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Investigating the effect of mechanical discontinuities on joint spacing

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Abstract

In rocks without systematic mechanical discontinuities (e.g., granite), joint spacing follows an approximately log-normal frequency distribution (i.e., the distribution has a kurtosis near zero). Joint spacing in rocks with systematic mechanical boundaries differs from the spacing in isotropic rocks, exhibiting consistently positive values of kurtosis (i.e., the distribution is more clustered around the mode than a perfectly log-normal distribution). We attribute this difference in joint-spacing distribution to mechanical boundaries such as bed partings in sedimentary rocks that constrain joint height and control joint spacing. Existing systematic joints can also act as mechanical boundaries during the development of later 'cross' joints. Two parallel mechanical boundaries determine the mechanical-layer thickness that influences joint spacing. In this paper, we investigate the effect of sampling geometry and mechanical discontinuities on joint-spacing statistics. In many situations, neither pavement surfaces nor properly oriented boreholes are available for measuring the spacing of cross joints that develop between existing systematic joints. When joint-spacing data come from scanlines that are oblique to a systematic joint set (e.g., crossing many systematic joints on a sub-vertical outcrop face), we consider whether the median spacing of the systematic joint set is statistically equivalent to the mechanical-layer thickness thought to control cross-joint spacing between individual pairs of systematic joints. The basis of our analysis is the fracture spacing index (FSI), which is the slope of a line fitted to a plot of mechanical-layer thicknesses vs. median joint spacing. We collected joint-spacing data along oblique scanlines from a large outcrop of the Devonian Brallier Formation, a distal turbidite sequence, near Huntingdon, PA. Our analysis indicates that the spacing of cross joints correlates better with a mechanical-layer thickness defined by the median systematic ('strike') joint spacing ($r^2 = 0.78$, FSI $= 1.02$) than with a mechanical-layer thickness defined by the stratigraphic bed thickness ($r^2 = 0.69$, FSI $= 0.97$). This is consistent with the conclusions of previous workers. We also note that the spacing data from the cross joints exhibit a higher (i.e., more positive) value of kurtosis than the data from the earlier strike joints (1.66 vs. 0.98). This is consistent with the idea that mechanical discontinuities alter joint spacing in a systematic manner. In this case, the cross joints may have been influenced by two sets of mechanical boundaries (bedding and existing joints), whereas the earlier strike joints were constrained by only one set (bedding).
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1. Introduction

1.1. Mechanical controls on joint propagation

When propagating in an isotropic, homogeneous rock, the rupture front of a vertical joint grows in all directions, moving concentrically outward from an initiation point to form a circular crack tip (Bankwitz and Bankwitz, 1984). In sedimentary rocks, bedding planes commonly act as strong mechanical discontinuities, and joint growth is less likely to be uniform in all directions (Woodworth, 1896; Lacazette and Engelder, 1992). Crack-tip propagation in the direction normal to bedding is commonly arrested at bedding interfaces, leading to stunted bedding-normal growth (Fig. 1).

Another discontinuity that may affect joint growth is a set of existing joints (Dyer, 1988). When the effective normal stress across an earlier formed joint is sufficiently low, which is typically the state under near-surface conditions, lateral joint growth may be arrested at an existing joint face in a manner similar to the arrest of vertical growth at a bedding interface (Gross, 1993). However, if the effective normal stress on an existing joint is sufficiently compressive, a propagating joint will cross through the earlier formed joint as if the rock was intact (Engelder et al., 1999). Joint arrest at bedding interfaces and pre-existing joints occurs largely because both types of boundaries suppress the transmission of the crack-tip stress field necessary for continued crack propagation (Helgeson and Aydin, 1991; Laubach et al., 1998).

The perpendicular distance between two parallel mechanical boundaries defines the mechanical-layer thickness, and these boundaries can be two adjacent bedding planes or two adjacent members of a systematic joint set (Gross, 1993). In sedimentary rocks, stratigraphic bed thickness defines the mechanical-layer thickness, and joint growth normal to bedding is commonly confined to this mechanical layer (Price, 1966; McQuillan, 1973; Ladeira and Price, 1981; Huang and Angelier, 1989; Narr and Suppe, 1991). The bed thickness responsible for constraining joint growth is known as the lithology controlled mechanical-layer thickness (Gross et al., 1995).

1.2. Characteristics of cross joints

Growth of parallel joints within the same bed leads to the development of a systematic joint set, commonly with a spacing approximately equal to

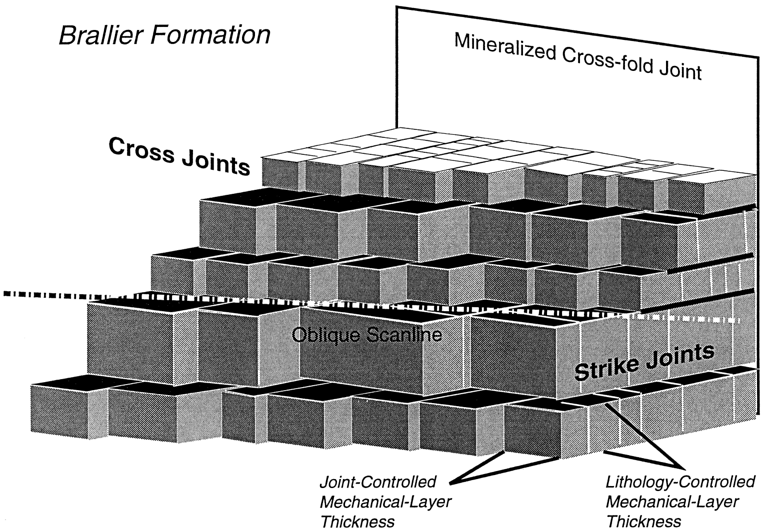

Fig. 1. Schematic drawing of joints in several beds of the Upper Devonian Brallier Formation near Huntingdon, Pennsylvania. The systematic strike joints have a large horizontal dimension relative to their vertical dimension. The cross joints have essentially equal horizontal and vertical dimensions. This sketch depicts the general nature of the outcrop face at Huntingdon, and shows a scanline cutting obliquely to the systematic joint set.

