



## RESEARCH LETTER

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## Key Points:

- The relationship between  $b$  value and stress is verified and calibrated
- Subduction zone  $b$  values correlate with plate tectonic forces
- The  $b$  value can be used to map variations in stress

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On the stress dependence of the earthquake  $b$  valueChristopher H. Scholz<sup>1</sup><sup>1</sup>Lamont-Doherty Earth Observatory, Columbia University, Palisades, New York, USA

**Abstract** Laboratory experiments have shown that the  $b$  value in the size distribution of acoustic emission events decreases linearly with differential stress. There have been a number of observations that indicate that this relation may also hold for earthquakes. Here using a simple frictional strength model for stresses in the continental lithosphere combined with earthquake  $b$  values measured as a function of depth in a wide variety of tectonic regions, we verify and calibrate that relation, finding  $b = 1.23 \pm 0.06 - (0.0012 \pm 0.0003)(\sigma_1 - \sigma_3)$ , where the stress difference  $(\sigma_1 - \sigma_3)$  is in megapascal. For subduction zones, we find that  $b$  value correlates linearly with the slab pull force and with the net reduction of plate interface normal force, both of which also indicate a negative linear relation between  $b$  value and differential stress.

## 1. Introduction

Earthquakes obey a power law size distribution, which when described in terms of magnitude is given by  $\log N = a - bM$ , where  $N$  is the number of earthquakes greater or equal to magnitude  $M$ , and  $a$  and  $b$  are the constants [Gutenberg and Richter, 1944]. The parameter  $a$  describes the total number of earthquakes, and the parameter  $b$ , often referred to as the  $b$  value, describes their relative size distribution.

Acoustic emissions from microfracturing in rock fracture experiments were found to obey the same form of size distribution, and further, that  $b$  value decreases linearly with increasing differential stress  $(\sigma_1 - \sigma_3)$  [Scholz, 1968]. This result on the stress dependence of  $b$  value in rock fracture experiments was reproduced more recently by Amitrano [2003], and the same stress dependence was found in stick-slip friction experiments by Goebel *et al.* [2013]. Scholz [1968] interpreted this behavior in terms of a positive influence of stress on the transition probability of incremental fracture growth leading to an increased probability of larger fractures with larger stress.

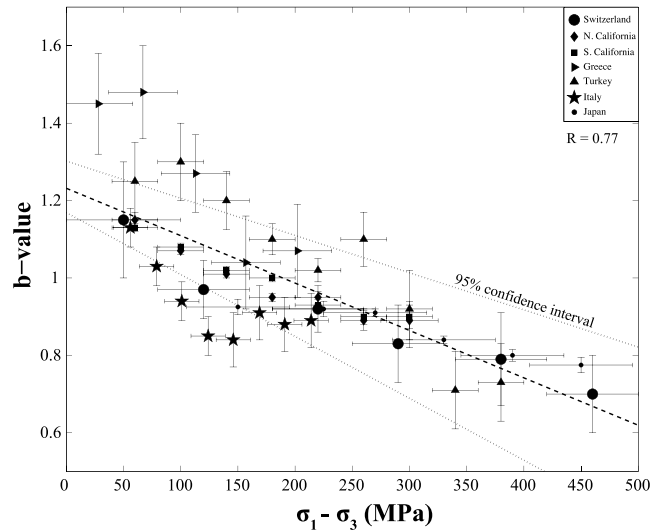
There have been a number of studies that suggest that the same stress dependence applies to  $b$  values of earthquakes. It has been found that the  $b$  value for earthquakes in the continental crust decreases approximately linear with depth [Mori and Abercrombie, 1997; Spada *et al.*, 2013]. It has also been found that  $b$  value depends systematically on earthquake focal mechanism, being smaller for thrust than for normal faulting events and having an intermediate value for strike-slip earthquakes [Gulia and Wiemer, 2010; Schorlemmer *et al.*, 2005].

Subduction zones are tectonic areas where there are a great variety of tectonic styles [Uyeda, 1982]. These have long been suggested to be due to large variations in tectonic stress owing to local variations in the plate tectonic driving forces [e.g., Ruff and Kanamori, 1980]. Nishikawa and Ide [2014] have recently investigated variations in  $b$  value among subduction zones. They found a correlation between  $b$  value and the age of the subducted lithosphere, which is related to one of the main driving forces of subduction zones.

