Mechanics of Earthquakes and Faulting

www.geosc.psu.edu/Courses/Geosc508

- Overview
- Milestones in continuum mechanics
- Concepts of modulus and stiffness.
- Stress-strain relations
- Elasticity
- Surface and body forces
- Tensors, Mohr circles.
- Theoretical strength of materials
- Defects
- Stress concentrations
- Griffith failure criteria

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Overview

- Course expectations
- Web site
- Lecture Materials: Should you print them every time? What happens when we don't finish the materials in a given set of lecture slides? Should I take notes separately or on the slides?
- Book
- Reading
- Problem Sets
- Project
- Presentations

Some Topics in the Mechanics of Earthquakes and Faulting

- What determines the size of an earthquake? That is, why were the 2003 San Simeon & 1994 Northridge events ~ M 6.6, the 1992 Landers event 7.3 and the 2011 Tōhoku event 9.0 ?

- What physical features and factors of faulting control the extent of dynamic earthquake rupture? *--Fault Area, Seismic Moment*

- What is the role of fault geometry (offsets, roughness, thickness) versus rupture dynamics ?

- What controls the amount of slip in an earthquake? *Average Slip, Slip at a point*

- Why is rise time a useful concept in earthquake rupture?

- What's the difference between rupture velocity and particle (slip) velocity?

- What controls whether fault slip occurs dynamically or quasi-statically? *Mechanics of Earthquakes vs. Mechanics of Faulting*

Some Topics in the Mechanics of Earthquakes and Faulting

- What factors determine the stability of frictional sliding? When does *stick-slip occur vs. stable sliding?*

- Nucleation: How does the earthquake process get going?

- What is the size of a nucleation patch at the time that slip becomes dynamic? How do we define dynamic versus quasi-dynamic and quasi-static? *Nucleation patch: physical size, seismic signature*

- What controls dynamic rupture velocity?

- How do faults grow and evolve with time?

- Ductile faulting? What happens at the base of the seismogenic zone?

- Shallow versus intermediate and deep earthquakes.

- Fault complexity and branching;

Continuum Mechanics, Historical. (See Love, 1926).

- 1638: Theory of elasticity starts with Galileo and his work on beams. For a beam extending from a wall, how long can it be before it breaks when loaded by: its own weight, a mass at the end?
- 1660: Hooke's law (published as an anagram in 1678: ceiiinosssttuv: *Ut tensio sic vis*)
- 1821: Navier's general equations (of motion for elastic materials) --also known by Cauchy's name.
- 1860: Young (Lord Kelvin) Concept of modulus introduced.

Hooke's law in simple form:

F = k x, where F is force, k is stiffness and x is displacement.

This was later generalized to $\sigma = E\,\varepsilon$, where σ is stress, E is Young's modulus and ε is linear strain.

In Hooke's time the generality of strain was not understood in terms of linear and shear components. Strain was simply referred to as "tension," probably reflecting the difficulty of separating the application of stress and strain in the laboratory.

Continuum Mechanics.

200 years later, Young posited the notion of modulus –*in a way that made it seem to have dropped from the sky.*

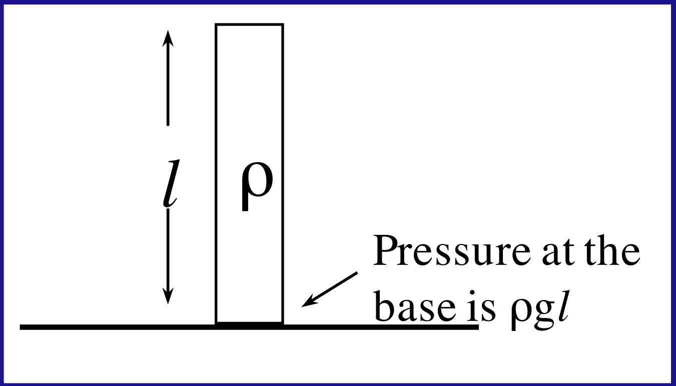
In Young's words: modulus is defined by the following statement.