the lithology controlled mechanical-layer thickness (Hobbs, 1967). If adjacent systematic joints serve as mechanical-layer boundaries, the distance between these joints defines a joint-controlled mechanical-layer thickness. 'Cross' joints propagate between and abut adjacent systematic joints (Hodgson, 1961; Hancock, 1985; Gross, 1993), and can have a spacing roughly equivalent to the joint-controlled mechanical-layer thickness (Gross, 1993). Because the spacing of systematic joints is correlated to bed thickness, cross-joint spacing is related to bed thickness as well.

On pavement surfaces, cross joints that are orthogonal to systematic joints form a characteristic ladder-like map pattern, which is one of three typical map-view patterns involving cross joints (Fig. 2). Closely spaced systematic joints are the most likely to bound orthogonal cross joints (Engelder and Gross, 1993). When systematic joints are more widely spaced, cross joints can propagate at an angle significantly different than 90º in the region away from the systematic joints. Closer to the existing joint, the propagating cross joints curve either parallel or normal to the systematic joints, depending on the local stress conditions (Dyer, 1988).

Cross joints are easily recognized on pavement surfaces, and are described in rocks of varying age and lithology: Silurian dolomite at Lannon, Wisconsin (La Pointe and Hudson, 1985), Devonian turbidites of the Appalachian Plateau, New York (Engelder and Gross, 1993; Zhao and Jacobi, 1997), Pennsylvanian coal beds of the Appalachian Plateau, Pennsylvania (Nickelsen and Hough, 1967), Jurassic sandstone at Arches National Park, Utah (Dyer, 1988), Jurassic carbonates along the English coastline (Rawnsley et al., 1992), Cretaceous chalk in Texas (Wiltschko et al., 1991), and Miocene carbonate near Santa Barbara, California (Gross, 1993).

The abutting of cross joints against systematic joints is clear evidence that cross joints postdate the systematic joints (Hodgson, 1961). Cross joints also seem to form under near-surface conditions, as indicated by their appearance in outcrop but not in the subsurface [e.g., in the Austin Chalk (Laubach et al., 1995)].

Near the surface, cross joints may play an important role in enhancing joint interconnectivity and concomitant fluid flow in rocks. Important applications under near-surface conditions include understanding groundwater migration and the draining of shallow coal-bed methane reservoirs penetrated by horizontal drilling. Modeling fluid flow through interconnected joints requires data on parameters such as joint spacing. Because the spacing between adjacent systematic joints appears to control the spacing of cross joints,

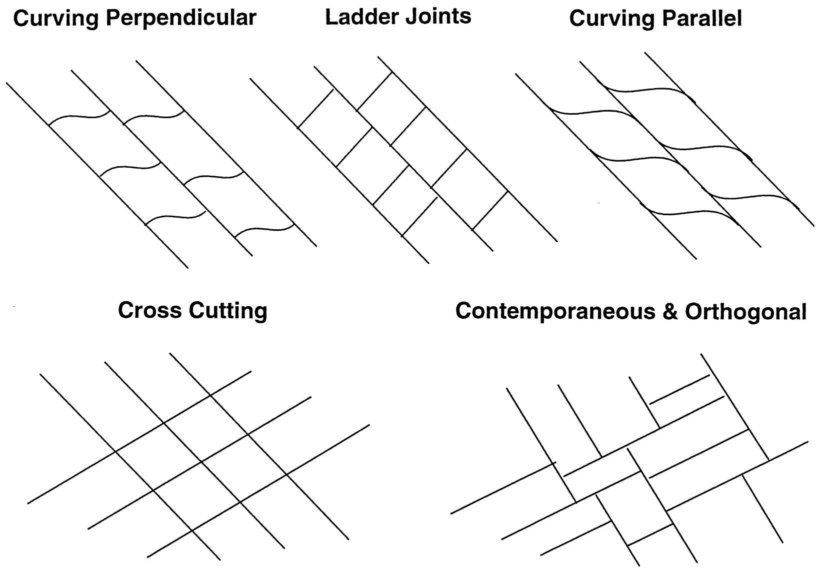

Fig. 2. Typical joint patterns on pavement surfaces. Cross joints propagate between pairs of systematic joints. Two systematic joint sets may be either cross-cutting or contemporaneous and orthogonal.

the most relevant data on cross-joint spacing come from individual cross joint sets found between pairs of adjacent systematic joints. So far, pavement outcrops are the primary source for such information on cross-joint spacing (e.g., Gross, 1993).

A more common exposure in the vegetated regions of the world is an approximately cross-sectional outcrop face. On such surfaces, the exposure of a single, long systematic joint surface would allow large sample populations of cross-joint spacing data to be collected; unfortunately, this geometry is rare. Typically, outcrop faces are at an oblique angle to both systematic and cross joints (Fig. 1). In such cases, just a few cross-joint spacing data are available between any individual pair of adjacent systematic joints. One objective of this study is to examine the statistical characteristics of cross-joint spacing data acquired from outcrop faces cutting obliquely to a systematic joint set.

