{a}}$$

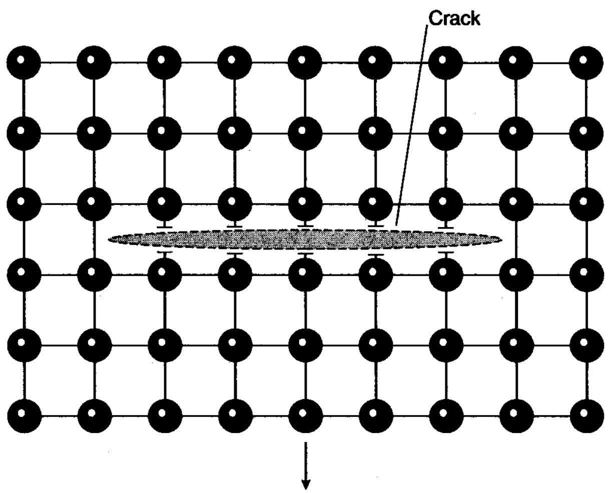
If we take crack radius as approx. equal to *a, the lattice dimension*, then setting σ_t, equal to σ at the crack tip, we have:

$$2\sigma_\infty \sqrt{\frac{c}{a}} = \sqrt{\frac{E\gamma}{a}}, \text{ which yields: } \sigma_\infty = \sqrt{\frac{E\gamma}{4c}}$$

Taking σ_∞ of 10 MPa, E= 10 GPa and γ of 4×10^{-2} J/m^2, gives a crack half length c of 1 micron.

•Griffith proposed that all materials contain preexisting microcracks, and that stress will concentrate at the tips of the microcracks

•The cracks with the largest elliptical ratios will have the highest stress, and this may be locally sufficient to cause bonds to rupture



- As the bonds break, the ellipticity increases, and so does the stress concentration
- The microcrack begins to propagate, and becomes a real crack
- Today, microcracks and other flaws, such as pores or grain boundary defects, are known as Griffith defects in his honor