…a column of the same substance capable of producing a pressure at its base which is to the weight causing a certain degree of compression, as the length of the substance is to the diminution of its length.

$\sigma = F/A$

$\varepsilon = dl/L$

$E = \sigma / \varepsilon$



It's a statement that you have to read over a *few* times in order to get the gist of…

But with these words, Young introduced for the first time a **definite physical concept associated with the coefficient of elasticity.**

Continuum Mechanics.

Think about how modulus differs from stiffness. Consider a simple experiment.

- Consider two identical springs of equal length *l*.

  Cut one in two, so that it's sections are of length *l*/2.

  Although each of the three sections are of identical material, the longer one will deform to a greater extent under a given load.

The same could be said for any material –including lengths of granite and columns of concrete. **Stiffness is a useful concept, but it is not a material property.**

Seismic moment, the passage of elastic waves, the strain field around a fault, and the velocity of a propagating rupture all depend on modulus, not stiffness.

To make things confusing, we sometimes refer to a generalized modulus as the 'stiffness' tensor, as in:

$\sigma_{ij} = c_{ijkl}\,\varepsilon_{kl}$, C is the stiffness tensor, σ is stress and ε is strain.

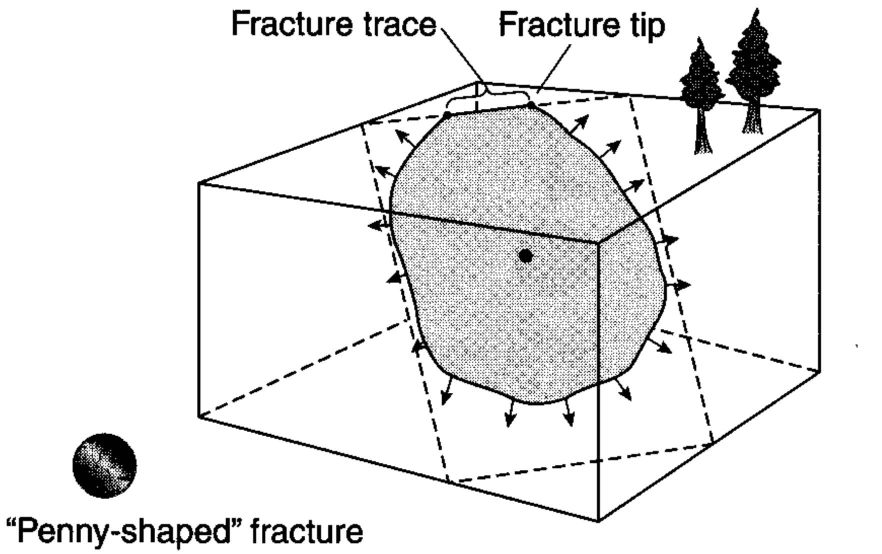
But this is not the same as Hooke's stiffness: $F = k\,x$, where k is a spring constant.

Why elasticity and continuum mechanics? What can we do with it in Fault Mechanics and Earthquake Physics?

Crack mechanics, e.g.:
What is the relationship between applied stress, expansion of the crack, and slip between the crack faces?

$$\sigma_{ij} = c_{ijkl}\,\varepsilon_{kl}$$

σ is stress and ε is strain.



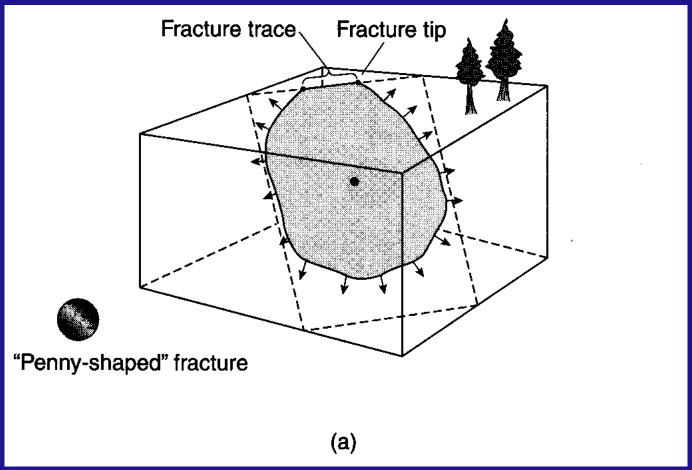
(a)