1.3. Joint-spacing statistics

The heart of our analysis is a statistical test to examine the conclusion that cross-joint spacing is critically controlled by the spacing of systematic joints (Gross, 1993). This is in contrast to the idea that cross-joint spacing depends on the thickness of the bed in which they occur. The relationship between joint height and spacing can be quantified as a fracture spacing index (FSI), which is the slope of the regression line fitted to a plot of mechanical-layer thicknesses vs. median joint spacing derived from scanline data (Narr and Suppe, 1991; Gross, 1993; Engelder et al., 1997). FSI is a measure of joint density. Although plotted on the abscissa, median spacing is the dependent variable (Narr and Suppe, 1991). Typical values for FSI range from about 0.8 to about 1.5 (e.g., Narr and Suppe, 1991; Gross, 1993; Engelder et al., 1997).

The fracture spacing ratio (FSR) is defined as the mechanical-layer thickness of a bed divided by the median spacing of a joint set along a scanline within that bed (Gross, 1993; Becker and Gross, 1996). In other words, an FSR represents one data point on an FSI plot. Joint spacing ratio (JSR) is our term for an individual cross-joint spacing measurement along a scanline normalized by the median spacing of the systematic joints crossed by that scanline.

The frequency distribution of joint spacing in mechanically isotropic rocks such as the granite at Stripa, Sweden (Rouleau and Gale, 1985) and the thick (on the order of 50 m) Devonian shales of the Appalachian Plateau (Engelder et al., 1999) is approximately log-normal, with values of kurtosis near zero (see Rives et al., 1992, for a discussion of reported joint-spacing distributions). Kurtosis (k) is a measure of the peakedness or flatness of a frequency distribution relative to a normal (Gaussian) distribution, which, by definition, has zero kurtosis. A frequency distribution with a positive kurtosis is more clustered ('peaked') about the statistical mode than a perfectly normal distribution, and a negative value of kurtosis indicates a 'flatter' distribution.

In the Stripa Granite data of Rouleau and Gale (1985), joint set #2 is a nearly vertical joint set with a mean spacing of less than 0.5 m. Joint-spacing data from the 'R' borehole suite have a kurtosis of -0.24 (Fig. 3). Sections of the Devonian shale on the Appalachian Plateau are so thinly bedded that they are isotropic to the growth of joints (i.e., their gross mechanical character is similar to granite). Like the Stripa Granite, joint-spacing data from the shales of the Moscow, Geneseo, and Middlesex formations (Engelder et al., 1999) have values of kurtosis relatively close to zero (Fig. 3): $k = -0.11$ (Moscow), $k = 0.06$ and 0.22 (two outcrops of the Geneseo), and $k = 0.15$ (Middlesex).

The distribution of joint-spacing data from well-bedded sedimentary rocks produces uniformly more positive values of kurtosis than the spacing data from mechanically isotropic rocks (Fig. 3); the frequency distributions from these rocks are more clustered (peaked) about the mode spacing value than expected for a log-normal distribution. This apparent difference between joint spacing in mechanically isotropic and anisotropic rocks is discussed below.

2. Methodology

The original joint-spacing data for this paper are from a large outcrop of the Upper Devonian Brallier Formation, a distal turbidite sequence, near Huntingdon, Pennsylvania (Frakes, 1967). The scanlines were set parallel to individual beds, on outcrop faces that cut obliquely to a systematic ('strike') joint

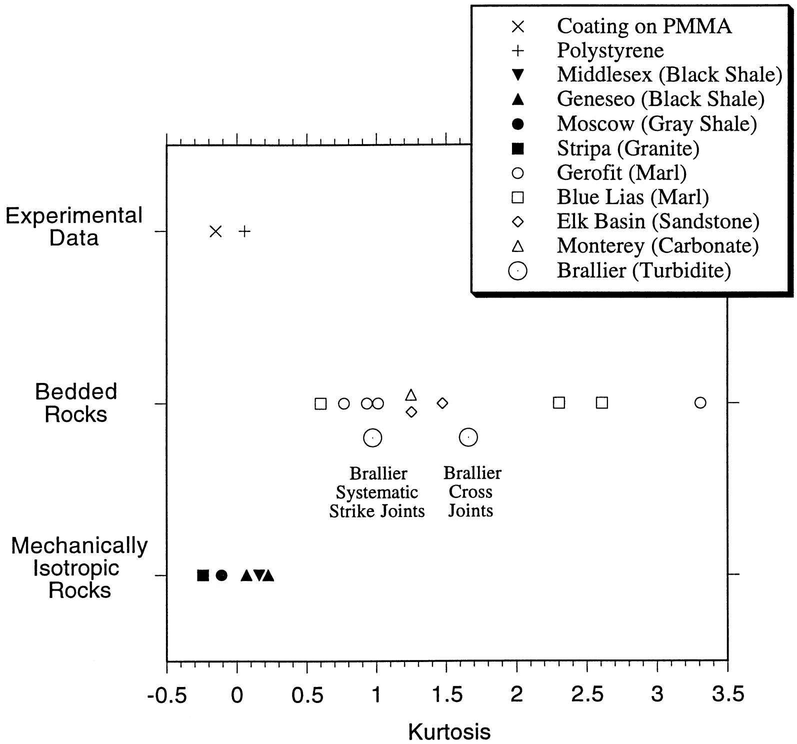

Fig. 3. Values of kurtosis calculated for joint-spacing data from several sources. Mechanically isotropic rocks have values near zero, while joints within bedded rocks have spacing distributions with values of kurtosis greater than 0.5. The data include the Stripa Granite (Rouleau and Gale, 1985), three formations of Devonian shale from the Appalachian Plateau (Engelder et al., 1999), the Cretaceous Gerofit Formation (Becker and Gross, 1996), the Jurassic Blue Lias Formation (Engelder et al., 1998), Cretaceous clastic rocks of Elk Basin (Engelder et al., 1997), the Miocene Monterey Formation (Gross, 1993), and the Devonian Brallier Formation (this study). Experimental data on joint spacing include a coating on PMMA (Wu and Pollard, 1995) and polystyrene (Rives et al., 1992).

set (Fig. 1). Along each scanline, the orientation of all joints and the spacing between adjacent joints were measured, and the abutting relationship between non-parallel joints was noted.