This note is meant to provide some clarification as to how these observations relate to the dependence of  $b$  value on stress.

2. Stress and  $b$  Values in the Continental Crust

The results of stress measurements in deep boreholes in the continental crust in various tectonic regimes indicate that stress is governed by the frictional strength of preexisting faults with friction coefficients in the range of 0.6–1.0 and with the vertical stress being given by the lithostatic gradient minus a hydrostatic pore pressure gradient [M. D. Zoback and Townend, 2001; M. L. Zoback and Zoback, 2007]. As a simple example, if we assume a friction coefficient of 0.75, a rock density of 2500 kg/m<sup>3</sup>, and hydrostatic pore pressure, we obtain, for compressional regions dominated by thrust faulting, a vertical gradient of differential stress  $(\sigma_1 - \sigma_3)$  of



**Figure 1.** The  $b$  value plotted versus stress difference for a variety of tectonic regions. The  $b$  value data are from Spada et al. [2013] and the stresses computed from the stress model discussed in the text.

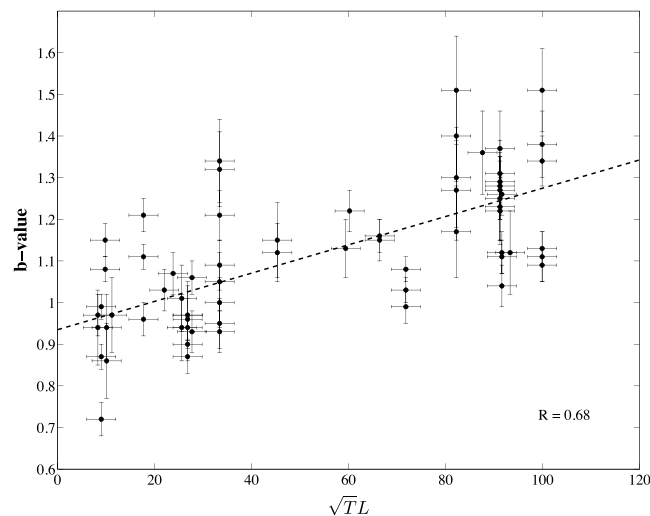
45 MPa/km; for extensional regimes dominated by normal faulting, one of 11.25 MPa/km; and for a strike-slip-dominated region, one of about 20 MPa/km. If  $b$  value decreases with differential stress as in the experimental results, this simple model can explain both the depth dependence of  $b$  value and the focal mechanism dependence described above.

Spada et al. [2013] presented results for  $b$  value as a function of depth for a number of tectonic regions. They found that  $b$  value decreases down to a depth, which they identify as the brittle-ductile transition, below which the  $b$  value was observed to increase dramatically (note that this interpretation agrees with the observation of Scholz [1968], who found much higher  $b$  values in the deformation of ductile

than of brittle rocks). Spada et al. also noted that the  $b$  value gradient with depth implies a similar gradient with stress.

To make a direct examination of the correlation, I calculated the stress difference for each of the  $b$  value determinations of Spada et al. using the simple stress model described above, down to the reversal of trend at the brittle-ductile transition. The normal faulting stress gradient was used for the data for Italy and Greece and the strike-slip profile for California and Turkey. Switzerland has normal faulting focal mechanisms in the south and thrust faulting mechanisms in the extreme north, but otherwise is dominated by strike-slip focal mechanisms [Kastrup et al., 2004], so I used the strike-slip gradient there. Intraplate Japan is dominated by thrust mechanism in NE Honshu and strike-slip mechanisms in SW Honshu and Shikoku [Wesnousky et al., 1982], so for the combined data set for Japan, I used a gradient intermediate between strike slip and thrust of 30 MPa/km. The result is given as a plot of  $b$  value versus stress difference in Figure 1. A very nice