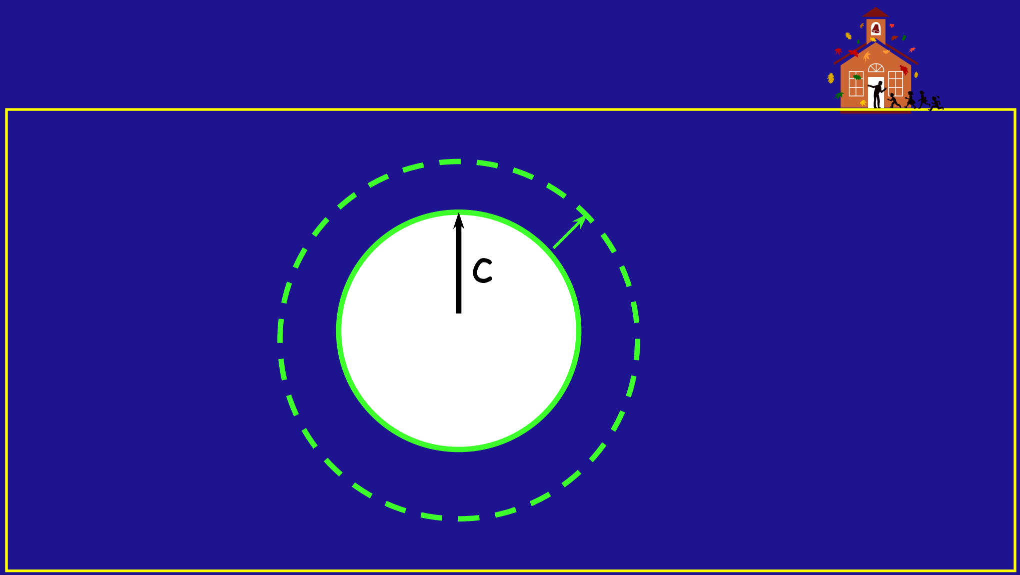
Dislocation model for fracture and earthquake rupture

Dislocation model, circular crack

For an increment of stress ($\Delta\sigma$), how much slip occurs between the crack faces (Δu), and how does that slip vary with position (x, y) and crack radius (c)

$$\Delta u(x,y) = \frac{24}{7\pi}\,\frac{\Delta\sigma}{\mu}\sqrt{c^2 - \left(x^2 + y^2\right)}$$





Relation between stress drop and slip for a circular dislocation (crack) with radius r
For ν =0.25, Chinnery (1969)