For the cross joints, we constructed two fracture spacing index (FSI) plots: one using a mechanical-layer thickness defined by the stratigraphic thickness of the bed containing the joints, and a second assuming the mechanical-layer thickness was equivalent to the median spacing of the systematic joint set crossed by the scanline. Our premise is based on the conclusion of Gross (1993): if the mechanical-layer thickness defined by the existing systematic joint set is more significant than bed thickness in controlling cross-joint spacing, the regression line fit to an FSI plot constructed with the former would have a higher coefficient of determination (r^2).

3. Characteristics of joints in the Brallier Formation

The outcrop of Brallier Formation we studied is on the northwest limb of the Broadtop Syncline, within the Valley and Ridge Province of central Pennsylvania (Fig. 4). The Brallier Formation correlates with the Ithaca Formation of the Genesee Group in the Finger Lakes District of the Appalachian Plateau of New York (Van Tyne, 1983). The Geneseo Formation (Fig. 3) constitutes the lower part of the Genesee Group. Throughout the Appalachian Plateau, the earliest systematic joints in siltstone beds strike in the cross-fold (i.e., dip) direction (Engelder and Geiser, 1980; Engelder et al., 1999). This is also true of systematic joints in the Brallier Formation near Port Matilda, Pennsylvania (Kovach, pers. commun.).

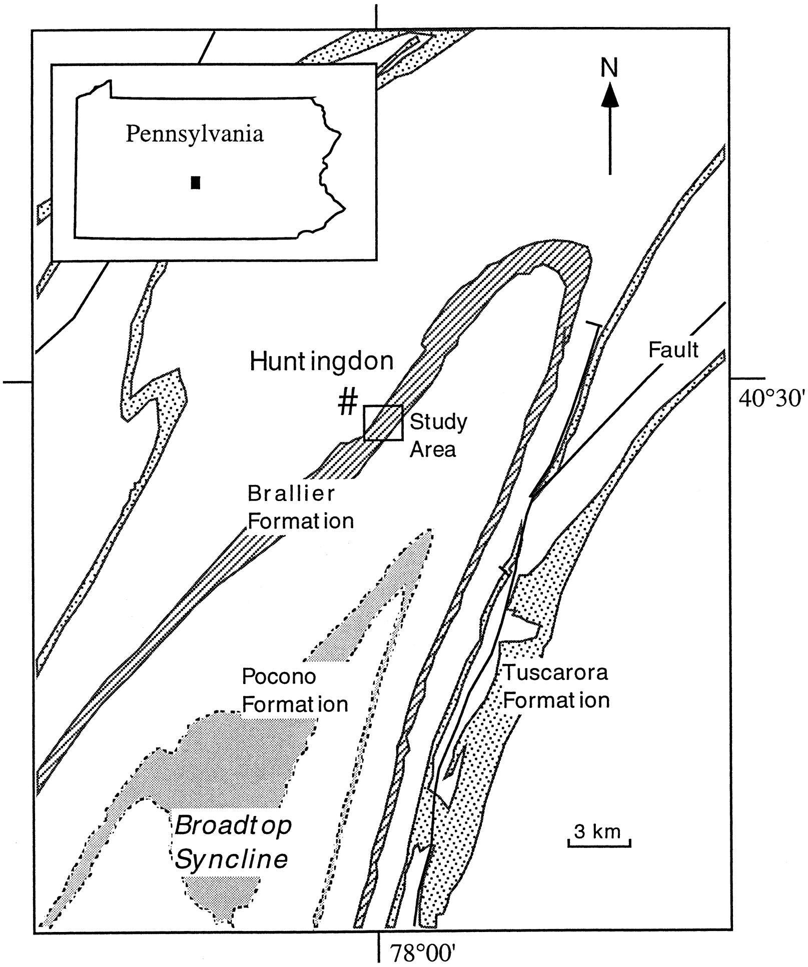

Fig. 4. A simplified geologic map of the study area showing the northeastern end of the Broadtop Syncline. The outcrop pattern of three sandstones is shown on this map: the Tuscarora Formation is Silurian, the Brallier Formation is Devonian, and the Pocono Formation is Mississippian.

The Brallier outcrop near Huntingdon contains three systematic joint sets: early cross-fold ('dip') joints coated or filled with euhedral crystals of quartz, 'strike' joints that are unmineralized or coated with a delicate pattern of microscopic crystals of unknown composition, and later-formed, unmineralized joints in the cross-fold orientation (Fig. 1). While the early mineralized cross-fold joints cut through many beds, the second and third joint sets are confined to siltstone beds (Fig. 5). One difference between the two later joint sets is that a distinct plumose pattern characteristic of Appalachian Plateau siltstones (e.g., Bahat and Engelder, 1984; Lacazette and Engelder, 1992) decorates the surfaces of the strike joints, whereas the surfaces of the unmineralized joints in the cross-fold orientation are relatively smooth and without character. The earliest mineral-coated joints are so widely spaced (typically greater than 5 m) that our scanlines rarely crossed a significant number of them.