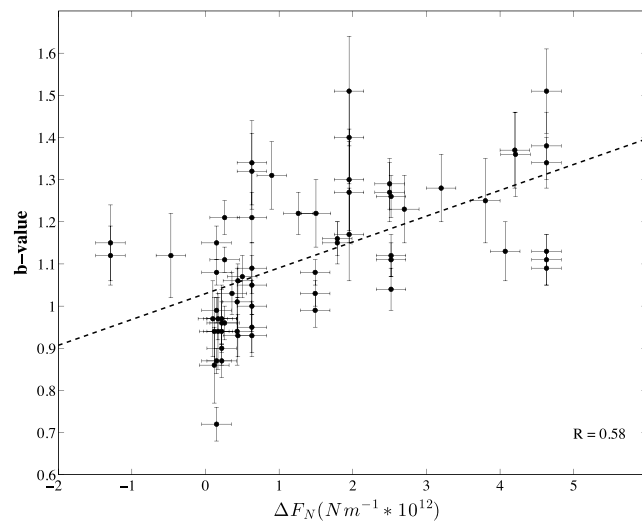
data collapse is seen: the data for all the regions combine to define a very good negative correlation with stress,  $b = 1.23 \pm 0.06 - (0.0012 \pm 0.0003)(\sigma_1 - \sigma_3)$ , where stress is in MPa. This plot simultaneously explains both the depth and focal mechanism dependence of  $b$  value discussed earlier.



**Figure 2.** The  $b$  value for subduction zones plotted versus  $L\sqrt{T}$ . The  $b$  values are from Nishikawa and Ide [2014], and the slab length  $L$  and plate age  $T$  are from Scholz and Campos [1995].

### 3. Stress and $b$ Values for Subduction Zones

Nishikawa and Ide [2014] determined the  $b$  value for a global collection of subduction zones and attempted to find correlations between  $b$  value and a number of plate tectonic parameters. They found a good correlation with age of the subducted plate for plates with ages less than 80 Ma. They pointed out



**Figure 3.** The  $b$  value plotted versus the net reduction of interface normal force  $\Delta F_N$  for subduction zones.

that plate age is related to the negative buoyancy of the subducting plate, one of the main driving forces of subduction. Plate age is related to that force, called the slab pull force, by  $F_{SP} = cL\sqrt{T}$ , where  $T$  is the age of the subducting slab and  $L$  is the length of the slab [Carlson *et al.*, 1983]. We plot the  $b$  values of Nishikawa and Ide versus  $L\sqrt{T}$  in Figure 2, where the values of  $L$  and  $T$  were obtained from Table 1 of Scholz and Campos [1995]. We find a good linear correlation that now extends to all plate ages. Note that increasing  $L\sqrt{T}$  corresponds to decreasing values of the vertical component of the normal stress acting across the plate interface and hence, because the interface is

frictional, to decreasing differential stress in the adjacent plates. Thus, the negative correlation of  $b$  value with stress is the same as found in continental regions.

Variations in the horizontal component of the plate interface normal force is supplied by the sea anchor force,  $F_{SA}$ , which is proportional to the slab length and the velocity of the upper plate relative to the mantle. The reduction of the net plate interface normal force from a reference state is given by  $\Delta F_N = F_{SA} \sin \varphi + F_{SP} \cos \varphi$ , where  $\varphi$  is the interface dip [Scholz and Campos, 1995]. The correlation of  $\Delta F_N$  with  $b$  value is shown in Figure 3, where the  $\Delta F_N$  values were taken from Figure 7 and Table 2 of Scholz and Campos [1995]. Here again, increasing values of  $\Delta F_N$  correspond with decreasing normal force and hence differential stress so the negative correlation with  $b$  value is the same as before.

#### 4. Discussion

This analysis shows that the  $b$  value for earthquakes decreases linearly with stress for both continental and subduction zone environments. The data collapse shown in Figure 1 demonstrates that both the depth dependence and focal mechanism dependences of  $b$  values are the result of the underlying stress dependence. This rules out other interpretations, such as that of rock heterogeneity [Mori and Abercrombie, 1997] in explaining the variation in  $b$  value.

The robustness of this result leads to greater confidence in using  $b$  value to map out stress variations in fault zones [De Gori *et al.*, 2012; Goebel *et al.*, 2012; Tormann *et al.*, 2014; Westerhaus *et al.*, 2002; Wiemer and Wyss, 1997] and interpreting temporal  $b$  value variations during the seismic cycle [Imoto, 1991; Nakaya, 2006; Smith, 1981].

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