- Importance of slip: e.g., $M_o = \mu A u$

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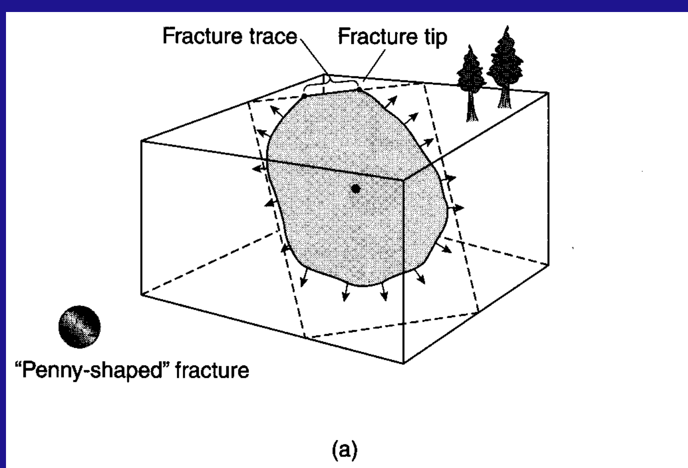
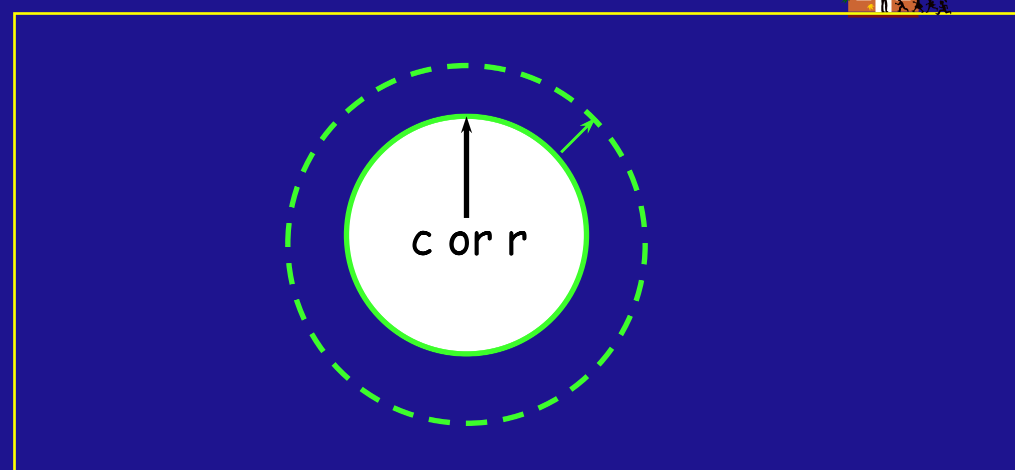




$$\Delta\sigma = \frac{7\pi}{24} G \frac{\Delta u_{max}}{r}$$

$$\Delta\sigma = \frac{7\pi}{16} G \frac{\Delta\bar{u}}{r}$$

Relation between stress drop and slip for a circular dislocation (crack) with radius r

For ν =0.25, Chinnery (1969)

- Importance of slip: e.g., $M_o = \mu A u$

Body forces act on every mass element of a body.

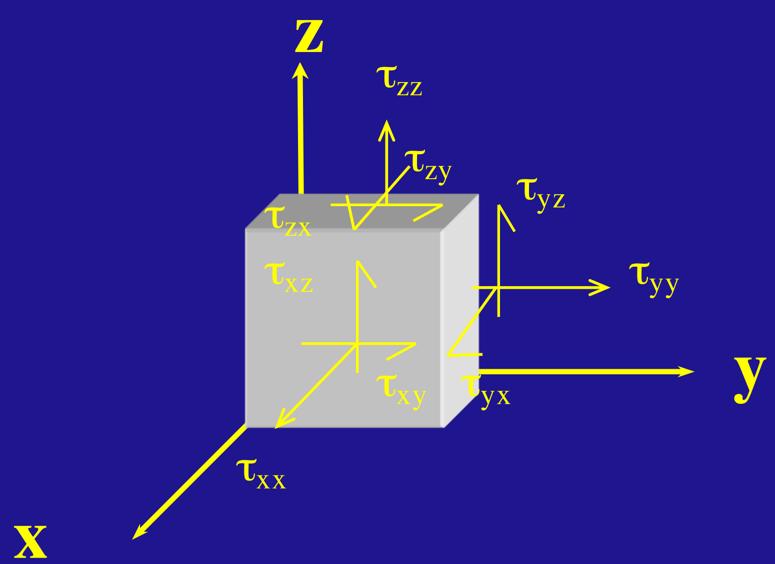
Surface forces, or tractions, act only along boundaries of a body.

Stress and Transformation of Stress (From One Coordinate System to Another)

In general we have 9 components of stress in 3d; and six of these are independent.

Why only 6 independent?