Relative ages were determined from the abutting relationship between joints. For well-exposed cases along our scanlines, we found that strike joints abut unmineralized joints in the cross-fold orientation 29 times (13% of the cases), while the unmineralized joints in the cross-fold orientation abut strike joints 185 times (85%). In four cases (2%), the two joints were mutually cross cutting. Given that 85% of all

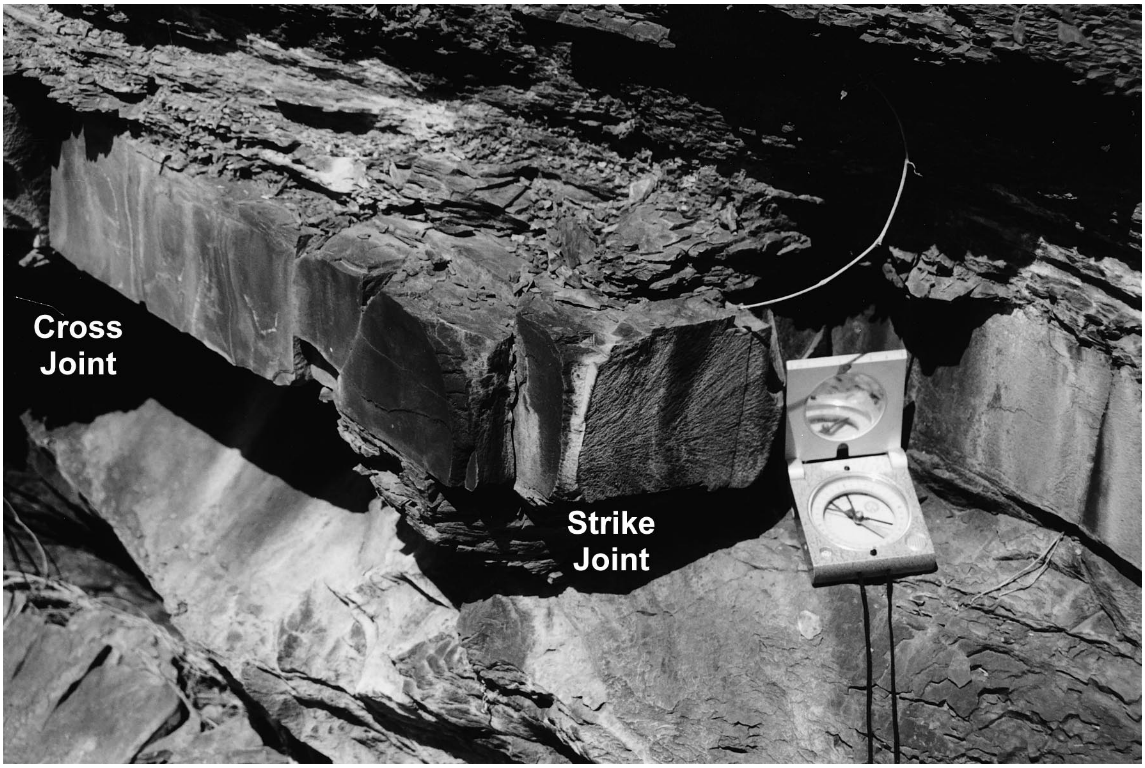

Fig. 5. A photo of joints in one bed of the Devonian Braillier Formation at Huntingdon, Pennsylvania. A plumose pattern typically decorates the surface of the strike joints whereas the surface of the cross joints is smooth.

clear intersections involved joints in the cross-fold orientation abutting strike joints, we considered all unmineralized joints confined to one bed and in the cross-fold orientation to be 'cross' joints. By the same reasoning, we included all joints confined to one bed and in the strike orientation within the population of strike joints. The spacing of these systematic strike joints defines the joint-controlled mechanical-layer thickness for cross joints within a given bed (Fig. 1).

One problem with mapping outcrop faces that are oblique to a systematic joint set is the difficulty of distinguishing cross joints from a second systematic joint set (Fig. 2). The evidence for abutting or cross-cutting joint sets is often missing (e.g., the rock removed to form the outcrop) or obscured by erosion or soil development. Later formed systematic joints can cut across earlier systematic joints, as in the case of the Devonian black shales of the Appalachian Plateau (Engelder et al., 1999). Such a cross-cutting pattern of systematic joints occurs when propagation of the later set takes place at a depth where there is a significant traction across the surfaces of the earlier systematic joints. In such a case, the cross-cutting map pattern can give the erroneous impression that the two joint sets formed contemporaneously.

The interpretation of the timing of jointing in the field is further complicated by the mutually abutting geometry that develops when two systematic, orthogonal joint sets do propagate contemporaneously (Fig. 2). While examples of this are reported from the Spanish Pyrenees (Turner and Hancock, 1990) and from southwestern Wales (Dunne and North, 1990), the mechanics of the process are not well understood. Elastic effects associated with opening and closing of joints may contribute to the formation of contemporaneous orthogonal joints, but these effects seem insufficient to fully account for contemporaneous orthogonal jointing (Martel, 1994). We feel the different surface morphologies on the strike and cross joints in the Brallier Formation is further evidence that they did not form contemporaneously.

Bedding within the Huntingdon outcrop has a fairly uniform dip of 14° to the southeast, with a strike that ranges from about 220° to about 230°. The mean vector of the poles to strike joints indicates that the average attitude of the strike joints is 231°, 81° NW (Fig. 6a). The strike joints are approximately parallel to the strike of bedding, and hence parallel to northern end of the Broadtop Synclinorium fold axis (Fig. 4). The average dip of the strike joints is about 5° (81° + 14° = 95°) from being exactly perpendicular to bedding; this lack of orthogonality between the bed partings and strike joints is visually apparent in the outcrop. The cross joints have an average attitude of 145°, 87° SW, and, unlike the strike joints, are essentially orthogonal to bedding (Fig. 6b).

4. Joint-spacing statistics in the Brallier Formation

The Huntingdon outcrop allowed for scanlines longer than 10 m in 42 beds. Of these beds, 32 had nine or more spacing measurements from strike joints, and the FSI plots for strike joints were constructed from these data. Similarly, 19 beds had nine or more spacing measurements from cross joints, and these data were used to make the FSI plots for the cross joints. To the eye, the two joint sets have about the same spacing and, hence, seem equally abundant in outcrop. The difference in the size of the two sample populations is due to a systematic change in the orientation of the outcrop face, which curves to become subparallel to the cross joints.