Need 9 components to fully specify the stress state.
These components make up a tensor.
Stress is a 2nd rank tensor.
Vector is a 1st rank tensor
Scalar is a 0th rank tensor



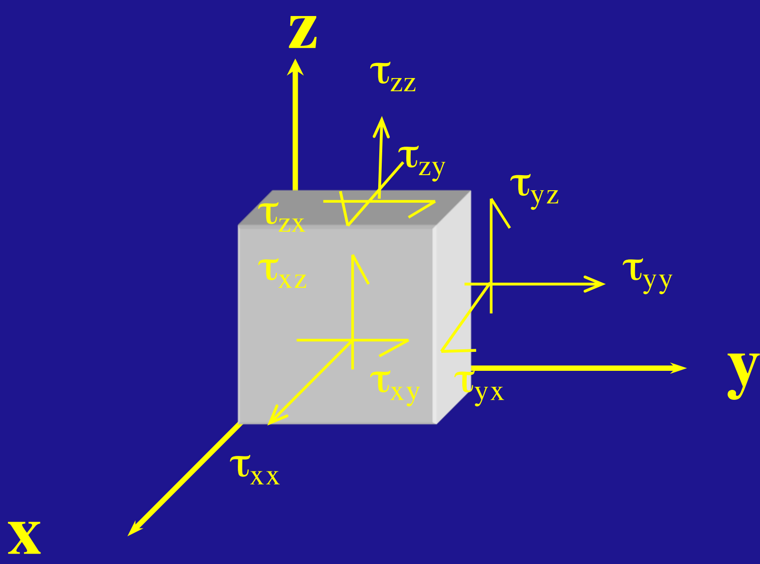
Nine components of the stress tensor

$$\tau_{xx}, \tau_{xy}, \tau_{xz}$$

$$\tau_{yx}, \tau_{yy}, \tau_{yz}$$

$$\tau_{zx}, \tau_{zy}, \tau_{zz}$$

Right-handed cartesian system and a cube of dimensions *dx, dy, dz*



Nine components of the stress tensor

$$\tau_{xx},\ \tau_{xy},\ \tau_{xz}$$
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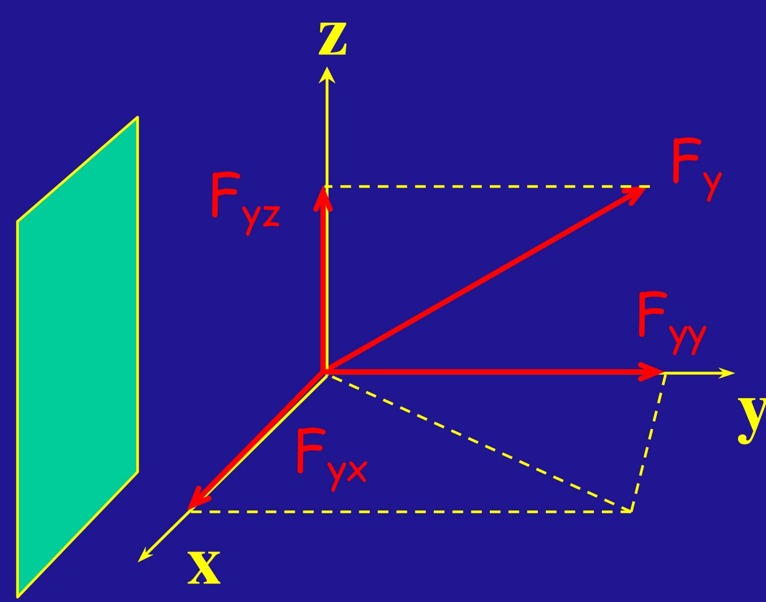
$$\tau_{zx},\ \tau_{zy},\ \tau_{zz}$$

Convention: first index refers to plane (face perpendicular to that axis), second index refers to resolved direction of force, τ_{yx}, τ_{12}