We began our analysis by normalizing each spacing measurement. This was done by dividing each measured joint spacing by the median spacing of the corresponding joint set along each scanline. The

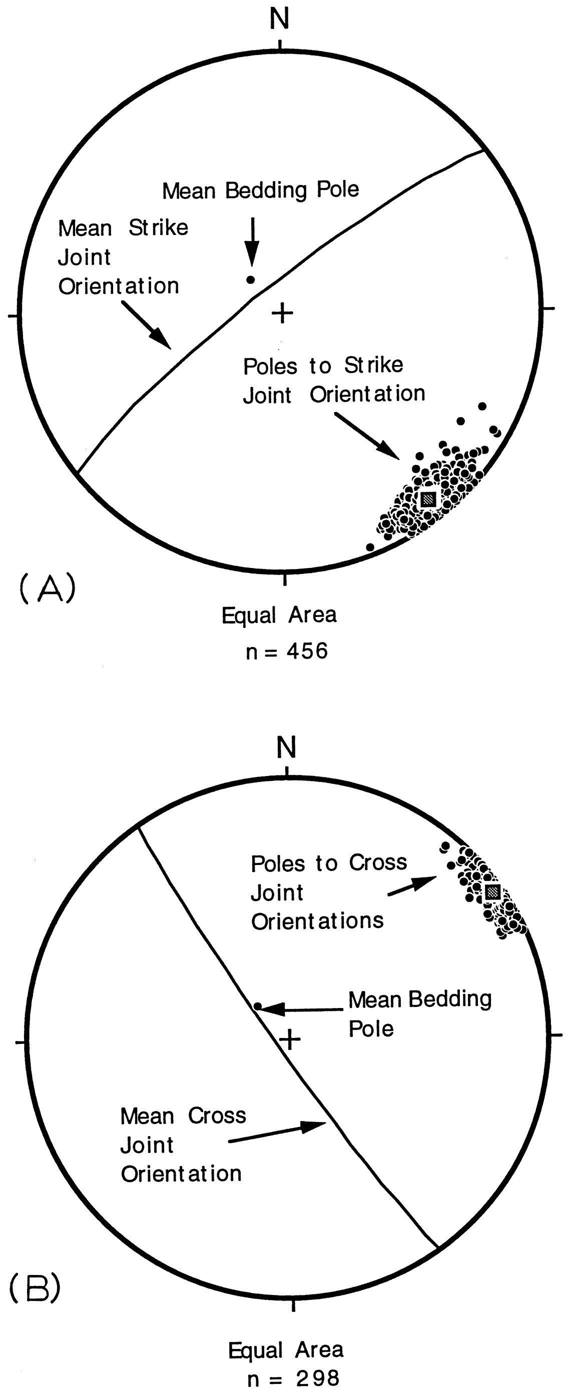

Fig. 6. (A) Stereonet plot (equal-area, lower-hemisphere projection) of strike-joint data and the mean pole of the bedding measurements. The mean strike-joint orientation is 231°, 81° NW and the mean orientation of bedding is 227°, 14° SE. Notice that the mean bedding pole does not lie along the mean strike-joint orientation, indicating that the strike joints are not perpendicular to bedding. (B) Stereonet plot of cross-joint data and the mean pole of bedding. The mean orientation of cross joints is 145°, 87° SW and the mean orientation of bedding is the same as above (227°, 14° SE).

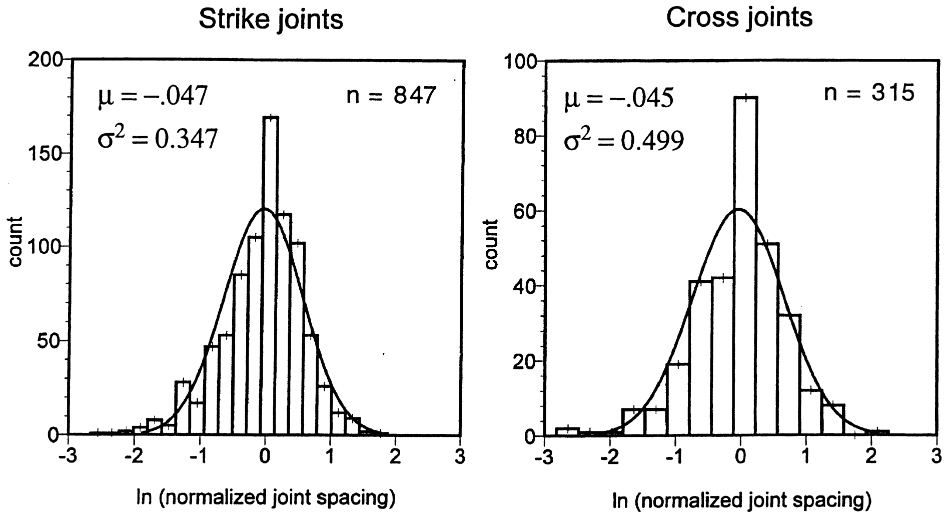

Fig. 7. (a) Histogram of spacing data for strike joints. (b) Histogram of spacing data for cross joints. In both plots, the data have been normalized by the median spacing of that joint set along each scanline. The smooth curves represent normal distributions with the same mean and variance as the raw data.

normalized data for strike joints therefore represent spacings from a single mechanical-layer thickness (i.e., a single bed). The normalized spacings of the cross joints, however, constitute a data set that crossed several systematic strike joints along a given scanline, and therefore includes several (joint-controlled) mechanical-layer thicknesses.