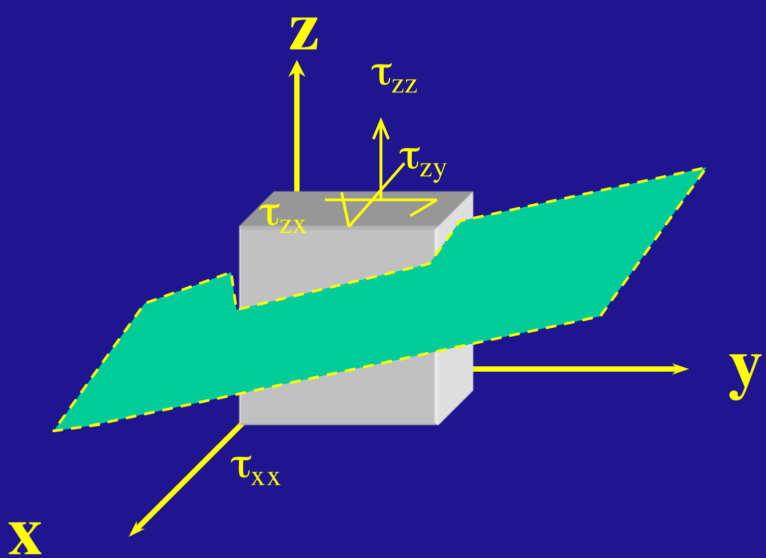
We can apply an independent force to each of the surfaces.

F_y is the force on the surface perpendicular to the y face. Force is a vector, so it can be decomposed into it's components in the x, y, and z directions.



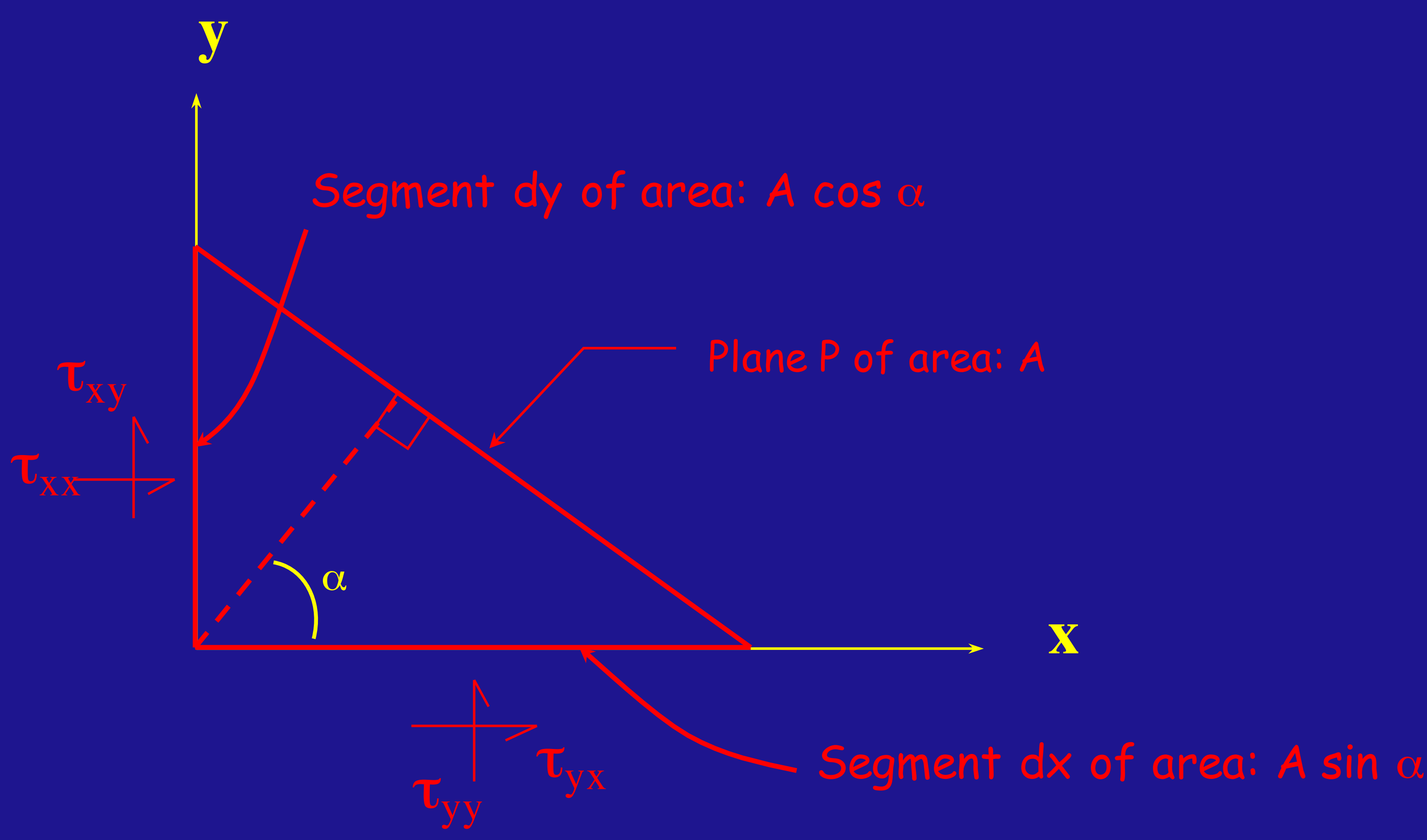
Transformation of Stress From One Coordinate System to Another

- Resolving the applied stress onto another plane, or set of planes



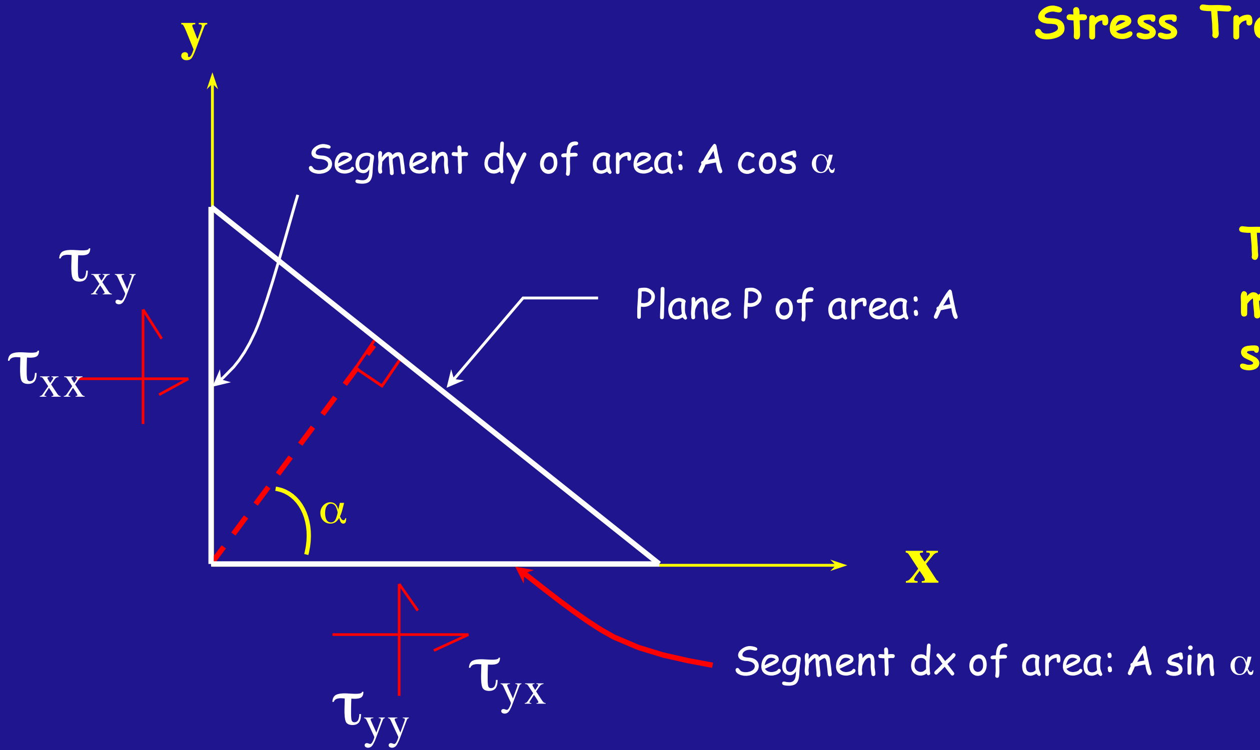
Transformation of Stress From One Coordinate System to Another