The two data sets differ from a log-normal distribution in a similar manner, with negative skewness and positive kurtosis (Fig. 7). The deviation is large enough that both data sets fail a standard test (Kolmogorov–Smirnov) of comparison with a log-normal distribution at a significance level of 0.05. We have assumed, however, that the distributions of the two data sets are similar enough that they can be compared statistically. Specifically, the distribution of the cross-joint data has a larger kurtosis ($k = 1.66$) than the strike-joint data ($k = 0.98$). Our interpretation of the significance of this is discussed below.

The FSI plot for strike joints was constructed assuming that stratigraphic bed thickness is the appropriate mechanical-layer thickness (Narr and Suppe, 1991). The FSI for the strike joints is 0.91, with a coefficient of determination (r^2) of 0.86 (Fig. 8a). Two FSI plots were constructed for the cross joints: one assuming that stratigraphic bed thickness is the pertinent mechanical-layer thickness (FSI $=$ 0.97, $r^2 = 0.69$; Fig. 8b), and a second assuming that the median spacing between systematic strike joints is the critical thickness (FSI $=$ 1.02, $r^2 = 0.78$; Fig. 8c).

Our use of the median strike-joint spacing as the mechanical-layer thickness in determining the second cross-joint FSI merits explanation. As opposed to a pavement-type exposure, the Huntingdon outcrop did not allow us to gather adequate cross-joint data along a scanline between (and parallel to) adjacent systematic joints (i.e., within a single joint-controlled mechanical-layer thickness). To overcome this difficulty, we used the median spacing of the systematic strike joints within a bed of interest as the mechanical-layer thickness for this FSI plot (Fig. 1). This FSI was determined using data from the 16 beds in which we recorded at least nine spacing measurements of both strike joints and cross joints.

While the FSI data indicate that the cross joints are more closely spaced than strike joints, the question remains whether or not this difference is statistically robust. An F-test indicates that the variance of the fracture spacing ratio (FSR) data from the strike joints and the variance of the FSR data from the cross joints (assuming a joint-controlled mechanical-layer thickness) are statistically different at the 95% confidence level. We also examined our two ‘sets’ of cross-joint data: the joint spacing ratio (JSR) data from cross joints normalized using a joint-controlled mechanical-layer thickness and the JSR data from cross joints normalized using stratigraphic bed thickness as the mechanical-layer thickness. An *F*-test indicates that the variances of these two cases are statistically different at the 85% confidence level.

Student's *t*-test was then performed on the FSR data, with the assumption that the variances for the strike joints and cross joints were different. The null hypothesis was rejected at the 95% confidence level, indicating that the mean FSR values for the strike joints and cross joints are statistically different. Student's *t*-test was also performed on the JSR data described above (different normalizing thicknesses). The null hypothesis was again rejected, indicating that the mean JSR values for these two situations is statistically different at the 85% level of confidence.

5. Discussion

In the Brallier Formation at Huntingdon, the earliest cross-fold joints, which are now mineralized, are widely spaced (typically greater than 5 m), so that the horizontal growth of later systematic strike joints was relatively unhindered by mechanical boundaries. With bed thicknesses generally less than 0.5 m, most strike joints could develop a horizontal dimension at least an order of magnitude larger than the vertical dimension before extending completely between early a pair of early cross-fold joints. If the cross-fold joints were mineralized (i.e., closed) before the strike joints propagated, horizontal growth could have been even larger relative to vertical growth.

In contrast, the horizontal dimension of the cross joints was stopped by the existing set of strike joints, resulting in a typical ratio of horizontal to vertical dimension near unity. As described above, our observation that the cross joints are more closely

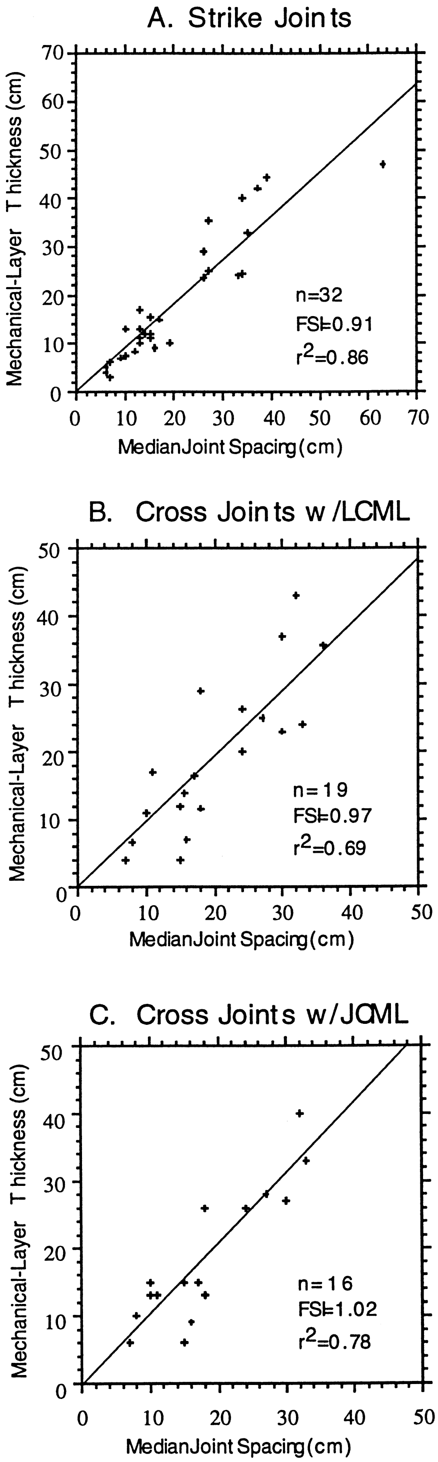

Fig. 8. (A) An FSI (fracture spacing index) plot for strike joints, with a mechanical-layer thickness equal to bed thickness. Here three data points fall on top of others. (B) An FSI plot for cross joints, with a mechanical-layer thickness equal to bed thickness. (C) An FSI plot for cross joints, with a mechanical-layer thickness equal to the median spacing of the systematic (strike) joints crossed by a given scanline.

spaced (FSI = 1.02) than the strike joints (FSI = 0.91) appears to be statistically robust, indicating that the two joint sets formed under different conditions. More work is necessary, however, to determine the specific mechanical parameters that control joint spacing.

Our data provided an opportunity to consider the relative influences of the lithology-controlled mechanical-layer thickness (i.e., bed thickness) and the joint-controlled mechanical-layer thickness (i.e., strike joint spacing) on cross-joint development. The regression lines fit to the cross-joint FSI plots (Fig. 8b and c) constructed using these different mechanical-layer thicknesses indicate that the joint-controlled mechanical-layer thickness produces a higher coefficient of determination ($r^2 = 0.78$) than the bed thickness ($r^2 = 0.69$). This result is consistent with the conclusion of Gross (1993): the spacing of cross joints is controlled primarily by the joint-controlled mechanical-layer thickness that constrains them.

We were also faced with the issue of determining the appropriate joint-controlled mechanical-layer thickness for use in our calculations. Because our outcrop exposure is sub-vertical and oblique to the systematic strike joint set, we encountered relatively few cross joints (typically less than 5) between any given set of strike joints. The preferred scanline orientation for measuring cross-joint spacing is parallel to the systematic joints that confine the cross joints. The appropriate mechanical-layer thickness with such a scanline is simply the absolute spacing of the adjacent systematic joints.

Instead, for all cross joints within a given bed, we used the median spacing of the strike joints within that bed as the joint-controlled mechanical-layer (JCML) thickness in our determination of the JCML-based FSI for cross joints. We feel that the use of this ‘proxy’ thickness is the reason for the low coefficient of determination ($r^2 = 0.78$) for the cross-joint FSI relative to the strike-joint FSI ($r^2 = 0.86$) in the Brallier Formation.

The larger value of kurtosis for the cross-joint data (1.66 vs. 0.98 for the strike-joint data) may also be due to the mechanical influence of a joint-controlled mechanical-layer thickness. This interpretation is based on the assumption that the (unknown) mechanical process fundamentally responsible for joint spacing in isotropic rocks results in a log-normal spacing distribution. This assumption is partly based on the values of kurtosis we have calculated for the joint spacings in mechanically isotropic rocks such as granite and thick sequences of thinly bedded shales (Fig. 3). These values are near zero, indicating that the spacing distribution in these rocks closely resembles a log-normal distribution. In addition, the Kolmogorov–Smirnov test fails to reject the hypothesis of a log-normal distribution for these data at a significance level of 0.05.

We have also calculated the kurtosis of joint spacings from several bedded units (Fig. 3). Without exception, these values are larger (i.e., more positive) than the values from the granite or thin-bedded shale. The Anscombe–Glynn kurtosis test rejects the hypothesis of log-normality for all the data from well-bedded rocks at a significance level of 0.05. The kurtosis test, however, fails to reject the hypothesis of log-normality for the data from the isotropic rocks at the same level of significance (0.05).

Following Gross and Engelder (1995), a hypothesis we are currently working to test is that systematic mechanical discontinuities (e.g., bed partings) alter the ‘fundamental’ log-normal joint spacing distribution in a predictable manner. Several workers have suggested that joint spacing in bedded rocks may be controlled by a ‘stress-shadow’ effect, with the expected result that the spacing will be regular, and approximately equal to bed thickness (Pollard and Segall, 1987; Narr and Suppe, 1991; Gross et al., 1995). Statistically, this should cluster the spacing distribution more tightly around the central (periodic) value, and this ‘peaked’ distribution would have a positive kurtosis.

In our Brallier data, the strike joints propagated under the influence of one systematic set of mechanical discontinuities (bed partings). The spacing of the strike joints correlates well with bed thickness (FSI = 0.91, $r^2 = 0.86$). The cross joints, however, experienced two sets of mechanical discontinuities (bedding and the existing strike joints). In addition, these two sets of discontinuities represent similar mechanical-layer thicknesses (within about 10%), and may have controlled the cross-joint spacing to a considerable degree. The larger value of kurtosis for the cross-joint data may be evidence of this.

Other workers have used mechanical models to simulate the development of systematic joint sets in thin brittle plates (e.g., Rives et al., 1992; Wu and Pollard, 1995). The spacing distribution of cracks in polystyrene sheets and in a brittle coating on PMMA resembles those found in mechanically isotropic rocks, with values of kurtosis near zero (Fig. 3). The data from the brittle coating on PMMA (Wu and Pollard, 1995) are interesting because the cracks are widely spaced, with an FSR on the order of 0.1. With such a wide spacing, it is likely that the 'stress shadows' adjacent to existing cracks had little effect on the location of later cracks (Fischer, 1994). While these cracks developed in a physical situation apparently analogous to sedimentary layering, they probably grew independently of any effect from neighboring cracks, and are therefore distributed in an essentially log-normal manner.

6. Conclusions

Our analysis of cross joints within the Brallier Formation supports the conclusion that cross-joint spacing correlates better with the spacing of the systematic joints they grow between, rather than the thickness of the bed in which they propagate. When cross-joint data are gathered along scanlines that are not parallel to the systematic joints between which they propagate, the median spacing of the systematic joint set is a valid approximation of the mechanical-layer thickness confining each individual cross joint. Finally, joint spacing in well-bedded sedimentary rocks is more tightly clustered about the mode than a perfectly log-normal distribution and is, therefore, statistically different than joint spacing in mechanically isotropic rocks.

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