- Resolving the applied stress onto a plane, or set of planes, in a different orientation



The forces on plane A must balance those on segments dx and dy

Stress Transformation



The forces on plane A must balance those on segments dx and dy

The force in a direction normal to P (σ A) has contributions from each of the four stress components:

- Force = Stress x Area
- 1) the shear force along dx is τ_{yx} A sin α and it's component normal to P is τ_{yx} A sin α cos α
- 2) the normal force along dx is τ_{yy} A sin α and it's component normal to P is τ_{yy} A sin α sin α
- 3) the shear force along dy is τ_{xy} A cos α and it's component normal to P is τ_{xy} A cos α sin α
- 4) the normal force along dy is τ_{xx} A cos α and it's component normal to P is τ_{xx} A cos α cos α

Force Normal to P:

$$A\sigma = \tau_{yx} A \sin\alpha \cos\alpha + \tau_{yy} A \sin\alpha \sin\alpha + \tau_{xy} A \cos\alpha \sin\alpha + \tau_{xx} A \cos\alpha \cos\alpha$$

Stress Transformation

Force Normal to P:

$$A\sigma = \tau_{yx} A \sin\alpha \cos\alpha + \tau_{yy} A \sin\alpha \sin\alpha + \tau_{xy} A \cos\alpha \sin\alpha + \tau_{xx} A \cos\alpha \cos\alpha$$

This can be simplified by eliminating A, and using $\tau_{xy} = \tau_{yx}$ and using the identity $2 \sin\alpha \cos\alpha = \sin 2\alpha$

Normal Stress on Plane P:

$$\sigma = \tau_{xx} \cos^2\alpha + \tau_{xy} \sin 2\alpha + \tau_{yy} \sin^2\alpha$$

Shear Force on P:

$$A\tau = \tau_{yx} A \sin\alpha \sin\alpha - \tau_{yy} A \sin\alpha \cos\alpha - \tau_{xy} A \cos\alpha \cos\alpha + \tau_{xx} A \cos\alpha \sin\alpha$$

This can be simplified to:

Shear Stress on Plane P:

$$\tau = (\tau_{xx} - \tau_{yy}) \cos\alpha \sin\alpha + \tau_{xy} (\sin^2\alpha - \cos^2\alpha)$$

- Stress components are a function of coordinate frame and orientation
- Principal Stresses
  - Shear stresses vanish, only normal stresses
  - By convention, maximum principal stress is σ_1 and $\sigma_1 > \sigma_2 > \sigma_3$, compression is positive

in 2D

τ_{xx}, 0

0, τ_{yy},

Stress Transformation

Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of Principal Stresses:

$$\sigma = \tau_{xx} \cos^2 \alpha + \tau_{xy} \sin 2\alpha + \tau_{yy} \sin^2 \alpha$$
$$\tau = (\tau_{xx} - \tau_{yy}) \cos \alpha \sin \alpha + \tau_{xy} (\sin^2 \alpha - \cos^2 \alpha)$$

2D

τ_{xx}, 0

0, τ_{yy},

$\sigma = \tau_{xx} \cos^2 \alpha + \tau_{yy} \sin^2 \alpha$ $\quad$ $\sigma = \sigma_1 \cos^2 \alpha + \sigma_2 \sin^2 \alpha$, $\quad$ Normal Stress

$\tau = (\tau_{xx} - \tau_{yy}) \cos \alpha \sin \alpha$ $\quad$ $\tau = (\sigma_1 - \sigma_2) \cos \alpha \sin \alpha$, $\quad$ Shear Stress

Use trig. identities such as $\cos 2\alpha = 1 - 2 \sin^2\alpha$ and $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$

Note that these relations make use of the mean stress and the differential stress

Shear and Normal Stress on a Plane of Arbitrary Orientation --written in terms of Principal Stresses:

Mohr Circle.

$$\sigma = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2}\cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2} sin(2\alpha)$$